Pauta: 01 de septiembre 2022.

①

a)

$2x-a$		$ax+2b$		$b-5$
	-3		3	

Para que sea continua en $\mathbb{R}$, debe cumplirse que:

i) $\lim\limits_{x\to -3^-} 2x-a = \lim\limits_{x\to -3^+} ax+2b = f(x=-3)$

ii) $\lim\limits_{x\to 3^-} ax+2b = \lim\limits_{x\to 3^+} b-5 = f(x=3)$

Sol:

i) $\lim\limits_{x\to -3^-} 2x-a = -6-a$

$\lim\limits_{x\to -3^+} ax+2b = -3a+2b$

$f(x=-3) = -3a+2b$

Condicion: $-6-a = -3a+2b$

$2a-2b = 6$

$$\boxed{a-b=3}$$

ii) $\lim\limits_{x\to 3^-} ax+2b = 3a+2b$

$\lim\limits_{x\to 3^+} b-5 = b-5$

$f(x=3) = 3a+2b$

condicion: $3a+2b = b-5$

$$\boxed{3a+b=-5}$$

resolver $\begin{array}{l} a-b=3 \\ 3a+b=-5 \end{array} \Rightarrow \left.\begin{array}{l} a=-1/2 \\ b=-7/2 \end{array}\right\}.$

b)
$$\underset{\hspace{6em}}{\dfrac{e^{2x}-1}{x}} \quad \underset{0}{|} \quad \dfrac{2}{\ln(2)}\,\dfrac{2^x-1}{x}$$

Sol:

veamos los limits

$$\lim_{x\to 0^-} \frac{e^{2x}-1}{x} = \left(\frac{0}{0}\right) \text{ indeterminado.}$$

$$\lim_{x\to 0} \frac{2^x-1}{x} = \ln(2)$$

$$\lim_{x\to 0^-} \frac{(e^2)^x-1}{x} = \ln(e^2) = 2\ln(e) = 2$$

$$\lim_{x\to 0^+} \frac{2}{\ln 2}\cdot\frac{2^x-1}{x} = \frac{2}{\ln 2}\cdot\ln(2) = 2$$

limites laterales iguales

$$f(x=0) = 3/2 \neq 2.$$

$\therefore$ es discontinua en $x=0$

1:-

$$\frac{kx^3+8k}{3x+6} \quad | \quad 2-3kx$$

$$-2.$$

$$\underbrace{\lim_{x\to -2^-} \frac{kx^3+8k}{3x+6}}_{\frac{-8k+8k}{-6+6} = \left(\frac{0}{0}\right)} = \underbrace{\lim_{x\to -2^+} 2-3kx}_{2+6k} = f(x=-2) = 2+6k.$$

$$\underbrace{\qquad}_{?}$$

$$\lim_{x\to -2} \frac{kx^3+8k}{3x+6} = \lim_{x\to -2} \frac{k(x^3+2^3)}{3(x+2)}$$

$$= \lim_{x\to -2} \frac{k\cancel{(x+2)}(x^2-2x+4)}{3\cancel{(x+2)}}$$

$$= \lim_{x\to -2} \frac{k(x^2-2x+4)}{3}$$

$$= \frac{k}{3}12 = 4k$$

$$4k = 2+6k. \rightarrow \boxed{k=-1}$$

2.

$$\lim_{x\to 1} \frac{x-1}{\sqrt{5x-1}-2}$$

$$= \lim_{x\to 1} \frac{(x-1)(\sqrt{5x-1}+2)}{(\sqrt{5x-1}-2)(\sqrt{5x-1}+2)}$$

$$= \lim_{x\to 1} \frac{(x-1)(\sqrt{5x-1}+2)}{5x-1-4}$$

$$= \lim_{x\to 1} \frac{\cancel{(x-1)}(\sqrt{5x-1}+2)}{5\cancel{(x-1)}}$$

$$= \frac{4}{5}.$$

$f(x=1) = 4/5.$

$$\lim_{x\to 1} \frac{3x^2-2x-1}{2x^2+x-3}$$

$$3x^2-2x-1 = \frac{2\pm\sqrt{4+12}}{6} = \frac{2\pm 4}{6} = \begin{cases} 1 \\ -1/3 \end{cases}$$

$$= 3(x-1)(x+1/3)$$

$$= (x-1)(3x+1)$$

$$2x^2+x-3 = \frac{-1\pm\sqrt{1+24}}{4} = \begin{cases} 1 \\ -3/2. \end{cases}$$

$$= 2(x-1)(x+3/2)$$

$$\lim_{x\to 1} \frac{3\cancel{(x-1)}(x+1/3)}{2\cancel{(x-1)}(x+3/2)} = 4/5.$$

$\Rightarrow$ es continua en $x=1$.

12.

$$\lim_{x\to 1^-} \frac{\sqrt{x}-1}{x-1} = \underbrace{\lim_{x\to 1^+} \frac{1}{2}x^3+x-1}_{\frac{1}{2}} = A.$$

$$\frac{1}{2} = A.$$

$$\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1} = \lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x-1}\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}+1}\right)$$

$$= \lim_{x\to 1} \frac{\cancel{x-1}}{(\cancel{x-1})(\sqrt{x}+1)}$$

$$= \lim_{x\to 1} \frac{1}{\sqrt{x}+1}$$

$$= \frac{1}{2}.$$

$$\Rightarrow \quad A = 1/2$$

1.b.

en $x=5$.

$$\lim_{x\to 5^-} x^2-x+1 = 21.$$

$$f(x=5) = 21$$

$$\lim_{x\to 5^+} \frac{\sqrt{x-1}-\sqrt{9-x}}{x-5} = \lim_{x\to 5} \frac{x-1-(9-x)}{(x-5)(\sqrt{x-1}+\sqrt{9-x})}$$

$$= \lim_{x\to 5} \frac{2x-10}{(x-5)(\sqrt{x-1}+\sqrt{9-x})}$$

$$= 2\lim_{x\to 5} \frac{1}{\sqrt{x-1}+\sqrt{9-x}}$$

$$= \frac{2}{2+2}$$

$$= 1/2.$$

no es continua en todo $\mathbb{R}$.