$$\int 2x\sqrt{1+x^2}\,dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$$\int \sqrt{u}\,du = \int u^{1/2}\,du = \frac{2}{3}u^{3/2} + c$$

$$= \frac{2}{3}(1+x^2)^{3/2} + c$$

---

$$\int (x^2 - 2x + 1)^6 (x-1)\,dx$$

$u = x^2 - 2x + 1$

$du = 2x - 2\,dx$

$\frac{du}{2} = (x-1)dx$

$$\int \frac{u^6}{2}\,du = \frac{u^7}{14} + c$$

$$= \frac{(x^2-2x+1)^7}{14} + c$$

---

$$\int \frac{x}{x+1}\,dx$$

$u = x + 1$

$du = dx$

$$\int \frac{u-1}{u}\,du = \int \left(1 - \frac{1}{u}\right)du$$

$$= u - \ln|u| + c$$

$$= x + 1 - \ln|x+1| + c.$$

$$\int \frac{x}{\sqrt{1-4x^2}}\,dx$$

$u = 1-4x^2$

$du = -8x\,dx$

$\frac{du}{-8} = x\,dx$

$$\int \frac{-1}{8\sqrt{u}}\,du = \int \frac{-1}{8} u^{-1/2}\,du$$

$$= \frac{-1}{8} \cdot u^{1/2} \cdot 2 + c$$

$$= \frac{-1}{4}\sqrt{1-4x^2} + c$$

---

$$\int \frac{\sqrt{2x+1}}{3}\,dx$$

$u = 2x+1$

$\frac{du}{2} = dx$

$$\int \frac{\sqrt{u}}{6}\,du = \int \frac{u^{1/2}}{6}\,du$$

$$= \frac{2u^{3/2}}{3 \cdot 6} + c$$

$$= \frac{(2x+1)^{3/2}}{9} + c$$

$$\int \sqrt[3]{x^4+2x}\,(2x^3+1)\,dx$$

$u = x^4+2x$

$du = (4x^3+2)\,dx$

$\frac{du}{2} = (2x^3+1)\,dx$

$$\int \frac{\sqrt[3]{u}}{2}\,du = \int \frac{u^{1/3}}{2}\,du$$

$$= \frac{2}{3}\,u^{4/3} + c$$

$$= \frac{2}{3}\,(x^4+2x)^{4/3} + c$$

---

$$\int e^{5x}\,dx$$

$u = 5x$

$\frac{du}{5} = dx$

$$\int \frac{e^u\,du}{5} = \frac{e^u}{5} + c$$

$$= \frac{e^{5x}}{5} + c.$$

$$\int \frac{2x^2-1}{\sqrt{2x^3-3x+1}}\,dx$$

$$u = 2x^3 - 3x + 1$$

$$du = (6x^2-3)\,dx$$

$$\frac{du}{3} = (2x^2-1)\,dx$$

$$\int \frac{1}{3\sqrt{u}}\,du = \int \frac{1}{3} u^{-1/2}\,du$$

$$= \frac{u^{1/2}}{6} + C$$

$$= \frac{(2x^3-3x+1)^{1/2}}{6} + C$$

---

$$\int \frac{\ln^2(x)}{x}\,dx$$

$$u = \ln(x)$$

$$du = \frac{1}{x}\,dx$$

$$\int u^2\,du = \frac{u^3}{3} + C$$

$$= \frac{\ln^3(x)}{3} + C.$$

$$\int \frac{2x^5}{\sqrt{x^2+3}}\,dx$$

$u = x^2+3 \rightarrow x^2 = u-3$

$du = 2x\,dx$

$$\int \frac{(2x)\,x^4\,dx}{\sqrt{x^2+3}} = \int \frac{(u-3)^2}{\sqrt{u}}\,du$$

$$= \int \left(\frac{u^2-6u+9}{\sqrt{u}}\right) du$$

$$= \int \left(u^{3/2} - 6u^{1/2} + 9u^{-1/2}\right) du$$

$$= \frac{2}{5}u^{5/2} - 6 \cdot u^{3/2} \cdot \frac{2}{3} + 9 \cdot u^{1/2} \cdot 2 + C$$

$$= \frac{2}{5}u^{5/2} - 4u^{3/2} + 18u^{1/2} + C$$

$$= \frac{2}{5}(x^2+3)^{5/2} - 4(x^2+3)^{3/2} + 18(x^2+3)^{1/2} + C$$

---

$$\int x \ln(x)\,dx$$

$u = \ln(x)$ $\qquad dv = x\,dx$

$du = \frac{1}{x}\,dx$ $\qquad v = \frac{x^2}{2}$

$$= \ln(x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx$$

$$= \ln(x) \cdot \frac{x^2}{2} - \int \frac{x}{2}\,dx$$

$$= \ln(x)\frac{x^2}{2} - \frac{x^2}{4} + C$$

---

$$\int x e^{2x}\,dx$$

$u = x$ $\qquad dv = e^{2x}\,dx$

$du = dx$ $\qquad v = \frac{e^{2x}}{2}$

$$= x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx$$

$$= \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$$

---

$$\int x^2 \ln(x)\,dx$$

$u = \ln x$ $\qquad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\qquad v = \frac{x^3}{3}$

$$= \ln(x)\frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx$$

$$= \ln(x)\frac{x^3}{3} - \frac{x^3}{9} + C$$

$$\int \frac{\ln(x)}{x^3}\,dx$$

$u = \ln x$ $\qquad dv = \frac{1}{x^3}\,dx$

$du = \frac{1}{x}\,dx$ $\qquad v = \frac{x^{-2}}{-2}$

$$= \ln(x)\,\frac{x^{-2}}{-2} - \int \frac{x^{-2}}{-2}\cdot\frac{1}{x}\,dx$$

$$= \frac{-\ln(x)}{2x^2} + \int -\frac{x^{-3}}{2}\,dx$$

$$= \frac{-\ln(x)}{2x^2} + \frac{x^{-2}}{-4} + C$$

$$= \frac{-\ln(x)}{2x^2} - \frac{1}{4x^2} + C$$

---

$$\int x^3 e^x\,dx$$

$u = x^3$ $\qquad dv = e^x\,dx$

$du = 3x^2\,dx$ $\qquad v = e^x$

$$= x^3 e^x - \underbrace{\int e^x\,3x^2\,dx}$$

$u = 3x^2$ $\qquad dv = e^x\,dx$

$du = 6x\,dx$ $\qquad v = e^x$

$$= 3x^2 e^x - \underbrace{\int e^x\,6x\,dx}$$

$u = 6x$ $\qquad dv = e^x\,dx$

$du = 6\,dx$ $\qquad v = e^x$

$$= 6xe^x - \underbrace{\int e^x\,6\,dx}_{6e^x}$$

$$= x^3e^x - \left(3x^2e^x - \left(6xe^x - 6e^x\right)\right) + C.$$

---

$$\int (3x^2 - 12x + 1)\ln(x)\,dx$$

$u = \ln x \qquad dv = (3x^2 - 12x + 1)\,dx$

$du = \frac{1}{x}\,dx \qquad v = x^3 - 6x^2 + x$

$$= \ln(x)(x^3 - 6x^2 + x) - \int \frac{x^3 - 6x^2 + 1}{x}\,dx$$

$$= \ln(x)(x^3 - 6x^2 + x) - \int \left(x^2 - 6x + \frac{1}{x}\right)dx$$

$$= \ln(x)(x^3 - 6x^2 + x) - \left(\frac{x^3}{3} - \frac{6x^2}{2} + \ln|x|\right) + C$$

---

$$\int \ln(x)\,dx$$

$u = \ln(x) \qquad dv = dx$

$du = \frac{1}{x}\,dx \qquad v = x$

$$= \ln(x)\cdot x - \int x\,\frac{1}{x}\,dx$$

$$= \ln(x)\,x - x + C$$

$$\int \frac{3x^2}{e^{-2x-1}}\,dx$$

$u = 3x^2 \qquad dv = e^{2x+1}\,dx$

$du = 6x\,dx \qquad v = \frac{e^{2x+1}}{2}$

$$= 3x^2 \cdot \frac{e^{2x+1}}{2} - \int \frac{e^{2x+1}}{2} \cdot 6x\,dx$$

$$= \frac{3}{2}x^2 e^{2x+1} - \underbrace{\int 3x\, e^{2x+1}\,dx}$$

$u = 3x \qquad dv = e^{2x+1}\,dx$

$du = 3\,dx \qquad v = \frac{e^{2x+1}}{2}$

$$= 3x \cdot \frac{e^{2x+1}}{2} - \int 3\,\frac{e^{2x+1}}{2}\,dx$$

$$= \frac{3x^2}{2} e^{2x+1} - \left(\frac{3x}{2} e^{2x+1} - \frac{3}{4} e^{2x+1}\right) + C.$$