Control #2 200

1

a) $F(u$ $\int$ os d

$\frac{\partial F}{\partial u}$ $\cosh$ $\frac{\partial F}{\partial v}$ $\cosh$ v

$\frac{\partial F}{\partial w}$ $\operatorname{senh} wy \; y \, dy$

Sea G $\int^{sen} \cosh \; dy$

$G(x$ $\int_{e^x}^{\operatorname{sen} x} \operatorname{senh} \; \ln x) \, dy$ $\cos x \cosh(\operatorname{sen}$ e $\cosh(e^{2x}$

$\cos$ e π e

$\nabla z($ y e $\cos$ sen $\cos$

∇z 2π e

Sea ∇z $\Rightarrow$ Max crec d $\frac{d}{d}$

Como z es diferenciable

∇ d

b)

ii) El vector normal a la superficie es $\vec{N} = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right)$

$$\Rightarrow \vec{N} = \left(2\pi\, e^{-\pi^2-1}, 2\, e^{-\pi^2-1}, -1 \right)$$

Luego el plano tangente en $(\pi, 1)$ tiene ecuación

$$\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} \pi \\ 1 \\ -e^{-\pi^2-1} \end{pmatrix} \right] \cdot \begin{pmatrix} 2\pi\, e^{-\pi^2-1} \\ 2\, e^{-\pi^2-1} \\ -1 \end{pmatrix} = 0$$

iii) $z(x,y) \approx z(\pi,1) + \nabla z(\pi,1) \begin{pmatrix} x-\pi \\ y-1 \end{pmatrix} + \frac{1}{2} (x-\pi, y-1)\, H_z(\pi,1) \begin{pmatrix} x-\pi \\ y-1 \end{pmatrix}$

donde

$$H_z(x,y) = e^{-x^2-y^2} \begin{pmatrix} -3\cos x + 4x\,\mathrm{sen}\,x + 4x^2\cos x & 4xy\cos x + 2y\,\mathrm{sen}\,x \\ 4xy\cos x + 2y\,\mathrm{sen}\,x & -2\cos x + 4y^2\cos x \end{pmatrix}$$

$$\Rightarrow H_z(\pi,1) = e^{-\pi^2-1} \begin{pmatrix} 3-4\pi^2 & -4\pi \\ -4\pi & -2 \end{pmatrix}$$

Control #2 2003-3

3-

a) $u = \frac{y^2-x^2}{2}$ $\quad v = \frac{y^2+x^2}{2}$

PDQ $\quad y^2\frac{\partial^2 F}{\partial x^2} - x^2\frac{\partial^2 F}{\partial y^2} = 0 \iff 2(u^2-v^2)\frac{\partial^2 F}{\partial u\partial v} = v\frac{\partial F}{\partial u} - u\frac{\partial F}{\partial v}$.

$\frac{\partial F}{\partial x} = \frac{\partial F}{\partial u}\cdot -x + \frac{\partial F}{\partial v}x \qquad \frac{\partial F}{\partial y} = y\frac{\partial F}{\partial u} + y\frac{\partial F}{\partial v}$

$\frac{\partial^2 F}{\partial x^2} = x^2\left(\frac{\partial^2 F}{\partial u^2} - 2\frac{\partial^2 F}{\partial u\partial v} + \frac{\partial^2 F}{\partial v^2}\right) + \frac{\partial F}{\partial v} - \frac{\partial F}{\partial u}$

$\frac{\partial^2 F}{\partial y^2} = y^2\left(\frac{\partial^2 F}{\partial u^2} + 2\frac{\partial^2 F}{\partial u\partial v} + \frac{\partial^2 F}{\partial v^2}\right) + \frac{\partial F}{\partial u} + \frac{\partial F}{\partial v}$

Luego

$y^2\frac{\partial^2 F}{\partial x^2} - x^2\frac{\partial^2 F}{\partial y^2} = -4x^2y^2\frac{\partial^2 F}{\partial u\partial v} + (y^2-x^2)\frac{\partial F}{\partial v} - (y^2+x^2)\frac{\partial F}{\partial u}$

Pero

$x^2y^2 = v^2-u^2$

$\Rightarrow -2(v^2-u^2)\frac{\partial^2 F}{\partial u\partial v} + u\frac{\partial F}{\partial v} - v\frac{\partial F}{\partial u} = 0$

$\Rightarrow 2(v^2-u^2)\frac{\partial^2 F}{\partial u\partial v} = u\frac{\partial F}{\partial v} - v\frac{\partial F}{\partial u} \quad / \cdot -1$

$2(u^2-v^2)\frac{\partial^2 F}{\partial u\partial v} = v\frac{\partial F}{\partial u} - u\frac{\partial F}{\partial v}$ ▣

F

Δf F y/x $\left[\quad \frac{y^2}{} \quad \frac{y}{x} \right.$ F $y/$

$\Delta f \quad 0$

$\Rightarrow \quad \left[F \quad \frac{1}{2} \quad 2 \right] c$

Se

$\Rightarrow F'' \quad F' \quad 0 \quad \infty \quad 0$

Re lo F

F A arctg B

lo 0 0

$\Rightarrow F \quad \frac{1}{\pi} a \quad g$

Lu

$f \quad y \quad \frac{}{\pi} a \quad tg \quad y/$ (*)

lo $\frac{\partial f}{2} \quad \partial f$ pa * p

Pauta Control #2

Pregunta 2:

a) i) Se tiene que:

$$g(x(t), y(t)) = \alpha \qquad \Big/ \frac{d}{dt}$$

$$\frac{\partial g}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial t} = 0 \quad \text{(Por regla de la cadena)}$$

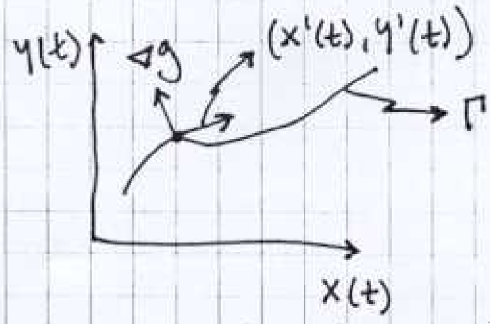
Además $\frac{\partial x}{\partial t} = x'(t)$ y $\frac{\partial y}{\partial t} = y'(t)$

$$\Rightarrow \frac{\partial g}{\partial x}(x(t), y(t))\, x'(t) + \frac{\partial g}{\partial y}(x(t), y(t))\, y'(t) = 0$$

$$\Rightarrow \langle \nabla g(x(t), y(t)), (x'(t), y'(t)) \rangle = 0$$

$$\Rightarrow \nabla g(x(t), y(t)) \perp (x'(t), y'(t)) \qquad (1.0)$$

ii)



Si $g: \mathbb{R}^2 \to \mathbb{R}$ y $(x(t), y(t))$ es una parametrización de Γ.
(Γ es la curva de nivel de $g(x,y)$ en α)

(1.0) $\Rightarrow$ ∇g siempre es perpendicular a las curvas de nivel.

iii) Sin pérdida de generalidad supongamos $a > 0$ y $b \geq 0$ (los otros casos son análogos)

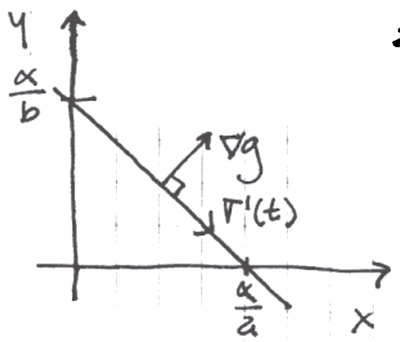
$$\Rightarrow \text{si } g(x,y) = ax + by \Rightarrow \nabla g(x,y) = \begin{pmatrix} a \\ b \end{pmatrix}$$

Sea $\alpha \in \mathbb{R} \Rightarrow ax + by = \alpha$ es una curva de nivel en α para g.

$$y = \frac{\alpha}{b} - \frac{a}{b}x$$

Γ, por su parte, puede ser parametrizada por:

$$\vec{r}(t) = \left(t, \frac{\alpha}{b} - \frac{a}{b}t\right) \quad t \in \mathbb{R}$$



$$ax + by = \alpha$$

$$y = \frac{\alpha}{b} - \frac{a}{b}x$$

$$\nabla g = \begin{pmatrix} a \\ b \end{pmatrix}$$

$(\ ,0)$

b)

i) Encontramos el o los puntos de intersección:

$$4t + 2t^2 + t^3 - 24 = 0$$

Claramente, $t=2$ es raiz del polinomio anterior

$\Rightarrow$ dividiendo:

$$\begin{array}{l} t^3 + 2t^2 + 4t - 24 : t - 2 = t^2 + 4t + 12 \\ t^3 - 2t^2 \\ \hline \quad 4t^2 + 4t \\ \quad 4t^2 - 8t \\ \hline \qquad 12t - 24 \\ \qquad 12t - 24 \\ \hline \qquad\qquad 0 \end{array}$$

$$\Rightarrow t^3 + 2t^2 + 4t - 24 = (t-2)(t^2 + 4t + 12)$$

Notemos que en $t^2 + 4t + 12 = 0$

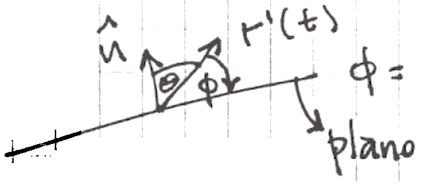
$$t = \frac{-4 \pm \sqrt{16 - 48}}{2} \in \mathbb{C}$$

$\Rightarrow t=2$ único punto de intersección

$$\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix}$$

$$r'(t) = (1, 2t, 3t^2)$$



$$\phi = \frac{\pi}{2} - \theta$$

$$\cos\theta = \frac{\vec{n}\cdot\vec{r}'}{\|\vec{n}\|\|\vec{r}'\|}$$

$$\vec{n} = \begin{pmatrix}4\\2\\1\end{pmatrix} \quad \|\vec{n}\| = \sqrt{16+4+1} = \sqrt{21}$$

$$r'(2) = (1,4,12)$$

$$\|r'(2)\| = \sqrt{1+16+144} = \sqrt{161}$$

$$\vec{n}\cdot\vec{r}' = 4+8+12 = 24$$

$$\cos\theta = \frac{24}{\sqrt{21}\sqrt{161}} \Rightarrow \phi = \frac{\pi}{2} - \arccos\left(\frac{24}{\sqrt{21}\sqrt{161}}\right)$$

(1.0)

ii)

$$\vec{r}(t) = 2\cos 2t\,\hat{\imath} + 3\cos t\,\hat{\jmath} \quad \text{con } t \geq 0$$

Veamos que lo anterior parametriza un arco de la parabola:

$$4(3\cos t)^2 - 9\cdot 2\cos 2t = 4\cdot 9\cos^2 t - 18\cos 2t$$

$$\Rightarrow 36\cos^2 t - 18(\cos^2 t - \operatorname{sen}^2 t)$$

$$= 36\cos^2 t - 18\cos^2 t + 18\operatorname{sen}^2 t = 18\cos^2 t + 18\operatorname{sen}^2 t$$
$$= 18$$

~~es~~ entonces cumple con $4y^2 - 9x = 18$.

Ademas la particula oscilara pues $t \geq 0$ y al estar compuesto con cosenos ~~a~~ la ~~parametrizacion~~ particula repetira su trayectoria cada vez que t avanze en 2π.

$r(2\pi) = r(0)$ por ejemplo ... así sucesivamente.

$$r'(t) = 2\cdot -\operatorname{sen}2t\cdot 2\,\hat{\imath} + 3\cdot -\operatorname{sen}t\,\hat{\jmath}$$
$$= -4\operatorname{sen}2t\,\hat{\imath} - 3\operatorname{sen}t\,\hat{\jmath}$$

(1.0)

$$r''(t) = -4\cos 2t\cdot 2\,\hat{\imath} - 3\cos t\,\hat{\jmath}$$
$$= -8\cos 2t\,\hat{\imath} - 3\cos t\,\hat{\jmath} \Rightarrow r''(0) = \begin{pmatrix}-8\\-3\end{pmatrix}$$

$$\cos\theta = \frac{\vec{n}\cdot\vec{r}\,'}{\|\vec{n}\|\|\vec{r}\,'\|}$$

$$\vec{n} = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix} \quad \|\vec{n}\| = \sqrt{16+4+1} = \sqrt{21}$$

$$r'(2) = (1, 4, 12)$$

$$\|r'(2)\| = \sqrt{1+16+144} = \sqrt{161}$$

$$\vec{n}\cdot\vec{r}\,' = 4+8+12 = 24$$

$$\cos\theta = \frac{24}{\sqrt{21}\sqrt{161}} \Rightarrow \phi = \frac{\pi}{2} - \arccos\left(\frac{24}{\sqrt{21}\sqrt{161}}\right)$$

(1.0)

ii) $\vec{r}(t) = 2\cos 2t\,\hat{\imath} + 3\cos t\,\hat{\jmath}$ con $t \geq 0$

Veamos que lo anterior parametriza un arco de la parabola:

$$4(3\cos t)^2 - 9\cdot 2\cos 2t = 4\cdot 9\cos^2 t - 18\cos 2t$$

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$$r'(t) = 2\cdot -\operatorname{sen} 2t \cdot 2\,\hat{\imath} + 3\cdot -\operatorname{sen} t\,\hat{\jmath}$$
$$= -4\operatorname{sen} 2t\,\hat{\imath} - 3\operatorname{sen} t\,\hat{\jmath}$$

(1.0)

$$r''(t) = -4\cos 2t\cdot 2\,\hat{\imath} - 3\cos t\,\hat{\jmath}$$

$$= -8\cos 2t\,\hat{\imath} - 3\cos t\,\hat{\jmath} \Rightarrow r''(0) = \begin{pmatrix} -8 \\ -3 \end{pmatrix}$$