

Capítulo del libro:

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9

Log Rules and Volume Tables

A *log rule* is a table or formula that gives the estimated volume of logs of specified diameters and lengths. A *volume table* is a tabulated statement of the average volumes of trees by one or more tree dimensions. In the United States most log rules, and a majority of volume tables, give volumes in board feet of lumber, although tables that give volume in cubic units are available. In nations where the International System of Units is used, log rules and volume tables usually give volumes in cubic meters.

9-1 BOARD-FOOT LOG RULES

One might think that it would be easy to prepare a board-foot log rule that would be universally applicable. This is not the case, however, because of variations in the dimension of lumber produced from logs, in the equipment used to saw logs, in the skill of operators, in computer programs used to saw logs, and in the logs. All of these things affect the amount of lumber obtained from any run of logs.

In the early years of the lumber industry in the United States and Canada, there were a number of independent marketing areas, and no industrial organization or governmental agency had control over the measurement of lumber or logs. As a result, different areas devised different rules to fit specific operating conditions. As a consequence, as Freese (1973) points out in his excellent report on log rules, "In the United States and Canada there are over 95 recognized rules bearing about 185 names." (Most of these "over 95 recognized rules" have long since been forgotten. Only five or six rules are important in present-day use. There are perhaps half a dozen others that may be encountered in certain localities.)

Board-foot log rules are used to estimate the contents of logs. This constitutes an attempt to estimate, before processing, the amount of lumber in logs. Thus, we must distinguish between the measurement of the board-foot contents of sawn lumber, that is, *mill tally*, and the estimation of the board-foot contents of logs, that is, *log scale*. The board-foot mill tally, though not exact, is a well-defined unit; the board-foot log scale is an ambiguous unit. Thus, the amount of lumber sawed from any run of logs rarely agrees with the scale of the logs. This variation O_v may be expressed in board feet.

$$O_v \text{ (in board feet)} = \text{Mill tally} - \text{Log scale}$$

When O_v is positive, a mill has produced an *overrun*; when O_v is negative, a mill has produced an *underrun*.

Foresters, however, have typically expressed O_v as a percentage of log scale.

$$\begin{aligned} O_v \text{ (in percent)} &= \left(\frac{\text{Mill tally} - \text{Log scale}}{\text{Log scale}} \right) 100 \\ &= \left[\left(\frac{\text{Mill tally}}{\text{Log scale}} \right) - 1 \right] 100 \end{aligned} \quad (9-1)$$

where the ratio Mill tally/Log scale is the overrun ratio, that is, the number of board-feet mill tally per board-foot log scale.

In the construction of all known board-foot log rules, three basic methods have been used: (1) mill study, (2) diagram, and (3) mathematical.

For the log rules in present-day use, generally the yield of logs V in board feet is estimated in terms of lumber 1-inch thick, from average small-end diameter inside bark D in inches, and log length L in feet.

9-1.1 Mill-Study Log Rules

In this method of constructing log rules a sample of logs is first measured on the log deck. Then, as each log is sawed, the boards are measured to determine the board-foot volume of the log. The log rule is prepared by relating board-foot yields, the dependent variable, with log diameters and lengths, the independent variables. The problem may be solved graphically or by the method of least squares.

A rule of this type should give good estimates for mills that cut timber with peculiar characteristics, or for those that use specific milling methods. The method, however, has never been widely used. The Massachusetts log rule, one of the few rules constructed by this method that is still in use, was based on 1200 white pine logs. This rule was constructed for round- and square-edged boards sawed from small logs (4-inch saw kerf). Some boards over 1-inch thick were included, so the values are slightly high for 1-inch boards.

9-1.2 Diagram Log Rules

The procedure for the construction of a diagram log rule is simple.

1. Draw circles to scale to represent the small ends of logs of different diameters inside bark. Assume logs are cylinders of a specific length, such as 8 feet.
2. Use definite assumptions on saw kerf and shrinkage, and board width, and draw boards (rectangles) 1-inch thick within the circles.
3. Compute the total board-foot content for each log diameter.
4. Determine the board-foot contents of other log lengths by proportion.

When a diagram log rule is prepared for any given log length, it will be found that increases in volume, from one diameter to the next, will be slightly irregular. These irregularities may be eliminated by preparing a freehand curve, or a regression equation, to predict volume from diameter for each length.

The Scribner log rule, the most widely used diagram log rule, was first published in 1846 by J. M. Scribner, a country clergyman. The rule was prepared for 1-inch lumber with a 1/4-inch allowance for saw kerf and shrinkage. The minimum board width is unknown. The original table gave board-foot contents for logs with scaling diameters (diameters inside bark at small end) from 12 to 44 inches, and with lengths from 10 to 24 feet. Log taper was not considered. A few years after the original rule was published, Scribner modified the rule by increasing the slab allowance on larger logs. This is the rule in use today. The Scribner rule gives a relatively high overrun (up to 30 percent) for logs under 14 inches. Above 14 inches the overrun gradually decreases and flattens out around 28 inches to about 3 to 5 percent.

A regression equation was prepared from the original Scribner table by Bruce and Schumacher (1950). The equation, which gives volume in board feet V , in terms of scaling diameter in inches D and log length in feet L , is

$$V = (0.79D^2 - 2D - 4) \frac{L}{16} \quad (9-2)$$

Some so-called Scribner tables contain values based on this equation. Values in these tables differ slightly from the original Scribner values because Scribner did not smooth the values he obtained from his diagrams. Note that the Scribner values in Table 9-1 are from the original Scribner table.

Since calculating machines were not generally available in the nineteenth century, scalers found the adding of long columns of figures laborious. Consequently, the Scribner rule was often converted into a *decimal rule* by dropping the units and rounding the values to the nearest 10 board feet. Thus, 114 board feet was written 11, and 159 board feet was written 16.

Because the original Scribner rule did not give values for logs less than 12 inches in diameter, a number of lumber companies extrapolated to derive volumes for small logs. Finally, the Lufkin Rule Company prepared three tables using different assumptions to extend the rule to cover small logs. They published these as decimal rules, and called them Scribner decimal A rule, Scribner decimal B rule, and Scribner decimal C rule. The decimal C rule is the only one of these rules still widely used.

There are other log rules based on diagrams. The best known are the Spaulding rule, which was devised by N. W. Spaulding of San Francisco in 1868, and the Maine rule, which was devised by C. T. Holland in 1856. The Spaulding rule is used on the Pacific Coast of the United States; the Maine rule is used in northeastern United States. The Spaulding rule, which closely approximates the values of the Scribner rule, may be expressed by the following regression equation, where V is volume in board feet, D is scaling diameter in inches, and L is log length in feet.

$$V = (0.778D^2 - 1.125D - 13.482) \frac{L}{16} \quad (9-3)$$

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9-1.3 Mathematical Log Rules

In this method of constructing log rules one makes definite assumptions on saw kerf, taper, and milling procedures and prepares a formula that gives board-foot yield of logs in terms of their diameters and lengths. As will be seen, this is not a regression analysis.

The *Doyle log rule*—one of the most widely used, one of the oldest, and one of the most cursed log rules—was first published in 1825 by Edward Doyle. The rule states: "Deduct 4 inches from the diameter of the log, D , in inches, for slabbing, square one-quarter of the remainder, and multiply by the length of the log, L , in feet." As Herrick (1940) pointed out, when Doyle deducted 4 inches from the diameter of the log for slabbing, he was squaring the log. Then, he calculated the board-foot contents of the squared log, or cant, as follows.

$$\frac{(D - 4)(D - 4)L}{12}$$

To allow for saw kerf and shrinkage, he reduced the volume of the cant by 25 percent to obtain the final rule.

$$V = \frac{(D - 4)^2 L}{12} (1.00 - 0.25) = \left(\frac{D - 4}{4} \right)^2 L \quad (9-4)$$

For 16-foot logs the *Doyle rule of thumb* is

$$V = (D - 4)^2$$

This all points up an important reason this rule gained wide acceptance; it was genuinely simple, and it could be easily applied.

When the Doyle rule is applied to logs between 26 and 36 inches in diameter, it gives good results. When the rule is applied to large logs, it gives an underrun; when the rule is applied to small logs, it gives a high overrun. This comes about because the 4-inch slabbing allowance is inadequate for large logs and excessive for small logs.

The *International log rule*, one of the most accurate mathematical log rules, was developed by Judson F. Clark in 1900 when he was working for the Province of Ontario. It was published in 1906 (Clark, 1906). The derivation of the original rule is logical and simple. One first computes the board-foot contents of a 4-foot cylinder in terms of cylinder diameter in inches D , assuming the cylinder will produce lumber at the rate of 12 board feet per cubic foot.

$$\text{Solid board-foot contents of 4-foot cylinder} = \frac{\pi D^2}{4(144)} (4)(12) = 0.262D^2$$

To allow for saw kerf and shrinkage one assumes that, for each 1-inch board cut, 1/8 inch will be lost in saw kerf and 1/16 inch in shrinkage. Thus, the proportion lost from saw kerf and shrinkage is

$$\left(\frac{3/16}{1 + 3/16} \right) \quad \text{or} \quad 0.158$$

When reduced by 0.158, the volume of the 4-foot cylinder becomes

$$0.262D^2(1.000 - 0.158) = 0.22D^2$$

Thus, the losses from saw kerf and shrinkage are proportional to the end area of the log (i.e., to D^2).

Clark determined that losses from slabs and edgings constitute a ring-shaped collar around the outside of the log and that they are proportional to the surface area, or the diameter D of the log. And from a careful analysis of the losses occurring during the conversion of sawlogs to lumber, Clark found that a plank 2.12 inches thick and D inches wide will give the correct deduction. (The thickness of the collar T is about 0.7 inches for all values of D . This can be determined from the following equation.

$$2.12D = \pi \frac{D^2}{4} - \frac{\pi(D - 2T)^2}{4}$$

which leads to $T^2 - DT + 0.6748D = 0$.) Therefore, in terms of cylinder diameter in inches D the board-foot deduction is computed using equation 7-2.

$$\frac{2.12(D)(4)}{12} = 0.71D$$

Thus, the net board-foot volume V of a 4-foot cylinder is

$$V = 0.22D^2 - 0.71D \quad (9-5)$$

After studying northeastern tree species, Clark decided to allow a taper of $\frac{1}{4}$ inch for each 4-foot section. With this assumption in mind, the basic formula was expanded to cover other log lengths.

$$V \text{ (8-foot logs)} = 0.44D^2 - 1.20D - 0.30 \quad (9-6)$$

$$V \text{ (12-foot logs)} = 0.66D^2 - 1.47D - 0.79 \quad (9-7)$$

$$V \text{ (16-foot logs)} = 0.88D^2 - 1.52D - 1.36 \quad (9-8)$$

$$V \text{ (20-foot logs)} = 1.10D^2 - 1.35D - 1.90 \quad (9-9)$$

Clark specified that lengths over 20 feet were to be scaled as two or more logs.

The volume of a log of intermediate length (e.g., 3, 10, 14, or 17 feet) can be calculated by application of the formula given by Grosenbaugh (1952a), or the following algorithm developed for use with programmable calculators (Beers, 1980).

1. Given log length L_i and scaling diameter D_i , calculate $L_i/4 = n.ff$; where n = integer part of the quotient and ff = fractional part of the quotient (.00, .25, .50, and .75).

2. Then, the volume of the i th log V_i is calculated

- a. For $4 \leq L_i \leq 20$, by

$$V_i = \sum_{j=1}^n \left[.22 \left(D_i + \frac{n-j}{2} \right)^2 - .71 \left(D_i + \frac{n-j}{2} \right) \right] + ff \left[.22 \left(D_i + \frac{n}{2} \right)^2 - .71 \left(D_i + \frac{n}{2} \right) \right]$$

- b. For $L_i < 4$, by

$$V_i = ff(.22D_i^2 - .71D_i)$$

- c. For $L_i > 20$, by dividing the log into shorter sections and following the above procedure.

The original International log rule may be modified to give estimates for saw kerfs other than $\frac{1}{4}$ inch. For example, for a kerf of $\frac{1}{8}$ inch (shrinkage of $\frac{1}{16}$ inch), the proportion lost is

$$\left(\frac{5/16}{1 + 5/16} \right) = 0.238$$

So the original rule may be converted to a $\frac{1}{8}$ -inch rule by multiplying the values by the following factor.

$$\frac{1.000 - 0.238}{1.000 - 0.158} = 0.905$$

The factor to convert to a $\frac{1}{8}$ -inch rule is 1.013; the factor to convert to a $\frac{1}{16}$ -inch rule is 0.950.

When Clark (1906) published his log rule in table form, he rounded all values to the nearest multiple of 5 board feet. This is the form in which most International log rule tables appear today. The rule may also be presented as a decimal rule (U.S. Forest Service, 1977).

Other formula rules have been constructed, but they have never enjoyed the popularity of the Doyle and International log rules.

9-1.4 Combination Log Rules

This type of rule combines values from different log rules. Such a rule takes advantage of the best, or the worst, features of the rules used. For example, the *Doyle-Scribner rule*, a combination of the Doyle and Scribner rules, was prepared for use in defective and overmature timber. Since the Doyle rule gives an overrun for small logs, its values were used for diameters up through 28 inches. Since the Scribner rule gives an overrun for large logs, its values were used for diameters greater than 28 inches. Thus, the

Doyle-Scribner rule, which is still used by some private operators in the South, gives a consistently high overrun that is supposed to compensate for hidden defects.

The Scribner-Doyle rule, exactly opposite to the Doyle-Scribner rule, gives a consistently low overrun.

9-1.5 Comparison of Log Rules

Because different methods and assumptions are used in the construction of log rules, different rules give different results, none of which will necessarily agree with the mill tally for any given log (Table 9-1). This points out that the board-foot log scale, by any rule, is a unit of estimate, not a unit of measure.

9-2 CUBIC-VOLUME LOG RULES

The cubic volume of a log may be determined by any of the methods given in Section 8-1. However, for log scaling, gross cubic volumes are usually computed by Huber's or Smalian's formula. The formulas could be used directly, but it is more convenient to prepare tables. In the preparation of cubic meter tables, volumes are most often computed for 2-centimeter diameter classes and 0.2-meter length classes. In the preparation of cubic foot tables, volumes are most often computed for 1-inch diameter classes and 1-foot length classes. Diameter is often given as the midpoint diameter inside bark. (When the midpoint diameter cannot be measured, the average of the diameters inside bark at the two ends of the log is generally used for midpoint diameter.) In some cases, volume is given in terms of small-end diameter inside bark.

Table 9-1
Volume of 16-Foot Logs

Log Diameter (inches)	Mill Tally ¹ (board feet)	Log Scale				
		Scribner	Maine	International 1/2-Inch Rule (board feet)	Doyle	Spaulding
6		18	20	20	4	
10	75	50	68	65	36	
14	145	114	142	135	100	114
18	229	213	232	230	196	216
22	382	334	363	355	324	341
26	578	500	507	500	484	488
30	665	657	706	675	676	656
34	862	800	900	870	900	845
38	1037	1068	1135	1095	1156	1064

¹ Average yield of logs sawed in an Indiana band mill.

9-3 LOG SCALING

Scaling is the determination of the gross and net volumes of logs in board feet, cubic feet, cubic meters, or other units. The determination of gross scale consists of measuring log length and diameter and determining the volume by a log rule. The volume values are usually read from a scale stick, a flat stick that has volumes for different log diameters and lengths printed on its face.

9-3.1 Board-Foot Scaling

In board-foot scaling a maximum length of 40 feet is standard for the western regions of the United States; 16 feet is standard for the eastern regions. When logs exceed the maximum scaling length, they are scaled as two or more logs. If a log does not divide evenly, the butt section is assigned the longer length. The scaling diameter for the assumed point of separation can be estimated from the taper of the log. Although logs are most commonly cut and measured in even lengths (i.e., 8, 10, 12, 14, and 16 feet), they may be cut and measured, particularly with hardwood logs, in both odd and even lengths (i.e., 8, 9, 10, 11, and 12 feet). Logs must be cut longer than standard lumber lengths because it is impossible to buck logs squarely and because there is logging damage to log ends. This extra length, which will range from 3 to 6 inches, depending on the size of timber, products sawed, and logging methods, is called *trim allowance*.

The *National Forest Log Scaling Handbook* (U.S. Forest Service, 1977) gives these instructions on trim allowance for board-foot scaling.

Contract trim allowances are normally the permissive maximums. Regularly tape-measure enough lengths to insure proper observance of trim. Scale logs overrunning the trim allowance to the next 1-foot scaling measure in length unless otherwise instructed. For example, if 6 inches is the contract trim allowance for logs 8 to 20 feet in length, a log measuring 20 feet 10 inches is scaled as a 21; one measuring 24 feet 10 inches, as a 24; but one measuring 25 feet 2 inches, as a 25-foot log; 32 feet 0 inches, as a 31; or 32 feet 2 inches, as a 32; 41 feet 2 inches, as a 41.

Most board-foot log rules call for diameter measurements inside bark, to the nearest inch, at the small end of the log. When a log is round, one measurement is enough. When a log is eccentric, as most logs are, the usual practice is to take a pair of measurements at right angles across the long and short axes of the log end and to average the results to obtain the scaling diameter.

To determine net scale one must deduct from gross scale the quantity of lumber, according to the log rule used, that will be lost due to defects. These deductions do not include material lost during manufacturing, or defects that affect the quality of the lumber. Instead, they include those defects that reduce the volume of lumber.

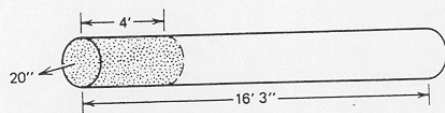
Detailed procedures have been worked out to estimate the volume of deductions in defective logs (see U.S. Forest Service, 1977). But the most logical and applicable system was proposed by Grosenbaugh (1952a). In this method the amount of material

lost in defect is estimated by multiplying the gross scale by the proportion of the log affected. The system works, with minor modifications, regardless of the units in which the log is measured: board feet by any log rule, cubic feet, cubic meters, cords, pounds, kilograms, and so forth. The procedures for common defects can be summarized in the following five rules, where scaling diameter is defined as the average inside bark diameter at the small end, and measurements are in English units.

1. When defect affects entire section, the proportion P lost is

$$P = \frac{\text{Length of defective section}}{\text{Log length}}$$

EXAMPLE

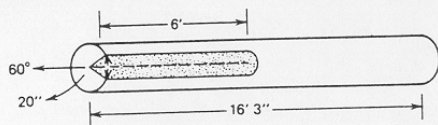


$$\begin{aligned} \text{Gross scale, Scribner decimal C rule} &= 28 \\ \text{Cull} &= (4/16)(28) = 7 & -7 \\ \text{Net scale} & & \overline{21} \end{aligned}$$

2. When defect affects wedge-shaped sector.

$$P = \left(\frac{\text{Length of defective section}}{\text{Log length}} \right) \left(\frac{\text{Central angle defect}}{360^\circ} \right)$$

EXAMPLE

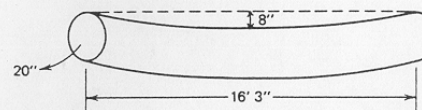


$$\begin{aligned} \text{Gross scale, Scribner decimal C rule} &= 28 \\ \text{Cull} &= \left(\frac{6}{16} \cdot \frac{60}{360} \right) (28) = 2 & -2 \\ \text{Net scale} & & \overline{26} \end{aligned}$$

3. When log sweeps (ignore sweep less than 2 inches).

$$P = \frac{\text{Maximum departure minus 2}}{\text{Scaling diameter}}$$

EXAMPLE

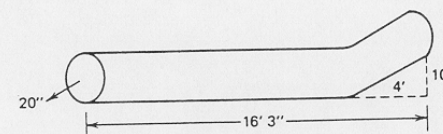


$$\begin{aligned} \text{Gross scale, Scribner decimal C rule} &= 28 \\ \text{Cull} &= \left(\frac{8 - 2}{20} \right) (28) = 8 & -8 \\ \text{Net scale} & & \overline{20} \end{aligned}$$

4. When log crooks.

$$P = \left(\frac{\text{Maximum deflection}}{\text{Scaling diameter}} \right) \left(\frac{\text{Length of deflecting section}}{\text{Log length}} \right)$$

EXAMPLE



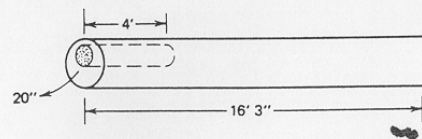
$$\begin{aligned} \text{Gross scale, Scribner decimal C rule} &= 28 \\ \text{Cull} &= \left(\frac{10}{20} \cdot \frac{4}{16} \right) (28) = 3 & -3 \\ \text{Net scale} & & \overline{25} \end{aligned}$$

5. When average cross section of interior defect is enclosable in an ellipse or circle.

$$P = \frac{(\text{Major diameter} + 1)(\text{Minor diameter} + 1)}{(\text{Scaling diameter} - 1)^2} \left(\frac{\text{Defect length}}{\text{Log length}} \right)$$

(Defect in peripheral inch of log (slab collar) can be ignored.)

EXAMPLE



Defect Diameters:
Major = 9 inches
Minor = 7 inches

$$\begin{aligned} \text{Gross scale, Scribner decimal C rule} &= 28 \\ \text{Cull} &= \frac{(9 + 1)(7 + 1)}{(20 - 1)^2} \left(\frac{4}{16} \right) (28) = 2 \quad -2 \\ \text{Net scale} &= \frac{26}{26} \end{aligned}$$

9-3.2 Cubic Volume Scaling

Cubic volume has its greatest utility when the volumes obtained give an accurate estimate of true volume. Thus, log measurement must be consistent. Then, log volumes in cubic units can be converted to the unit of measure appropriate to each manufacturing plant with less uncertainty than in converting from board-feet log scale to board feet of lumber, or from board-feet log scale to square feet of veneer.

In cubic volume scaling, log diameters and log lengths are taken as explained in Section 9-2. In making deductions for defects the five rules given in Section 9-3.1 are applicable. If cubic feet are used, diameter is average small-end diameter in inches and length is in feet, and the rules are used without modifications. If cubic meters are used, diameter is average small-end diameter in centimeters and length is in meters, and the third and fifth rules are changed as follows.

3. When log sweeps (ignore sweep less than 5 centimeters).

$$P = \frac{\text{Maximum departure minus 5}}{\text{Scaling diameter}}$$

5. When average cross section of interior defect is enclosable in an ellipse or circle.

$$P = \frac{(\text{Major diameter} + 2)(\text{Minor diameter} + 2)}{(\text{Scaling diameter} - 2)^2} \left(\frac{\text{Defect length}}{\text{Log length}} \right)$$

9-3.3 Unmerchantable Logs

The definition of a cull, or unmerchantable, log is largely a local matter. Merchantability varies with species, economic conditions, and other factors. However, no matter what units are employed, specifications for a merchantable log should give the minimum length allowed, and minimum diameter allowed, and the minimum percent of sound material left after deductions are made for cull. For example, a cull log might be defined as any log less than 8 feet long, less than 6 inches in diameter, or less than 50 percent sound.

9-3.4 Sample Scaling

Under conditions where the scaling operation interferes with the movement of the logs, or where scaling costs are high, sample scaling should be considered. Sample scaling is generally feasible when: (1) logs are fairly homogeneous in species, volume, and value, (2) logs are concentrated in one place so they can be scaled efficiently, and (3) total number of logs is large.

Once one has decided to use sample scaling, one is faced with two basic questions: How many logs must be scaled to determine the total scale within limits of accuracy acceptable to both buyer and seller? How should the sample logs be selected?

The number of logs n to measure can be calculated from the formula applicable to a finite population.

$$n = \frac{CV^2 t^2 N}{Na^2 + CV^2 t^2} \quad (9-10)$$

where

- CV = coefficient of variation expressed as a percent
- t = t -value corresponding to chosen probability
- N = total number of logs in population
- a = desired standard error of mean expressed as a percent of mean

An example will illustrate the use of equation 9-10. Let us assume CV is 50 percent for a population N of 10,000 logs, and the desired standard error is 3 percent (both CV and N are estimates). Then if we let t be 2, giving approximately 20 to 1 odds that a chance discrepancy between the estimated and true scale will not exceed 3 percent, we obtain

$$n = \frac{50^2(2^2)(10,000)}{10,000(3^2) + 50^2(2^2)} = 1000$$

A practical procedure to obtain the 1000-log sample would be to scale every tenth log—that is, take a systematic sample. Of course, to obtain total volume, every log must be counted since the total number of logs N used to calculate sample size n is an estimate.

Although random sampling is required if one desires to calculate valid sampling errors, it is not essential if the sole purpose of sampling is to obtain an unbiased estimate of the average volume per log and the total volume, for a given run of logs.

Johnson, Lowrie, and Gohlke (1971) describe how the 3P sample selection procedure can be applied to sample log scaling. This is a promising procedure that is probably more efficient in most situations than the conventional sample log scaling method described above. Indeed, even if the logs are not homogeneous in species, volume, and value, it can be used efficiently.

9-4 STACKED VOLUME

Stacked volume, which is discussed in Chapter 7, has traditionally been obtained for firewood, pulpwood, excelsior wood, charcoal wood, and other relatively low-value products that are assembled in stacks.

In scaling a stack of wood one first records the length—the average of measurements taken on both sides of the stack, to the nearest 0.1 foot. Then, stack height is obtained by averaging measurements taken at intervals of about 4 feet. The height, which is reduced about 1 inch per foot by some scalers to compensate for settling and shrinkage, is recorded to the nearest 0.1 foot. Finally, piece lengths are checked to see if they vary from the lengths specified in the sale or purchase contract (standard lengths for pulpwood cut in the United States are 4 feet, 5 feet, 5 feet 3 inches, and 8 feet 4 inches). If they do, the procedure given in the contract should be followed.

The volume in standard cords V_c of a stack of wood is calculated as follows.

$$V_c = \frac{L_s H_s L}{128} \quad (9-11)$$

where

L_s = stack length in feet
 H_s = stack height in feet
 L = stick length in feet

If stacks are piled on slopes, the length and height measurements should be taken at right angles to one another.

If the stacked volume is measured in cubic meters (1 cubic meter = 1 stere), stack length and height are measured in 2-centimeter classes, and piece lengths are checked as described above. Then, gross volume of stack in cubic meters V_m is

$$V_m = L \times H \times W \quad (9-12)$$

where

L = stack length in meters
 H = stack height in meters
 W = stick length in meters

Since the above procedure gives gross stacked volume, to obtain net volume deductions must be made for defective wood. The definitions of defects and the procedures of allowing for defects will vary from one organization to another. But in general deductions are made for *defective sticks* and *loose piling*.

Defective sticks include rotted sticks, burned sticks, undersized sticks, and peeled sticks with excessive bark adhering.

Loose piling may occur when knots have been improperly trimmed, when excessively crooked wood is present, and when sticks have been carelessly piled.

When making deductions for defective sticks, the scaler examines each stick in a pile and notes which sticks will not meet specifications. These sticks are then culled

by deducting the cubic space they occupy from the gross cubic space occupied by the pile—either a stick is acceptable or it is not acceptable. Deductions for loose piling are made by estimating the cubic space that would be occupied by sticks that could be included in the loose pile and subtracting this volume from the gross cubic space occupied by the pile.

The term *rough wood* is used to designate wood with bark in contrast to the term *peeled wood*, which refers to wood with bark removed. It should be made clear in a sales contract whether wood is to be measured *rough* or *peeled*. If the sale price is based on rough wood volume, then if peeled wood must be measured, volume must be increased 10 to 20 percent, depending on bark thickness (Section 8-3).

9-4.1 Determination of Solid Cubic Contents of Stacked Wood

It is often necessary to know the solid cubic contents (standard cord, long cord, stere, etc.) of wood that is stacked on the ground or on trucks. Although average converting factors, such as those given in Chapter 7, are often used, better factors are generally required. These can be determined by the following methods.

1. Direct Measurement. The cubic volume of individual sticks, or of groups of sticks, can be determined by displacement (Chapter 8). The cubic volume of individual sticks can also be computed by using Huber's, Smalian's, or Newton's formula. In any case, when the cubic space occupied by a pile is known, the ratio of solid cubic volume to total cubic volume can be calculated from the equation

$$f = \frac{\text{Solid cubic volume of pile}}{\text{Total cubic volume of pile}}$$

The factor f multiplied by the space occupied by a cord, or by an entire pile, will give the solid cubic volume of wood in the stack.

2. Photographic Methods. The factor f can be estimated from photographs of the ends of sticks in a pile. One sets up, or holds, the camera at a distance from the pile that will give a scale of about 1:30 (Chapter 13). The optical axis of the lens should be perpendicular to the side of the pile. Normally, only a portion of a stack or truck load is included in a single photograph. After a photograph is developed, a templet consisting of systematically spaced pinholes is placed over the photograph and perforated with a needle at each pinhole. Then the photograph is placed on a light table for counting (Fig. 9-1). (A transparent dot grid with systematically spaced dots may be used, but it is more difficult to count dots than pinholes.) The factor f is computed as follows.

$$f = 1 - \frac{(\text{Total dots in air spaces})}{(\text{Total dots in photograph})}$$

Although one photograph of a truck load will usually give an adequate sample, several photographs of large stacks may be required.

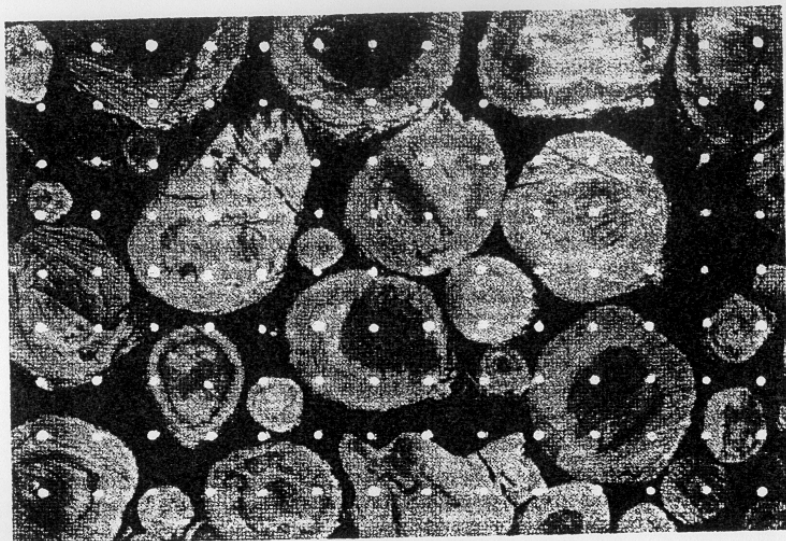


Fig. 9-1 Perforated polaroid photograph of portion of a truck load of low-quality hardwoods. Solid wood contents of this sample is 79 cubic feet per standard cord ($f = 0.62$). A 20 percent photo sample of each load is sufficient to give ± 2.4 percent accuracy at the 95 percent confidence level when a dot grid with 16 systematically spaced dots, or pinholes, per square inch is used (Garland, 1968).

Cameras with film dimensions of 2½ by 2½ inches to 4 by 5 inches are recommended, because contact prints prepared from these films may be used without magnification to obtain accurate dot counts. If 35-mm film is used, enlargement or magnification is required to obtain accurate dot counts.

Kallio, Lothner, and Marden (1973) tested another logical photographic method to obtain the solid cubic contents of stacked wood. They placed an 8-foot scale rule against the stack so scale could be determined, and took the photographs, as explained above, to obtain 35-mm colored slides. Then they projected the slides on a rear projection screen at about half actual size and measured each stick directly on the screen. They found that solid cubic volume can be determined more accurately by this method than by dot counting. The method, however, is slower.

In general, it is not feasible to determine bark volume by the photographic method, because the line between the inner bark and the wood is not always clear.

3. Angle-Gauge Method. By "projecting" an angle of about 23 degrees parallel to the face of a stack from randomly selected points, the conversion factor f may be

quickly and efficiently obtained. This method, which is a modification of horizontal point sampling, is discussed by Loetsch, Zöhrer, and Haller (1973).

9-5 ESTIMATION OF TREE VOLUMES

Tree volumes can be estimated from previously established relationships between certain tree dimensions and tree volume. Diameter, height, and form are the independent variables that are commonly used to determine the values of the dependent variable—tree volume. The final result is presented in formula or table form. The *volume formula* or *volume table*, then, gives the average contents of individual trees (in board feet, cubic feet, cubic meters, cords, or other units) in terms of one or more of the previously mentioned tree dimensions.

Local Volume Tables. Local volume tables give tree volume in terms of diameter at breast height only. The term *local* is used because such tables are generally restricted to the local area for which the height-diameter relationship hidden in the table is relevant. Although local volume tables may be prepared from raw field data—that is, from volume and diameter measurements for a sample of trees—they are normally derived from standard volume tables. Table 9-2 shows a typical local volume table.

Standard Volume Tables. Standard volume tables give volume in terms of diameter at breast height and merchantable or total height. Tables of this type may be prepared for individual species, or groups of species, and specific localities. The applicability of a standard volume table, however, depends on the form of the trees to which it is applied rather than on species or locality; for each diameter-height class the form of the trees to which the table is applied should agree with the form of the trees from which the table was prepared. Table 9-3 shows a typical standard volume table.

Form Class Volume Tables. Form class volume tables give volume in terms of diameter at breast height, merchantable or total height, and some measure of form, such as Girard form class or absolute form quotient. Such tables come in sets, with one table for each form class. The format of each table is similar to that of a standard volume table. Note that if a single form class table is chosen as representative of a stand, volume determinations may be in error because it is unlikely that all trees will be of the same form class. Furthermore, since form class varies with tree size, species, and site, it is unlikely that variations in form class will be random. Thus, it is difficult to obtain an accurate average form class for a stand and is therefore undesirable to use a single form class table for any extensive area.

9-5.1 Descriptive Information to Accompany Volume Tables

A volume table should include descriptive information that will enable one to apply it correctly. This information includes:

Table 9-2
Example of a Local Volume Table for Yellow Poplar
(Liriodendron tulipifera) in Stark County, Ohio, Using International Rule
 (1/4-Inch Kerf)—Merchantable Stem to a Variable Top Diameter

dbh Outside Bark (inches)	Volume per Tree (board feet)	Merchantable Length (feet)	Basis in Trees (number)
10	30	19.5	4
11	50	23	5
12	70	26.5	13
13	95	30	9
14	125	33	9
15	155	36.5	1
16	190	40	5
17	235	43	7
18	285	45.5	6
19	345	48	4
20	405	51	2
21	480	53.5	1
22	555	56	2
23	635	58	3
24	720	60	1
25	800	62	—
26	885	64	1
27	975	65.5	2
28	1065	67	1
29	1155	69	5
30	1245	70	9
31	1340	71.5	7
32	1435	72.5	7
33	1535	73.5	1
34	1630	74.5	1
35	1725	75	—
36	1825	76	—

Trees climbed and measured by personnel of Work Projects Administration Official Project 65-1-42-166—the Ohio Woodland Survey. Measurements taken at 16-foot log lengths above a 2.0-foot stump height. Scaled as 16-foot logs, and additional shorter top logs; top sections less than 8 feet in length scaled as fractions of an 8-foot log. Basis, 107 trees.

Table prepared, in 1939, by curving volume of merchantable length over dbh.

Aggregate difference: Table is 0.8% low. Average percentage deviation of basic data from table, 19.4%.

SOURCE: Diller and Kellog, 1940.

1. Species, or species group, to which the table is applicable, or the locality in which the table is applicable.
2. Definition of dependent variable, that is, volume, including unit in which volume is expressed.

Table 9-3
Example of a Standard Volume Table, Using Board-Foot Volume,
International 1/4-Inch Rule, for Red Oak (*Quercus rubra*) in Pennsylvania

dbh (inches)	Merchantable Height—Number of 16-Foot Logs											
	1/2	1	1 1/2	2	2 1/2	3	3 1/2	4	4 1/2	5	5 1/2	6
8	8	18	28	37	47	57						
9	11	23	35	48	60	73						
10	13	29	44	59	75	90						
11	17	35	54	72	91	109	105					
12	20	42	64	86	108	130	128	121				
13	24	50	76	102	127	153	153	175	197			
14	28	58	88	118	148	178	179	205	231			
15	33	67	102	136	170	205	208	238	268	298		
16	37	77	116	155	194	233	239	274	308	343	377	
17		87	131	175	219	264	273	312	351	390	429	469
18		97	147	197	246	296	308	352	396	441	485	529
19		109	164	219	275	330	345	395	445	494	544	594
20		121	182	243	304	366	385	440	496	551	606	662
21			201	268	336	403	427	488	549	611	672	733
22			221	295	369	443	471	538	606	673	741	809
23			241	322	403	484	517	591	665	739	813	888
24			263	351	439	527	565	646	727	808	889	970
25			285	381	477	572	616	704	792	880	968	1057
26			309	412	516	619	668	764	860	955	1051	1147
27				445	556	668	723	826	930	1033	1137	1240
28				478	598	718	780	891	1003	1114	1226	1338
29				513	642	771	839	959	1079	1199	1319	1439
30				549	687	825	900	1028	1157	1286	1415	1544
31					734	881	962	1101	1239	1376	1514	1652
32					782	939	1028	1175	1323	1470	1617	1764
33					832	999	1096	1253	1409	1566	1723	1880
34					883	1060	1165	1332	1499	1666	1833	1999
35					936	1124	1237	1414	1591	1768	1945	2122
36					990	1189	1311	1499	1686	1874	2062	2249
							1387	1586	1784	1983	2181	2380

Stump height, one foot. Top diameter, 8.0 inches, inside bark.

Block indicates extent of basic data. Basis, 210 trees.

Sample trees scaled as 16-foot logs; top section measured to nearest foot.

Standard error of regression coefficient = 0.00261.

Proportion of variation accounted for by the regression = 0.974.

Tabular values derived from regression $V = -1.84 + 0.01914D^H$.

SOURCE: Bartoo and Hutnik, 1962.

3. Definition of independent variables, including stump height and top diameter limit, if merchantable height is used.
4. Author.
5. Date of preparation.
6. Number of trees on which table is based.
7. Extent of basic data.
8. Method of determining volumes of individual trees.
9. Method of construction.
10. Appropriate measures of accuracy.

Table 9-2 and Table 9-3 include these items.

The first three items in the above list should always be given. The remaining items are of less interest and are sometimes omitted. When measures of accuracy are given, they should be understood to be measures of accuracy of the table when it is applied to the data used in its construction. Such measures give no assurance that a volume table will apply to other trees. Thus, when an accurate estimate is required, a table should be checked against the measured volumes of a representative sample of trees obtained from the stands to be estimated.

9-5.2 Checking Applicability of Volume Tables

In an applicability check, one should compare the volume of sample trees with the estimated volume from the volume table to be checked. Three conditions should be observed in selecting sample trees.

1. Sample trees for a given species, or species group, should be distributed through the timber to which the volume table will be applied.
2. No sizes, types, or growing conditions should be unduly represented in the sample.
3. If a sample of cut trees is used, this sample, if not representative of the timber, should be supplemented by a sample of standing trees. (Measurements on standing trees can be made by methods described in Chapter 2.)

Definite rules for measuring sample trees should be established. For example, the following rules are satisfactory for the eastern United States.

1. Diameters along the tree stem, inside and outside bark, should be taken at 8-foot intervals above a 1-foot stump, and at stump height, breast height, and merchantable height.
2. Diameter should be measured to nearest $\frac{1}{16}$ inch and bark thickness to nearest $\frac{1}{32}$ inch.
3. Knots, swellings, and other abnormalities should be avoided at points of measurement by taking measurements above or below them.

4. Total or merchantable heights should be measured to nearest foot. (Utilization standards for the timber in question should be considered in determining the upper limit of merchantable height.)

Table 9-4 illustrates how the comparison of measured and estimated volumes of sample trees should be made. For practical purposes, the aggregate difference of a test sample should not exceed $2CV/\sqrt{n}$, where CV is the coefficient of variation of the volume table being tested, and n is the number of trees used in the test. Since the coefficient of variation for the table tested in Table 9-4 is 15 percent, the table is applicable without correction because

$$\frac{2(15)}{\sqrt{62}} = 3.8\% > 1.0\%$$

If desired, checks may be made by diameter classes. And, of course, more complicated statistical tests, such as Chi square, might be used. The above procedure, however, is generally satisfactory.

When a table is judged to be inapplicable, one should adjust the table or obtain a better table. Practical methods of making adjustments are described by Gevorkiantz and Olsen (1955).

9-6 CONSTRUCTION OF VOLUME TABLES

The principles of volume table construction given by Cotta early in the nineteenth century are still valid.

Table 9-4
Comparison of Measured and Estimated Volumes of Sample of Red Oak

dbh Class (inches)	Sample Trees (number)	Measured Volume (board feet)	Estimated Volume ¹ (board feet)	Aggregate Difference (percent)
13.0-15.9	14	2,010	2,045	-1.7
16.0-18.9	10	2,003	1,943	+3.1
19.0-21.9	9	3,041	3,106	-2.1
22.0-24.9	21	9,257	8,895	+4.1
25.0-27.9	4	2,084	2,223	-6.3
28.0-30.9	3	2,130	2,110	+0.9
31.0-33.9	0	—	—	—
34.0-36.9	1	870	860	+1.2
All classes	62	21,395	21,182	+1.0

¹ From volume table.

SOURCE: Gevorkiantz and Olsen, 1955.

Tree volume is dependent upon diameter, height, and form. When the correct volume of a tree has been determined, it is valid for all other trees of the same diameter, height, and form.

Since the time of Cotta, hundreds of volume tables have been constructed and used. Numerous methods have been used to construct the tables. But since 1946 there has been a trend, particularly for hardwoods, to reduce the number of volume tables used by adopting composite volume tables, tables applicable to average timber, regardless of species. Indeed, where the same standards of utilization are employed, differences in tree volumes among species are often of no practical consequence. Excellent examples of composite volume tables are Beers' (1973) for hardwoods in Indiana, and Gevorkiantz and Olsen's (1955) for timber in the Lake States. These tables have been extensively tested and have been found to replace individual species tables, especially for the estimation of volume on large tracts. Adjustment factors can be used for individual species that vary from the average.

Why have so many volume tables been constructed? Why has so much research gone into the development of volume tables? The answer is that foresters have been looking for methods that are simple, objective, and accurate. However, because trees are highly variable geometric solids, no single table, or set of tables, could possibly satisfy all of these conditions, regardless of the method of construction. Consequently, one by one the older methods of volume table construction have been abandoned. For example, the once popular harmonized-curve method (Chapman and Meyer, 1949), which requires large amounts of data to establish the relationships and considerable judgment to fit the curves, is rarely used today. The alignment-chart method, another subjective method, has been generally discarded. Other discarded methods have been described by Spurr (1952). Today interest has focused on the use of mathematical functions, or models, to prepare volume tables. There is no advantage for the majority of foresters in using any other method.

9-6.1 Mathematical Models for Construction of Volume Tables

The following equations can serve as mathematical models for the construction of volume tables or as bases to develop other models.

$$\text{Local volume table: } V = aD^b \quad (9-13)$$

$$\text{Standard volume table: } V = bD^2H \quad (9-14)$$

$$V = a + bD^2H \quad (9-15)$$

$$V = aD^bH^c \quad (9-16)$$

$$\text{Form class volume table: } V = aD^bH^cF^d \quad (9-17)$$

where

- V = volume in cubic units or board feet
- D = dbh
- H = total or merchantable height
- F = a measure of form (Girard form class or absolute form quotient)
- a, b, c, d = constants

Equation 9-14 is called the *constant form factor volume equation*; equation 9-15 is known as the *combined variable volume equation*. However, both equations have combined variables. The nature of the equations becomes clear when they are rewritten: $V = bX$ and $V = a + bX$, where $X = D^2H$.

Both of these models are predicated on the assumption that, when volume is plotted over X (i.e., D^2H), the trend is linear. The constants a and b may be determined graphically, but the least squares solution is preferable. For equation 9-14 the line is forced through the origin; for equation 9-15 the Y intercept is computed. The theory and details of the calculations can be found in any standard text on regression.

Equation 9-16, which was proposed by Schumacher and Hall (1933), is given in its nonlinear form. Its logarithmic form, a form which has been utilized to fit nonlinear tree volume equations, is

$$\log V = \log a + b \log D + c \log H \quad (9-18)$$

A logarithmic equation is more compatible with the homogeneity of variance assumption for regression. On the other hand, a bias, called "logarithmic transposal discrepancy," is introduced in fitting the logarithmic equation and in recalculating the standard error in arithmetic units for comparison with nonlogarithmic equations. [This bias and its correction are discussed by Meyer (1953), Brownlee (1967), and Baskerville (1972).] By using nonlinear functions to estimate parameters, and by employing weighting methods to correct heterogeneous variance about the regression line, nonlinear tree volume equations may be developed that retain the statistical advantages and overcome the shortcomings of the logarithmic equation. In fact, with the availability of computers, there appears to be little justification for the use of logarithmic models, except to obtain initial estimates of the coefficients.

Moser and Beers (1969) give a method of utilizing equation 9-16 by nonlinear regression. Since their procedure is one of the most feasible methods of volume table construction, and since it is applicable to other nonlinear functions, it will be discussed in more detail than the other methods.

To fit sample data to equation 9-16, it is necessary to estimate values of the parameters that minimize

$$\sum_{i=1}^n e_i^2 = \sum_{i=1}^n (Y_i - \hat{a}D_i^bH_i^c)^2 \quad (9-19)$$

A rapid method of obtaining a convergent solution, which has been implemented in several standard statistical packages, is an algorithm developed by Marquardt (1963).

Table 9-5 (continued)
Tariff Table No. 24.5

dbh (inches)	Total Tree Volume						Volume to 6-Inch Top					
	Including Top and Stump (cubic feet)			Including Top Only (cubic feet)			Volume to 4-Inch Top (cubic feet)			Scribner (board feet)		
	Vol A	GM A	Vol B	GM B	Vol C	GM C	Vol C	GM C	Vol E	GM E	Vol F	GM F
2	0.3	0.2	0.2	0.2
3	0.7	0.7	0.6	0.6
4	1.5	1.0	1.4	1.0
5	2.6	1.4	2.5	1.3	1.4	1.5	1.4	1.5
6	4.1	1.7	4.0	1.6	3.0	1.8	3.0	1.8
7	5.9	2.0	5.7	2.0	4.9	2.1	4.9	2.1	6	7.2	9	10.7
8	8.1	2.4	7.8	2.3	7.1	2.4	7.1	2.4	14	9.8	21	13.9
9	10.5	2.7	10.2	2.6	9.6	2.7	9.6	2.7	25	11.9	36	16.2
10	13.3	3.0	12.9	2.9	12.3	3.0	12.3	3.0	38	13.7	53	18.0
dbh	V/BA Ratio			% of Vol A			V/BA Ratio			% of Vol B		
	V/BA Ratio			V/BA Ratio			V/BA Ratio			V/BA Ratio		
2	9.3	8.3	89.0
3	12.5	11.6	92.5
4	16.3	15.4	94.4
5	19.0	18.1	95.5	9.7	51.0
6	20.7	19.9	96.1	14.9	71.9
7	22.0	21.2	96.4	18.1	82.1	6.8	37.7	21.1	3.1	32.0	4.7	4.7
8	23.0	22.2	96.6	20.1	87.6	12.3	61.1	40.6	3.3	59.9	4.9	4.9
9	23.7	23.0	96.7	21.5	90.7	16.3	75.6	56.8	3.5	81.5	5.0	5.0
10	24.3	23.6	96.7	22.5	92.6	19.1	84.6	69.6	3.6	97.5	5.1	5.1

This tariff table gives volume in cubic feet for entire tree and volume in cubic and board feet to various merchantable limits. Volume/basal area ratios for horizontal point sampling and growth multipliers (GM) to determine growth are also given. The letters, A, B, C, etc., that follow Vol and GM are used for convenient identification of columns.

SOURCE: Turnbull and Hoyer, 1965.

Table 9-6
Average Distribution of Tree Volume by Logs According to Log Position

Usable Length (16-foot logs)	Percent of Total Tree Volume in Each Log, by Position					
	1st	2nd	3rd	4th	5th	6th
1	100					
2	58	42				
3	42	33	25			
4	34	29	22	15		
5	29	25	21	15	10	
6	24	23	20	16	11	6

SOURCE: Mesavage and Girard, 1946.

methods of expressing volume distribution. Although the percentages vary slightly with tree diameter and unit of volume, they may be used without serious error for merchantable trees of all sizes that are measured in cubic or board-foot volume. Note that Fig. 9-2, which is basically for 16-foot logs, provides a satisfactory guide when heights are measured in 8- or 12-foot lengths.

For the experienced timber estimator who can estimate the length deductions for various indicators, such as conks, cankers, and injuries, graphs, such as Fig. 9-2, provide convenient guides. For example, for a tree that contains three 16-foot logs, it is judged that 4 feet of the butt must be removed, and that an 8-foot section between 32 and 40 feet on the merchantable stem must be cut out. The cull, then, will be 12 percent (12 - 0) for the defective butt, and 13 percent (88 - 75) for the defective section, or 25 percent for the tree.

Since many estimators do not have the experience to estimate cull in this manner, they need a more systematic method. The following three methods are suggested.

1. Use indicator cull percentages developed by multiple regression analysis. The dependent variable is cull in percent of gross volume; independent variables are age, dbh, conks, cankers, basal injuries, and trunk injuries. (Variables such as conks, cankers, and injuries are given a value of 1 if one or more are present; 0 if none are present.)
2. Use tables of average length deductions for indicators such as conks, cankers, injuries, crooks, forks, and so forth, and apply these along with tables giving hidden defect percentages by dbh classes. (Tables of average length deductions for various indicators are normally used in conjunction with graphs, such as Fig. 9-2.)
3. Use cull classes that give, on the basis of visible defects, the percentage of deductions to apply to gross volume. For example, cull classes might be set up in the following manner for Central States Hardwoods.

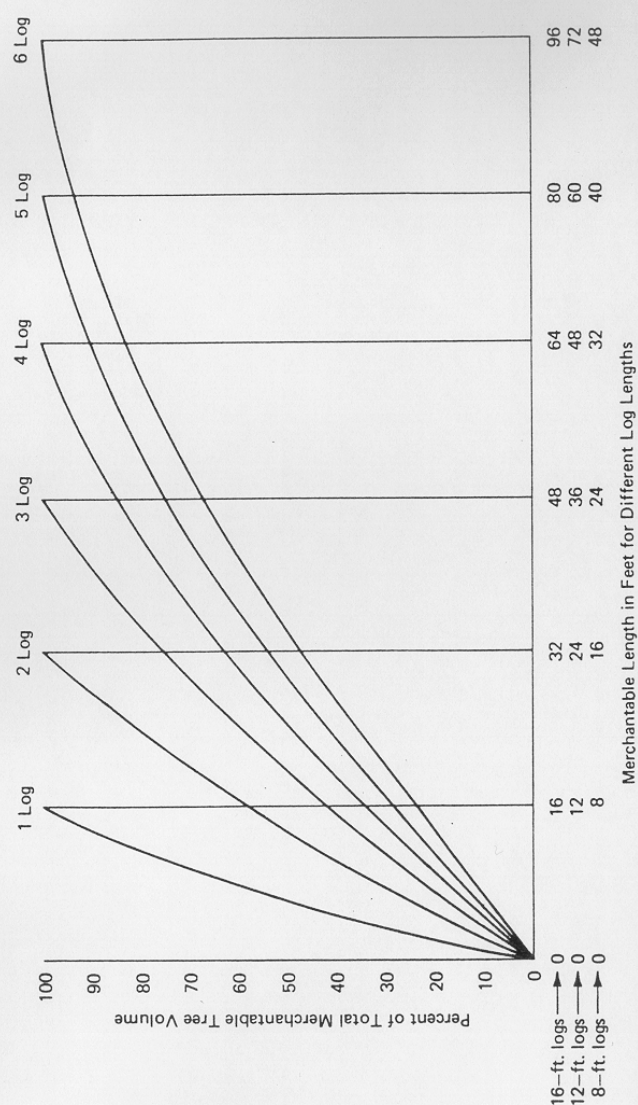


Fig. 9-2 Percentage of total merchantable volume at various heights for trees of different merchantable lengths (derived from Table 9-6).

Cull Class	Defect in Percent of Gross Board-Foot Volume (Scribner)	Maximum Number of Defects Permitted
1	5	Two minor defects confined to upper two-thirds of merchantable stem
2	15	One major defect in merchantable stem
3	25	Three major defects in entire merchantable stem, or 2 major defects in lower one-third of merchantable stem
4	40	Four major defects in entire merchantable stem, or 3 major defects in lower one-third of merchantable stem
5	100	Will not meet Class 4 specifications

To apply this table, one must have specifications for minor and major defects, and a set of curves to estimate major defect equivalents for defects, such as butt rot, in the lower one-third of the merchantable stem (Miller and Beers, 1981).

Aho and Roth (1978) used the first two methods to estimate cull in white fir. Miller and Beers (1981) used the third method to estimate cull in Central States Hardwoods. The authors feel the third method is the easiest to develop and the easiest to modify.

Whatever method is used, a cull percent applicable to a species, or species group, in a stand is normally determined by estimating the cull of a sample of 40 to 50 trees of the species, or species group. Then,

$$\text{Cull percent} = \left(\frac{\text{Cull volume of sample}}{\text{Gross volume of sample}} \right) 100 \quad (9-22)$$

This percentage is applied to all trees of the species, or species group, in the stand.