Pauta : Alan Amolef.

$$\lim_{a \to 1} \frac{\sqrt{a+3}-2}{a-1}$$

evaluar:

$$\lim_{a \to 1} \frac{\sqrt{a+3}-2}{a-1} = \left(\frac{0}{0}\right) \text{ indeterminado}$$

$$= \lim_{a \to 1} \frac{\sqrt{a+3}-2}{a-1} \cdot \left(\frac{\sqrt{a+3}+2}{\sqrt{a+3}+2}\right)$$

$$= \lim_{a \to 1} \frac{a+3-4}{(a-1)(\sqrt{a+3}+2)}$$

$$= \lim_{a \to 1} \frac{\cancel{a-1}}{(\cancel{a-1})(\sqrt{a+3}+2)}$$

$$= \lim_{a \to 1} \frac{1}{\sqrt{a+3}+2}$$

$$= \frac{1}{4}$$

$$\therefore \lim_{a \to 1} \frac{\sqrt{a+3}-2}{a-1} = 1/4$$

$$\lim_{x \to -2} \frac{bx^3 + 8b}{3bx^2 + 11bx + 10b}$$

evaluar:

$$\lim_{x \to -2} \frac{bx^3+8b}{3bx^2+11bx+10b} = \left(\frac{0}{0}\right) \text{ indeterminado}$$

$$= \lim_{x \to -2} \frac{\cancel{b}(x^3+8)}{\cancel{b}(3x^2+11x+10)}$$

$$= \lim_{x \to -2} \frac{x^3+2^3}{3\left(x+\frac{5}{3}\right)(x+2)}$$

$$= \lim_{x \to -2} \frac{\cancel{(x+2)}(x^2-2x+4)}{3(x+5/3)\cancel{(x+2)}}$$

$$= \lim_{x \to -2} \frac{x^2-2x+4}{3\left(x+\frac{5}{3}\right)}$$

$$= \frac{12}{-1} = -12$$

$$\therefore \lim_{x \to -2} \frac{bx^3+8b}{3bx^2+11bx+10b} = -12$$

Que sea continua en $x=-1$, $x=3$.

$$f(x)=\begin{cases} \dfrac{a(x^3+1)}{x-1}+b & x<-1 \\ 3ax-2 & -1\le x\le 3 \\ \dfrac{b(x^2+4x-21)}{x-3} & 3<x \end{cases}$$

Solución:

$$\frac{a(x^3+1)}{x-1}+b \quad \Big| \quad 3ax-2 \quad \Big| \quad \frac{b(x^2+4x-21)}{x-3}$$

$$-1 \qquad\qquad 3$$

en $x=-1$ debe cumplirse que:

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) = f(x=-1)$$

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1} \frac{a(x^3+1)}{x-1} + b = b$$

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1} (3ax-2) = -3a-2$$

$$f(x=-1) = 3a(-1)-2 = -3a-2$$

condición:

$$b = -3a-2$$

$$\boxed{3a+b=-2}$$

para $x=3$ debe cumplirse que:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x) = f(x=3)$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3} (3ax-2) = 9a-2$$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3} \frac{b(x^2+4x-21)}{x-3} = \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 3} \frac{b(x+7)\cancel{(x-3)}}{\cancel{(x-3)}}$$

$$= \lim_{x \to 3} b(x+7)$$

$$= 10b$$

$$f(x=3) = 3a(3)-2 = 9a-2$$

condición

$$9a - 2 = 10b$$

$$\boxed{9a - 10b = 2}$$

resolviendo simultaneamente

$$\begin{aligned} 3a + b &= -2 \\ 9a - 10b &= 2 \end{aligned}$$

$$a = -6/13$$

$$b = -8/13.$$

derivada con definición

$$f(x) = 5bx^2 + 4x - 1$$

$$f'(x) = \lim_{h \to 0} \frac{5b(x+h)^2 + 4(x+h) - 1 - (5bx^2 + 4x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{5b(x^2 + 2xh + h^2) + 4(x+h) - 1 - (5bx^2 + 4x - 1)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5bx^2} + 10bxh + 5bh^2 \cancel{+4x} + 4h \cancel{-1} \cancel{-5bx^2} \cancel{-4x} \cancel{+1}}{h}$$

$$= \lim_{h \to 0} \frac{10bxh + 5bh^2 + 4h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10bx + 5bh + 4)}{\cancel{h}}$$

$$= \lim_{h \to 0} (10bx + 5bh + 4)$$

$$= 10bx + 4$$

$$\therefore \; f'(x) = 10bx + 4$$

$$f(x) = \sqrt[4]{16x - x^4}$$

a) $f(x) = (16x - x^4)^{1/4}$

$$f'(x) = \frac{1}{4}(16x - x^4)^{-3/4}(16 - 4x^3)$$

$$= \frac{16 - 4x^3}{4(16x - x^4)^{3/4}}$$

$$= \frac{4 - x^3}{(16x - x^4)^{3/4}}$$

b)

$$f'(x=2) = \frac{4 - 2^3}{(16(2) - 2^4)^{3/4}} = -1/2$$

ec punto pendiente

en $x = 2 \to y = \sqrt[4]{16(2) - 2^4}$
$= 2$

$$y - 2 = \frac{-1}{2}(x - 2)$$

$$\boxed{y = -\frac{x}{2} + 3}$$

c) $$y - 2 = \frac{-1}{-1/2}(x - 2)$$

$$\boxed{y = 2x - 2}$$

$$p = \frac{2500}{10+2q}$$

a) I = ingreso total

$$I = p \cdot q = \frac{2500q}{10+2q}$$

b) $$I' = \frac{2500(10+2q) - 2500q \cdot 2}{(10+2q)^2}$$

$$I' = \frac{25000}{(10+2q)^2}$$

c) $$I'(q=5) = \frac{25000}{(10+2(5))^2} = 62,5$$