

$$a) \quad f(x) = 2x + 1$$

$$= \lim_{h \rightarrow 0} \frac{2(x+h) + 1 - (2x + 1)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x + 2h + 1 - 2x - 1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2h}{h}$$

$$= \lim_{h \rightarrow 0} 2$$

$$= 2$$

$$b) \quad f(x) = 3x^2 + 4$$

$$= \lim_{h \rightarrow 0} \frac{3(x+h)^2 + 4 - (3x^2 + 4)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) + 4 - 3x^2 - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 4 - 3x^2 - 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(6x + 3h)}{h}$$

$$= \lim_{h \rightarrow 0} (6x + 3h)$$

$$= 6x$$

$$c) \quad g(x) = x^4 + x^2$$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^4 + (x+h)^2 - x^4 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h)^2)^2 + (x+h)^2 - x^4 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{(x^2 + 2xh + h^2)^2 + (x^2 + 2xh + h^2) - x^4 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 4x^2h^2 + 2x^2h^2 + 4xh^3 + x^2 + 2xh + h^2 - x^4 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4x^3h + 4x^2h^2 + 2x^2h^2 + 4xh^3 + 2xh + h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(4x^3 + 4x^2h + 2x^2h + 4xh^2 + 2x + h)}{h}$$

$$= \lim_{h \rightarrow 0} (4x^3 + 4x^2h + 2x^2h + 4xh^2 + 2x + h)$$

$$= 4x^3 + 2x$$

$$d) \quad h(x) = \frac{2}{x}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x - 2(x+h)}{h(x+h)x}$$

$$= \lim_{h \rightarrow 0} \frac{-2h}{h(x+h)x}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(x+h)x}$$

$$= \frac{-2}{x^2}$$

$$e) f(x) = \frac{6}{x^2+1}$$

$$= \lim_{h \rightarrow 0} \frac{6}{(x+h)^2+1} - \frac{6}{x^2+1}$$

$$= \lim_{h \rightarrow 0} \frac{6}{h} \frac{(x^2+1) - (x^2+2xh+h^2+1)}{((x+h)^2+1)(x^2+1)}$$

$$= \lim_{h \rightarrow 0} \frac{6}{h} \frac{-2x-h}{((x+h)^2+1)(x^2+1)}$$

$$= \lim_{h \rightarrow 0} \frac{6(-2x-h)}{((x+h)^2+1)(x^2+1)}$$

$$= \frac{6(-2x)}{(x^2+1)(x^2+1)}$$

$$= \frac{-12x}{(x^2+1)^2}$$

$$f) f(x) = \frac{2x}{x^2-x} \quad \text{reescribimos, } f(x) = \frac{-2}{x-1}$$

$$= \lim_{h \rightarrow 0} \frac{2}{x+h-1} - \frac{2}{x-1}$$

$$= \lim_{h \rightarrow 0} \frac{2}{h} \frac{(x-1) - (x-h+1)}{(x+h-1)(x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{2}{h} \frac{-h}{(x+h-1)(x-1)}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= \frac{-2}{(x-1)(x-1)}$$

$$= \frac{-2}{(x-1)^2}$$

$$g(x) = \frac{3}{\sqrt{x-2}}$$

$$\lim_{h \rightarrow 0} \frac{3}{\sqrt{x+h-2}} - \frac{3}{\sqrt{x-2}}$$

$$\lim_{h \rightarrow 0} \frac{3}{h} \left(\frac{\sqrt{x-2} - \sqrt{x+h-2}}{\sqrt{x+h-2} \sqrt{x-2}} \right)$$

$$\lim_{h \rightarrow 0} \frac{3}{h} \left(\frac{\sqrt{x-2} - \sqrt{x+h-2}}{\sqrt{x+h-2} \sqrt{x-2}} \right) \left(\frac{\sqrt{x-2} + \sqrt{x+h-2}}{\sqrt{x-2} + \sqrt{x+h-2}} \right)$$

$$\lim_{h \rightarrow 0} \frac{3}{h} \frac{(x-2 - x-h+2)}{(\sqrt{x+h-2} \sqrt{x-2}) (\sqrt{x-2} + \sqrt{x+h-2})}$$

$$\lim_{h \rightarrow 0} \frac{-3h}{h (\sqrt{x+h-2} \sqrt{x-2}) (\sqrt{x-2} + \sqrt{x+h-2})}$$

$$\lim_{h \rightarrow 0} \frac{-3}{\sqrt{x+h-2} \sqrt{x-2} (\sqrt{x-2} + \sqrt{x+h-2})}$$

$$= \frac{-3}{\sqrt{x-2} \sqrt{x-2} (\sqrt{x-2} + \sqrt{x-2})}$$

$$= \frac{-3}{2(x-2) \sqrt{x-2}}$$

$$= \frac{-3}{2\sqrt{(x-2)^3}}$$

$$h) \quad f(x) = \sqrt{x^2 + 4}$$

$$\lim_{h \rightarrow 0} \frac{\sqrt{(x+h)^2 + 4} - \sqrt{x^2 + 4}}{h}$$

$$\lim_{h \rightarrow 0} \left(\frac{\sqrt{(x+h)^2 + 4} - \sqrt{x^2 + 4}}{h} \right) \left(\frac{\sqrt{(x+h)^2 + 4} + \sqrt{x^2 + 4}}{\sqrt{(x+h)^2 + 4} + \sqrt{x^2 + 4}} \right)$$

$$\lim_{h \rightarrow 0} \frac{(x+h)^2 + 4 - x^2 - 4}{h (\sqrt{(x+h)^2 + 4} + \sqrt{x^2 + 4})}$$

$$\lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h (\sqrt{(x+h)^2 + 4} + \sqrt{x^2 + 4})}$$

$$\lim_{h \rightarrow 0} \frac{h(2x + h)}{h (\sqrt{(x+h)^2 + 4} + \sqrt{x^2 + 4})}$$

$$\lim_{h \rightarrow 0} \frac{2x + h}{\sqrt{(x+h)^2 + 4} + \sqrt{x^2 + 4}}$$

$$= \frac{2x}{\sqrt{x^2 + 4} + \sqrt{x^2 + 4}}$$

$$= \frac{x}{\sqrt{x^2 + 4}}$$