Examen MA22A-01/07-12-02

1.-

a) Como la densidad es constante e igual a 1, numéricamente la Masa y el Volumen coinciden.

$$Vol=\int_0^{\frac{\pi}{2}}\int_0^3\int_0^{rsen\theta} rdzdrd\theta=9 \qquad \bar{x}=\frac{1}{9}\int_0^{\frac{\pi}{2}}\int_0^3\int_0^{rsen\theta} r\cos\theta rdzdrd\theta=\frac{9}{8}$$

$$\bar{y}=\frac{1}{9}\int_0^{\frac{\pi}{2}}\int_0^3\int_0^{rsen\theta} rsen\theta rdzdrd\theta=\frac{9\pi}{16} \qquad \bar{z}=\frac{1}{9}\int_0^{\frac{\pi}{2}}\int_0^3\int_0^{rsen\theta} zrdzdrd\theta=\frac{9\pi}{32}$$

b)

i) $\int_0^{\pi}\int_0^1 (rsen\theta)rdrd\theta=\frac{2}{3}$ ii) $\int_0^{\pi}\int_0^1 (r\cos\theta)(rsen\theta)rdrd\theta=0$ iii) $\int_0^{\pi}\int_0^1 (rsen\theta)^2 rdrd\theta=\frac{\pi}{8}$

iv) $\int_0^{\pi}\int_0^1 (r\cos\theta)^2(rsen\theta)rdrd\theta=\frac{2}{15}$ v) $\int_0^{\pi}\int_0^1 r^2(rsen\theta)rdrd\theta=\frac{2}{5}$

2.-

a) PDQ $Vol=\frac{AB}{4}(2aB+2bA+4c)$

Demostración

$$Vol=\int_0^B\int_0^A\int_0^{ax+by+c} dzdydx=\int_0^B(aAx+\frac{bA^2}{2}+cA)dx=\frac{AB}{4}(2aB+2bA+4c)$$

b) El problema a resolver es:

$$f(x)=\|x\|_2^2=x^t x$$
$$sa \qquad q(x)=x^t Ax=1$$

=> $L(x,\lambda)=x^t x-\lambda(x^t Ax-1)$

Derivando se obtiene $2x-\lambda 2Ax=0\Rightarrow Ax=\frac{1}{\lambda}x$,por lo tanto 1/λ es un valor propio de la matriz A, como A es definida positiva, todos sus valores propios son reales y positivos.

Como $f(x)=x^t x=x^t\lambda Ax=\lambda x^t Ax=\lambda$ luego el mínimo de *f* corresponderá al recíproco del máximo valor propio y el máximo de *f* será el recíproco del mínimo valor propio.

Para el caso de $q(x,y,z)=7x^2+4y^2+4z^2+4xy-4xz-2yz$, la matriz A será

$$A=\begin{bmatrix}7 & 2 & -2\\ 2 & 4 & -1\\ -2 & -1 & 4\end{bmatrix}$$

Los valores propios de A son 3 y 9. por lo tanto

$$f_{\min}=\frac{1}{9} \qquad f_{\max}=\frac{1}{3}$$

3.- $f(x,y)=g(xy,\frac{x}{y})$ dos veces diferenciable.

PDQ

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=0 \Leftrightarrow (uv+\frac{u}{v})\frac{\partial^2 g}{\partial u^2}+2(1-v^2)\frac{\partial^2 g}{\partial u\partial v}+\frac{v}{u}(1+v^2)\frac{\partial^2 g}{\partial v^2}+2\frac{v^2}{u}\frac{\partial g}{\partial v}=0$$

$$\left.\begin{array}{l} u=xy \\ v=\dfrac{x}{y}\end{array}\right\} \qquad \begin{array}{l} uv=x^2 \Rightarrow x=\sqrt{uv} \\ \dfrac{u}{v}=y^2 \Rightarrow y=\sqrt{\dfrac{u}{v}}\end{array}$$

$$\frac{\partial f}{\partial x}=\frac{\partial g}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial g}{\partial v}\frac{\partial v}{\partial x}$$

$$\frac{\partial^2 f}{\partial x^2}=\left(\frac{\partial^2 g}{\partial u^2}\frac{\partial u}{\partial x}+\frac{\partial^2 g}{\partial v\partial u}\frac{\partial v}{\partial x}\right)\frac{\partial u}{\partial x}+\frac{\partial g}{\partial u}\frac{\partial^2 u}{\partial x^2}+\left(\frac{\partial^2 g}{\partial u\partial v}\frac{\partial u}{\partial x}+\frac{\partial^2 g}{\partial v^2}\frac{\partial v}{\partial x}\right)\frac{\partial u}{\partial x}+\frac{\partial g}{\partial v}\frac{\partial^2 v}{\partial x^2}$$

$$\frac{\partial^2 f}{\partial x^2}=\frac{\partial^2 g}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x}\right)\frac{\partial^2 g}{\partial u\partial v}+\frac{\partial^2 g}{\partial v^2}\left(\frac{\partial v}{\partial x}\right)^2+\frac{\partial g}{\partial u}\frac{\partial^2 u}{\partial x^2}+\frac{\partial g}{\partial v}\frac{\partial^2 v}{\partial x^2}$$

análogamente se tiene:

$$\frac{\partial^2 f}{\partial y^2}=\frac{\partial^2 g}{\partial u^2}\left(\frac{\partial u}{\partial y}\right)^2+2\left(\frac{\partial u}{\partial y}\frac{\partial u}{\partial y}\right)\frac{\partial^2 g}{\partial u\partial v}+\frac{\partial^2 g}{\partial v^2}\left(\frac{\partial v}{\partial y}\right)^2+\frac{\partial g}{\partial u}\frac{\partial^2 u}{\partial y^2}+\frac{\partial g}{\partial v}\frac{\partial^2 v}{\partial y^2}$$

Por lo tanto

$$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=\frac{\partial^2 g}{\partial u^2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2\right]+2\frac{\partial^2 g}{\partial u\partial v}\left[\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}\right)\right]+\frac{\partial^2 g}{\partial v^2}\left[\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]$$

$$+\frac{\partial g}{\partial u}\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right]+\frac{\partial g}{\partial v}\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right]=0$$

donde

$$\frac{\partial u}{\partial x}=y=\sqrt{\frac{u}{v}} \Rightarrow \frac{\partial^2 u}{\partial x^2}=0$$

$$\frac{\partial u}{\partial y}=x=\sqrt{uv} \Rightarrow \frac{\partial^2 u}{\partial y^2}=0$$

$$\frac{\partial v}{\partial x}=\frac{1}{y}=\sqrt{\frac{u}{v}} \Rightarrow \frac{\partial^2 u}{\partial x^2}=0$$

$$\frac{\partial v}{\partial y}=\frac{-x}{y^2}=\frac{-v}{u}\sqrt{uv} \Rightarrow \frac{\partial^2 u}{\partial y^2}=\frac{2x}{y^3}=\frac{2\sqrt{uv}}{\left(\frac{u}{v}\right)^{3/2}}$$