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Dynamic growth model for Scots pine (*Pinus sylvestris* L.) plantations in Galicia (north-western Spain)

Ulises Diéguez-Aranda[a,*], Fernando Castedo Dorado[b], Juan Gabriel Álvarez González[a], Alberto Rojo Alboreca[a]

[a] *Departamento de Ingeniería Agroforestal, Universidad de Santiago de Compostela, Escuela Politécnica Superior, Campus Universitario, 27002 Lugo, Spain*
[b] *Departamento de Ingeniería Agraria, Universidad de León, Escuela Superior y Técnica de Ingeniería Agraria, Avenida de Astorga, 24400 Ponferrada (León), Spain*

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Abstract

In this study we developed a dynamic growth model for Scots pine (*Pinus sylvestris* L.) plantations in Galicia (north-western Spain). The data used to develop the model were obtained from a network of permanent plots, of between 10 and 55-year-old, which the *Unidade de Xestión Forestal Sostible* (Sustainable Forest Management Unit) of the University of Santiago de Compostela has set up in pure plantations of this species of pine in its area of distribution in Galicia. In this model, the initial stand conditions at any point in time are defined by three state variables (number of trees per hectare, stand basal area and dominant height), and are used to estimate stand volume, classified by commercial classes, for a given projection age. The model uses three transition functions expressed as algebraic difference equations of the three corresponding state variables used to project the stand state at any point in time. In addition, the model incorporates a function for predicting initial stand basal area, which can be used to establish the starting point for the simulation. This alternative should only be used when the stand is not yet established or when no inventory data are available. Once the state variables are known for a specific moment, a distribution function is used to estimate the number of trees in each diameter class, by recovering the parameters of the Weibull function, using the moments of first and second order of the distribution (arithmetic mean diameter and variance, respectively). By using a generalized height–diameter function to estimate the height of the average tree in each diameter class, combined with a taper function that uses the above predicted diameter and height, it is then possible to estimate total or merchantable stand volume.

Keywords: Stand growth model; Scots pine; Algebraic difference equations; Weibull function

1. Introduction

Forest growth models predict growth of a target forest stand using site characteristics and management

* Corresponding author. Tel.: +34 982 252303; fax: +34 982 285926.
E-mail address: udieguez@lugo.usc.es (U. Diéguez-Aranda).

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options as input variables, and constitute important tools for decision-making in sustainable forest management. Most of these models are empirical and can be organized around three types representing a broad continuum of model classes: whole stand models, size class models and individual tree models (Davis et al., 2001).

The choice of the type of model to develop depends on both the purposes of its application and the resources available (Vanclay, 1994). These factors also determine the data needed and the resolution of the estimates. Although individual tree models provide more detailed estimates, the complexity and the amount of input data increase from whole stand models to individual tree models (Gadow and Hui, 1999; Davis et al., 2001). Among the three model types described, whole stand models are generally recommended for the management of forest plantations (García, 1988; Vanclay, 1994), because they represent a good compromise between generality and accuracy of the estimates. In addition, the large state vectors that individual tree models require are likely to contain mostly redundant information, with consequent losses in precision (overparametrization), as well as being costly or the values being impossible to obtain accurately from routine field sampling (García, 1994).

Nevertheless, some whole stand models provide rather limited information about the forest stand (in some cases only stand volume) (Vanclay, 1994; Porté and Bartelink, 2002). Considering that forest management decisions require more detailed information about stand structure and volume, as classified by merchantable products, whole single-species, even-aged stand models can be disaggregated mathematically using a diameter distribution function, which may be combined with a generalized height–diameter equation and with a taper function to estimate commercial volumes that depend on certain specified log dimensions. Similar methodologies have been used by Cao et al. (1982), Burk and Burkhart (1984), Knoebel et al. (1986), Uribe (1997), Río (1999), Kotze (2003), and Castedo Dorado (2004) in the development of forest growth models, mainly for plantations.

Generally, the first step in constructing a whole stand model for a given species is the use of data from an initial inventory of plots covering a wide range of ages, densities and sites in the area of study. These data allow the construction of static models, which may provide good results in unthinned or scarcely thinned stands (García, 2001). A second inventory of these plots provides information about the trajectories of the main stand variables over time. These data allow the construction of dynamic growth models, which predict rates of change, i.e. increments in any stand variable. Integration of these equations (or summation when using discrete time) provides a better description of the development of the stand over time than static models (García, 1988).

Several growth models for Scots pine (*Pinus sylvestris* L.) stands have been developed in Spain: García Abejón (1981), García Abejón and Gómez Loranca (1984), García Abejón and Tella (1986), Rojo and Montero (1996), and Bravo (1998) developed yield tables for the main ecoregions in the Spanish natural stands; Río and Montero (2001) developed a dynamic stand growth model for the *Central System* Mountains and the *Iberian System* Mountains, for both natural stands and plantations; and Palahí et al. (2002, 2003) developed a dynamic stand growth model and a distance-independent individual-tree growth model for natural stands in north-eastern Spain. Preliminary studies (Martínez Chamorro et al., 1997) showed that the growth pattern of the plantations of Scots pine in Galicia (north-western Spain) was significantly different from those corresponding to the rest of Spain, mainly because of dissimilar ecoregional conditions. Nevertheless, although the oldest Scots pine stands in Galicia were planted in the 1940s, no full studies of the growth and yield of this species have been carried out to date in this region. Therefore, taking into account that forest growth models are important tools for decision-making in sustainable forest management, the objective of this study is to develop a dynamic whole stand model for simulating the growth of Scots pine plantations in Galicia. Because the model is constituted by different interconnected modules, the following submodels were developed: a site quality system (for both dominant height growth and site index prediction), an equation for reduction in tree number, a stand basal area growth function, and a disaggregation system composed of a diameter distribution function, a generalized height–diameter relationship and a total and merchantable volume equation.

2. Material and methods

2.1. Data

The data used to develop the model were obtained from three different sources. Initially, in the winters of 1996 and 1997, a network of 155 plots was established in pure Scots pine plantations in Galicia. Most of these plantations were established using 2-year-old bare root planting stock, spaced 2 m apart. Early mortality was generally 5–10%, but no replacement of dead plants was made because of the high initial density. Little weeding was carried out and few pre-commercial thinnings were applied. The sites are of moderate fertility and soil depth, and in most cases at elevations above 800 m. Summer droughts are not common in these areas.

The plots were located throughout the area of distribution of this species in the area of study, and were subjectively selected to represent the existing range of ages, stand densities and sites. The plot size ranged from 625 to 1200 m^2, depending on stand density, in order to achieve a minimum of 60 trees per plot. We adopted this procedure because the plots were established for developing a whole stand model and an adequate number of trees is required to accurately estimate yield and growth. All the trees in each sample plot were labelled with a number. Two measurements of diameter at breast height (1.3 m above ground level) at right angles were made using callipers, and the arithmetic mean of the two measurements calculated. Total height was measured in a 30-tree randomized sample and in an additional sample including the dominant trees (the proportion of the 100 thickest trees per hectare, depending on plot size). Descriptive variables of each tree were also recorded, e.g. if they were alive or dead.

After examination of the data for evidence of plots installed in poor-extreme site conditions and plots in which very intensive silvicultural treatments were carried out, a subset of 68 of the initially established plots was re-measured in the winter of 2003. These plots were selected for the dynamic components of the model. The interval between the measurements was considered sufficient to absorb the short-term effects of abnormal climatic extremes. In most cases, an interval of 5 years is appropriate (Gadow and Hui, 1999).

The first two sources of data were the two inventories carried out in 1996–1997 and 2003. The stand variables calculated for each inventory were: age (A, in years), dominant height (H, in meters) defined as the mean height of the 100 thickest trees per hectare, stand basal area (B, in m^2 ha^{-1}), number of trees per hectare (N), and relative spacing index in percent (RS). The latter variable was obtained for each plot as $RS = 10,000/(H\sqrt{N})$ (Wilson, 1946).

Mean, maximum, minimum and standard deviation for each of the main stand variables (for both inventories) are shown in Table 1.

Apart from the two inventories, two dominant trees were destructively sampled at 114 locations (where cutting was allowed) in the winters of 1996 and 1997. These trees were selected as the first two dominant trees found outside the plots but in the same plantations within ±5% of the mean diameter at 1.3 m above ground level and mean height of the dominant trees. The trees were felled leaving stumps of average height 0.11 m; total bole length was measured to the nearest 0.01 m. The logs were cut at 2–2.5 m intervals along the first 4–5 m of bole length and at 1 m intervals thereafter. The number of rings was counted at each cross-sectioned point, and then converted to age above stump height. As cross-section lengths do not coincide with periodic height growth, we adjusted height-age data from stem analysis to account for this bias using Carmean's method (1972), and the modification proposed by Newberry (1991) for the topmost

Table 1
Summarised data corresponding to the plots from the first and second inventories used for model development

Variable	1st inventory (155 plots)				2nd inventory (68 plots)			
	Mean	Maximum	Minimum	S.D.	Mean	Maximum	Minimum	S.D.
Stand age (years)	32.7	48	12	7.8	39.7	55	21	7.8
Dominant height (m)	12.1	22.6	4.0	4.3	15.5	24.0	9.0	3.9
Stand basal area (m^2 ha^{-1})	34.3	74.2	4.2	14.4	44.6	72.6	16.2	11.3
Stems per hectare	1433	2720	600	425.5	1247	2112	580	340.0
Relative spacing index (%)	26.1	80.7	12.2	12.1	20.0	42.3	11.0	5.6

Table 2
Summary statistics of the sample of 228 trees used for model development

Variable	Mean	Maximum	Minimum	S.D.
No. logs	10.7	21	6	2.9
d (cm)	22.8	36.5	8.7	5.7
h (m)	10.8	20.7	4.2	3.9
h_{st} (m)	0.11	0.21	0.03	0.03
v (m^3)	0.246	0.815	0.016	0.178
Age (years)	33.0	50.0	12.0	7.7

d: diameter at breast height over bark (cm); h: total tree height (m); h_{st}: stump height (m); v: total tree volume over bark (m^3) above stump level.

section of the tree. A test of six methods of estimating true heights from stem analysis data (Dyer and Bailey, 1987) showed that Carmean's algorithm provided the most accurate estimates. Log volumes (stem parts with merchantable size) were calculated by Smalian's formula. The top of the tree was considered as a cone. Tree volume above stump height was aggregated from the corresponding log volumes and the volume of the top of the tree. The third source of data corresponds to the 228 trees felled. Summary statistics, including number of logs, mean, minimum, maximum and standard deviation of each of the main tree variables, are shown in Table 2.

2.2. Model structure

This section describes the simulation of growth and dynamics of Scots pine plantations using the proposed model. The basic structure of the dynamic stand growth model is shown in Fig. 1. The model is based on the state-space approach (García, 1994), which is adequate for systems that evolve in time, as takes place in forest systems, and makes use of state variables and transition functions. The stand conditions at any point in time are defined by three state variables (dominant height, number of trees per hectare and stand basal area), and are used to estimate stand volume, classified by commercial classes. The model uses three transition functions expressed as algebraic difference equations of the three corresponding state variables, which are used to project the future stand state. Once the state variables are known for a given time, the model is disaggregated mathematically using a diameter distribution function, which is combined with a generalized height–diameter equation and with a taper function to estimate total and merchantable stand volumes. Stand growth is subsequently calculated by subtraction.

The state of a system at any given time may be roughly defined as the information needed to determine the behaviour of the system from that time on; given the present state the future does not depend on the past. In other words, it is assumed that two stands with the same values for these variables would behave in practically the same way, no matter how they happened to reach that state (García, 1994). Therefore, the first question is how to characterize the state of the system at any point in time. The state must be such that, to a sufficient degree of approximation, future states are determined by the current state and future actions, and any quantities of interest can be estimated from the state variables (García, 1994). In selecting the state variables, the principle of parsimony must be taken into account (Burkhart, 2003); the model should be the sim-

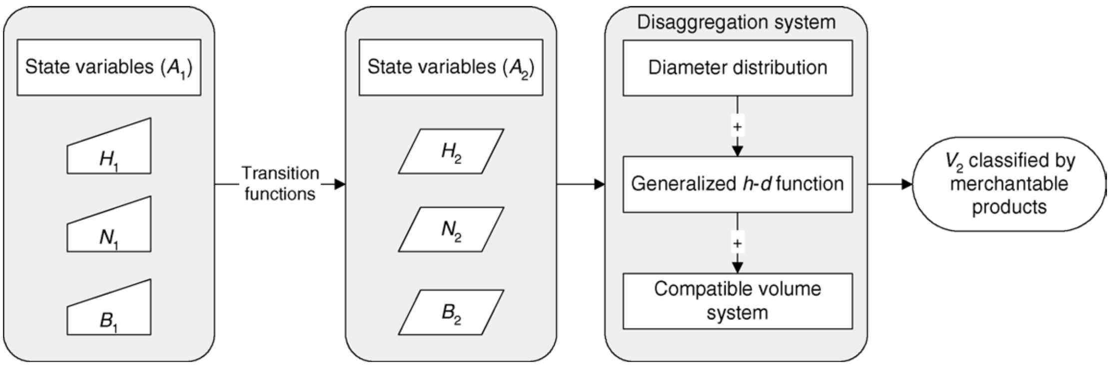

Fig. 1. Basic structure of the whole stand model. H_1, H_2, N_1, N_2, B_1, B_2 = dominant height, number of trees per hectare and stand basal area at ages A_1 and A_2, respectively.

plest one that describes the biological phenomena and remains consistent with the structure and function of the actual biological system (Milsum, 1966). A two dimensional vector including dominant height (*H*) or age (*A*) and stand basal area (*B*) as explanatory variables may be sufficient to describe the state of the stand at a given time (Pienaar and Turnbull, 1973), so that the advantage of including an additional variable may not be justified (Decourt, 1974; García, 1988). Nevertheless, in situations covering a wide range of silvicultural regimes, the inclusion of an additional variable such as the number of trees (*N*) is necessary (García, 1994; Amateis et al., 1995; Álvarez González et al., 1999). After thinning or pruning, it may take some time for trees to fully occupy the additional space that has been made available to them. It might be expected that, immediately after the intervention, a stand would grow less than another not recently treated stand with the same *H*, *B* and *N*. To account for this, a fourth variable representing relative site occupancy or canopy closure has been found to be useful in some instances, especially where very heavy thinning and pruning take place (García, 1990).

The transition functions must possess some obvious properties: (i) consistency, meaning no change for zero elapsed time; (ii) path-invariance, where the result of projecting the state first from A_0 to A_1, and then from A_1 to A_2, must be the same as that of the one-step projection from A_0 to A_2; (iii) causality, in that a change in the state can only be influenced by inputs within the relevant time interval (García, 1994). Transition functions generated by integration of differential equations (or summation of difference equations when using discrete time) automatically satisfy these conditions. To fit these functions, it is necessary to make use of data from plots installed in stands from different sites and of different ages, and densities, measured at least twice; hence, interval or permanent plots are necessary (Gadow and Hui, 1999).

The following sections describe how we developed each of the three transition functions and the disaggregation system.

2.3. Transition function for dominant height growth

The difference equation method (Clutter et al., 1983) was used to develop a site quality system, in which both dominant height growth and site index (*S*, defined as the dominant height of the trees at a specific base age) models are defined as special cases of the same equation. Although true site productivity may not be fully represented by site index, it is the most widely accepted and probably the simplest method for estimating site productivity. Two algebraic difference equations derived from the Sloboda (1971) and McDill and Amateis (1992) differential functions, and five algebraic difference forms derived from the Bertalanffy–Richards (Bertalanffy, 1949, 1957; Richards, 1959), Korf (cited in Lundqvist, 1957) and Hossfeld (cited in Peschel, 1938) growth functions were evaluated to model dominant height growth. These algebraic difference equations are base-age invariant and polymorphic, and one of these has multiple asymptotes. All of the models have been widely used to develop height/age curves (e.g. García, 1983; Cieszewski and Bella, 1989; Elfving and Kiviste, 1997; Trincado et al., 2002; Calama et al., 2003).

Data from stem analysis and dominant height of the sample plots measured twice were combined and used for modelling. Several data structures can be used to fit a model expressed in algebraic difference form. All possible growth intervals (Borders et al., 1988) typically produce fitted models with a better predictive performance as compared to, for example, forward moving first differences (Goelz and Burk, 1992; Huang, 1999). Therefore, this structure was selected for developing the site quality system. To correct the inherent autocorrelation of the longitudinal data and structure used, autocorrelation was modelled by expanding the error term using the proposal of Goelz and Burk (1992, 1996):

$$H_{ij} = f(H_j, A_i, A_j, \beta) + e_{ij} \quad \text{with}$$
$$e_{ij} = \rho e_{i-1,j} + \gamma e_{i,j-1} + \varepsilon_{ij} \qquad (1)$$

where H_{ij} depicts prediction of height i by using H_j (height j), A_i (age i), and A_j (age $j \neq i$) as predictor variables; β is the vector of parameters to be estimated; e_{ij} is the corresponding error term; the ρ parameter accounts for the autocorrelation between the current residual and the residual from estimating H_{i-1} using H_j as a predictor; the γ parameter accounts for the autocorrelation between the current residual and the residual from estimating H_i using H_{j-1} as a predictor; and ε_{ij} are independent and identically distributed errors. In using all possible differences, the number of observations is artificially inflated and the corresponding standard errors for the parameters are therefore too small. Thus, the standard errors were corrected by $\sqrt{n(\text{apd})/n(\text{fd})}$, where n(apd) is the number of observations using all possible differences and n(fd) is the number of observations if using only first differences (Goelz and Burk, 1996). This provided consistent estimates of the parameters and their standard errors. Fitting was carried out by modelling the mean and the error structure simultaneously using the SAS/ETS® MODEL procedure (SAS Institute Inc., 2000a), which allows for dynamic updating of residuals.

2.4. *Transition function for reduction in tree number*

We developed an equation for predicting the reduction in tree number due to density-dependent mortality, or self-thinning, which is mainly caused by competition for light, water and soil nutrients within a stand (Peet and Christensen, 1987). Density-dependent mortality reflects lowered vigour and decreased growth rate, and is at least partially predictable (Murty and McMurtrie, 2000). This equation is especially important if only low intensity thinnings are carried out, as was the case of the studied stands, which were mostly located in unthinned stands or stands thinned lightly from below (A- or B-grade of severity—Smith et al., 1997).

According to Clutter et al. (1983), most mortality analyses are based on the following variables obtained from re-measurement data: age and number of trees per hectare at the beginning and at the end of the period involved, and sometimes site quality represented by the site index value. Therefore, the model was constructed using data from the 68 plots measured twice.

Fourteen biologically based algebraic difference equations derived from the following four differential functions were used for model development:

$$\frac{\mathrm{d}N/\mathrm{d}A}{N} = K \tag{2}$$

$$\frac{\mathrm{d}N/\mathrm{d}A}{N} = \alpha N^{\beta} f(S) A^{\delta} \tag{3}$$

$$\frac{\mathrm{d}N/\mathrm{d}A}{N} = \alpha N^{\beta} \left(f(S) + \frac{\delta}{A} \right) \tag{4}$$

$$\frac{\mathrm{d}N/\mathrm{d}A}{N} = \alpha N^{\beta} f(S) \delta^{A} \tag{5}$$

where K is a constant, S site index, and α, β, and δ the model parameters.

Eq. (2) is the simplest assumption and considers the instantaneous mortality rate as a constant quantity. This approach was used by Tomé et al. (1997) and is appropriate for use in populations where the proportional mortality rate is constant for all ages, site indices and stand densities (Clutter et al., 1983).

On the other hand, Eqs. (3)–(5) consider the mortality rate related to number of trees per hectare, age and site index (Clutter and Jones, 1980; Pienaar and Shiver, 1981; Bailey et al., 1985; Da Silva—cited in Van Laar and Akça, 1997; Pienaar et al., 1990; Zunino and Ferrando, 1997; Woollons, 1998; Álvarez González et al., 2004). These equations differ only in the form in which the effect of age is included. The effect of site index was incorporated in the general form:

$$f(S) = c_0 + c_1 S^{c_2} \tag{6}$$

Parameter estimates were obtained by ordinary least squares using the Gauss–Newton's iterative method (Hartley, 1961) of the SAS/STAT® NLIN procedure (SAS Institute Inc., 2000b).

2.5. *Transition function for stand basal area growth*

Stand basal area growth was also modelled using algebraic difference equations. In this method, initial B and A are assumed to provide sufficient information about the growth trajectory of the stand, although the inclusion of other variables in the equation (e.g. site index) is also possible and sometimes necessary. Thus, the use of an algebraic difference equation to project stand basal area over time requires the knowledge of the initial B at age A, which may generally be calculated directly on the basis of inventory data. Otherwise, the previous use of an initialization equation to estimate the initial B is required.

Therefore, the stand basal area growth system is composed by two sub-modules: one for stand basal area initialization and another for projection. The integral form of growth functions was used to develop the initialization sub-module, while the stand basal area projection sub-module was fitted using algebraic difference equations.

The initialization sub-module was fitted using data from 223 plots (155 from the first inventory and 68 from the second one). The projection equation was fitted using pairs of data from the 68 plots measured twice. Compatibility between both sub-modules was ensured (i) by deriving the algebraic difference equations from the growth functions or vice versa; and (ii) by estimating independently the projection function parameters, substituting their values into the initialization function, and then fitting it to obtain the estimates of the remaining parameters. This form of proceeding gives priority to the projection function, and it was selected because it was considered that the model projection would be most frequently used to estimate stand basal area from any given initial stand conditions obtained from a plantation inventory.

Eight algebraic difference equations obtained from the available literature (Clutter, 1963; Bennett, 1970; Borders and Bailey, 1986; Pienaar and Shiver, 1986; Souter, 1986; Pienaar et al., 1990; Hui and Gadow, 1993; Forss, 1994) and their corresponding integral forms, and six algebraic difference equations derived from the integral forms of the Bertalanffy–Richards (Bertalanffy, 1949, 1957; Richards, 1959) and Korf (cited in Lundqvist, 1957) functions were used for model development. Taking into account the actual relationship between stand basal area and site quality, site index was included as an explanatory variable accompanying the solved parameter of the growth functions of Bertalanffy–Richards and Korf, in order to improve the predictions of the corresponding initialization equations.

Fitting of both sub-modules was carried out using the SAS/STAT® NLIN procedure (SAS Institute Inc., 2000b) over the logarithmic transformation of the equations to avoid problems associated with the estimation of standard errors of the coefficients due to heteroscedasticity.

2.6. Disaggregation system

2.6.1. Diameter distribution

Many parametric density functions have been used to describe the diameter distribution of a stand (e.g. Charlier, Normal, Beta, Gamma, Johnson S_B, Weibull). Among these, the Weibull function has been the most frequently used in forest growth models because of its flexibility and simplicity (Maltamo et al., 1995; Río, 1999; Kangas and Maltamo, 2000; Torres-Rojo et al., 2000). Expression of the Weibull density function is as follows:

$$f(x) = \left(\frac{c}{b}\right) \left(\frac{x-a}{b}\right)^{c-1} \mathrm{e}^{-((x-a)/b)^c} \tag{7}$$

where x is the random variable, a the location parameter which defines the origin of the function, b the scale parameter, and c the shape parameter that controls the skewness.

The Weibull parameters can be obtained using different methodologies, which can be classified in two groups: parameter estimation and parameter recovery (Hyink, 1980; Vanclay, 1994). Several studies (Cao et al., 1982; Reynolds et al., 1988; Borders and Patterson, 1990; Torres-Rojo et al., 2000) have found that the parameter recovery approach provides better results than parameter estimation, even in long-term projections.

The parameter recovery approach relates stand variables, generally stand basal area, dominant height and number of trees, to percentiles (Cao and Burkhart, 1984) or moments (Newby, 1980; Burk and Newberry, 1984) of the diameter distribution, which are subsequently used to recover the Weibull parameters. Nevertheless, the moment method is the only one which directly warrants that the sum of the disaggregated basal area obtained by the Weibull function equals the stand basal area provided by an explicit growth function of this variable, resulting in numeric compatibility (Hyink, 1980; Knoebel et al., 1986; Kangas and Maltamo, 2000; Torres-Rojo et al., 2000). It was therefore the method selected for this study.

In the moment method, the parameters of the Weibull function are recovered from the first three order moments of the diameter distribution (i.e. the mean, variance and skewness coefficient, respectively). Alternatively, the location parameter (a) may be set to zero. The use of this condition restricts the parameters of the Weibull function to two, thus making it easier to model, and providing similar results to the three-parameter Weibull (Maltamo et al., 1995; Álvarez González and Ruiz, 1998). Thus, to recover parameters b and c the following expressions were used:

$$\mathrm{var} = \frac{\bar{d}^2}{\Gamma^2\left(1+\frac{1}{c}\right)}\left(\Gamma\left(1+\frac{2}{c}\right)-\Gamma^2\left(1+\frac{1}{c}\right)\right) \quad (8)$$

$$b = \frac{\bar{d}}{\Gamma\left(1+\frac{1}{c}\right)} \quad (9)$$

where $\bar{d}$ is the arithmetic mean diameter of the observed distribution, var its variance, and Γ the Gamma function.

Once the mean and the variance of the diameter distribution are known, and taking into account that Eq. (8) only depends on parameter c, the latter can be obtained using iterative procedures. Parameter b can then be calculated directly from Eq. (9). Considering that the disaggregation system is developed for inclusion in a stand growth model, only the arithmetic mean diameter requires to be modelled, because the variance can be directly obtained from the arithmetic and the quadratic mean diameter (d_g) using the equations in Fig. 2.

Thus, the arithmetic mean diameter was modelled using the following expression (Frazier, 1981):

$$\bar{d} = d_g - e^{\mathbf{X}\beta} \quad (10)$$

where $\mathbf{X}$ is a vector of explanatory variables (e.g. dominant height, number of trees per hectare, age) that characterize the state of the stand at a specific time and must be obtained from any of the functions of the stand growth model, and β is a vector of parameters to be estimated. This procedure insures that predictions of average diameter ($\bar{d}$) are always lower than quadratic mean diameter (d_g), and has been widely used in diameter distribution modelling using the parameter recovery approach (Cao et al., 1982; Burk and Burkhart, 1984; Knoebel et al., 1986).

The diagram of the proposed disaggregation system is shown in Fig. 2.

2.6.2. Height estimation for diameter classes

Once the diameter distribution is known, it is necessary to estimate the height of the average tree in each diameter class. A local height–diameter relationship may be used for this. Nevertheless, because of the heterogeneity of site conditions and silvicultural state of the stands, a single height–diameter relationship cannot well represent all situations. Moreover, the height–diameter curve of an even-aged stand varies with age (Curtis, 1967; Assmann, 1970; Speidel, 1983; Lappi, 1997); that is, trees of different ages but with similar diameters belong to different sociological classes.

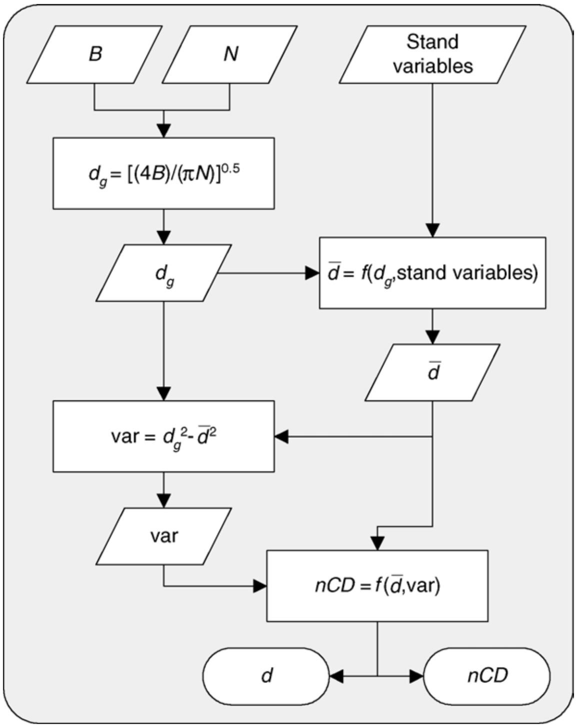

Fig. 2. Diagram of the disaggregation system. B = stand basal area; N = number of trees per hectare; d_g = quadratic mean diameter; $\bar{d}$ = arithmetic mean diameter; var = diameter variance; nCD = number of trees per hectare in the diameter class; d = diameter at breast height of the average tree in the diameter class.

A practical alternative is the use of a generalized height–diameter relationship, which predicts the height of a tree as a function of its diameter at breast height and one or more stand variables (e.g. dominant height, quadratic mean diameter, dominant diameter, number of trees per hectare, stand basal area) which take into account some basic characteristics common to all the local height regressions that represent each individual stand (Gadow and Hui, 1999; Staudhammer and LeMay, 2000).

Nine generalized height–diameter relationships that predict the dominant height of the stand (H) when the diameter at breast height of the subject tree (d) equals the dominant diameter of the stand (d_0) were used for model development (Mønness, 1982; Gaffrey, 1988; Tomé, 1988; Cañadas et al., 1999; Nilson, 1999). The equations of Gaffrey (1988) and Nilson (1999) were modified to ensure the above-mentioned compatibility. These equations include exponential terms in the form $1/d - 1/d_0$, $1 - d/d_0$ or $(1 - e^{b_0 d})/(1 - e^{b_0 d_0})$, or multiplicative terms such as d/d_0.

Parameter estimates were obtained by ordinary least squares using the Gauss–Newton's iterative method (Hartley, 1961) of the SAS/STAT® NLIN procedure (SAS Institute Inc., 2000b).

Practical use of the generalized height–diameter equation requires estimation of dominant diameter, which is difficult to project (Lappi, 1997). Therefore, it must be estimated from the diameter distribution. The remaining explanatory variables can be easily obtained at any point in time from dominant height, number of trees per hectare and stand basal area transition functions.

2.6.3. *Total and merchantable volume estimation*

Once the diameter and height of the average tree in each diameter class are estimated, the total tree volume can be calculated directly using a volume equation. If required, a volume ratio equation may be used to estimate the volume to a certain height or diameter as a percentage of total volume (Burkhart, 1977; Cao et al., 1980; Clutter, 1980; Reed and Green, 1984). Alternatively, merchantable volume from the stump to some fixed top diameter limit or bole length may be estimated from a taper equation, by integrating it over a specified height of the tree. The latter is the most commonly used approach (Kozak, 1988; Newnham, 1992; Riemer et al., 1995; Castedo Dorado and Álvarez González, 2000; Bi, 2000; Novo et al., 2003).

A volume equation and a taper function are numerically consistent if the total volume of a tree calculated using the volume equation is the same as the volume calculated by integrating the taper equation over the total height of the tree. In this case, the equations constitute a compatible system (Demaerschalk, 1972; Clutter, 1980; Rustagi and Loveless, 1991). Sometimes, the volume equation can be replaced by a merchantable volume equation, which provides the total volume (v) when the total height (h) is used as an input variable. An up-to-date review of compatible systems is provided by Diéguez-Aranda (2004). The advantage of compatible systems is that they allow the use of a very simple and common volume equation (e.g. $v = a_0 d^{a_1} h^{a_2}$, $v = a_0 + a_1 d^2 h$) if only total volume is required, thereby simplifying the calculations.

Data on diameter at different heights and total stem volume from 228 destructively sampled trees were used for fitting a compatible system. To correct the inherent autocorrelation of the longitudinal data used, and taking into account that observations within a tree were not equally distributed, the error term was expanded using an autoregressive continuous time model CAR(x) (Zimmerman and Núñez-Antón, 2001). The main purpose in using this error structure was to obtain consistent estimates of the parameters and their standard errors.

Fifteen equations were tested for comparison (Diéguez-Aranda, 2004). The fittings were carried out by simultaneously optimizing total tree volume, diameter at different heights and the error structure using the SAS/ETS® MODEL procedure (SAS Institute Inc., 2000a).

Aggregation of total (v) or merchantable (v_i) tree volume times number of trees in each diameter class provides total (V_{total}) or merchantable stand volume (V_{clasif}), respectively.

The diagram of the total and merchantable volume estimation system is shown in Fig. 3.

2.7. *Selection of the best equation in each module*

The comparison of the estimates of the different models fitted in each module was based on numerical and graphical analyses. Two statistical criteria obtained from the residuals were examined: the coefficient of determination for nonlinear models (R^2) (see Ryan, 1997, pp. 419 and 424), which shows the proportion of the total variance of the dependent variable that is explained by the model; and the root mean square error (*RMSE*), which analyses the accuracy of the estimates. Although several shortcomings have been stated against the use of the R^2 in nonlinear regression (Neter et al., 1996), the general usefulness of some global measure of model adequacy would seem to override some of those limitations (Ryan, 1997).

The importance of validation to those who construct and use models is well recognized (Gentil and

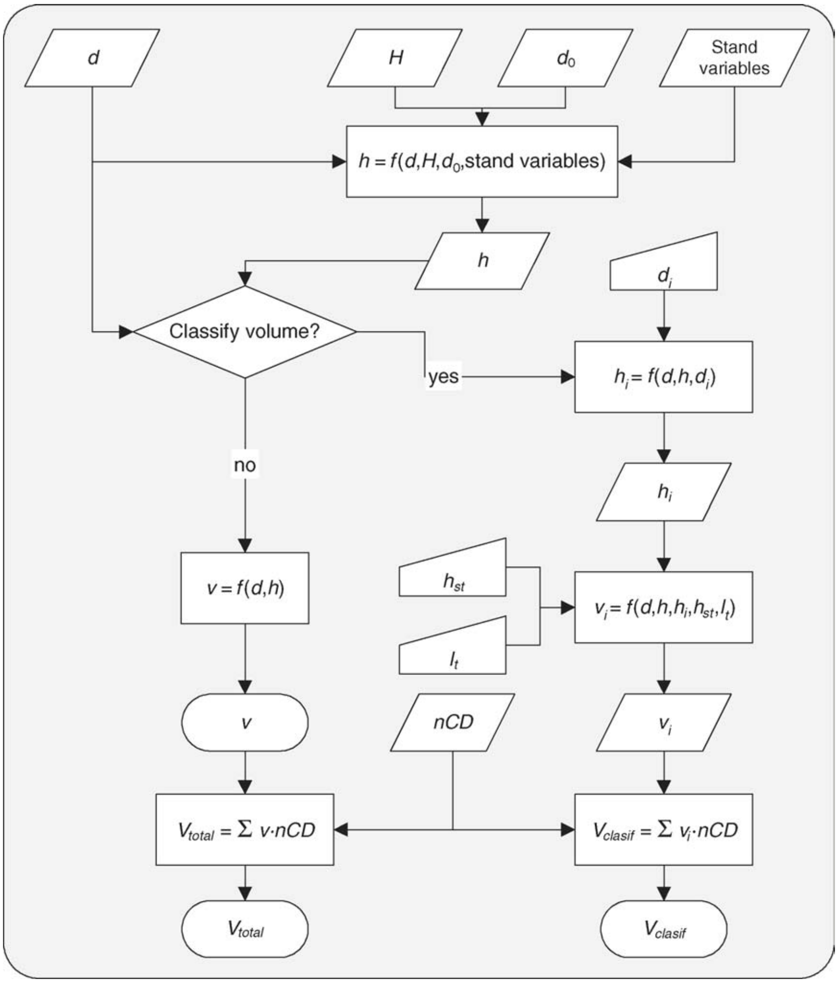

Fig. 3. Diagram of the total and merchantable volume estimation system. d = diameter at breast height of the average tree in the diameter class; H = dominant height; d_0 = dominant diameter; h = total tree height; d_i = top diameter at height h_i; h_{st} = stump height; h_i = height above ground level to top diameter d_i; l_t = log length; v = total tree volume above stump level; v_i = merchantable tree volume, the volume above stump level to a specified top diameter d_i; nCD = number of trees per hectare in the diameter class; V_{total} = total stand volume; V_{clasif} = merchantable stand volume.

Blake, 1981; Reynolds, 1984; Mayer and Butler, 1993; Rykiel, 1996; Vanclay and Skovsgaard, 1997; Huang et al., 2003). Therefore, the validation of each module was based on the analysis of the same statistical criteria but the residual of each observation (tree or plot, depending on the data set) was obtained by refitting the model without that observation. This technique, generally referred to as n-way cross-validation, is not really a true validation using an independent data set, but it is often useful as an additional criterion for selecting the best model.

Apart from these statistics, one of the most efficient ways of ascertaining the overall picture of model performance is by visual inspection. Therefore, graphical analyses, consisting of examination of plots of observed against predicted values of the dependent variable and of plots of studentized residuals against the estimated values, were carried out. These graphs are useful both for detection of possible systematic discrepancies and for selecting the weighting factor (Neter et al., 1996). Specific graphs suitable for each of the different components of the model were also examined.

2.8. Overall validation of the model

In addition to the validation of each component of the dynamic growth model, overall validation of the whole model was carried out. It must be taking into account that validation of a growth model is complicated because it consists of several submodels that may be estimated independently or simultaneously using different techniques (Huang et al., 2003).

The use of an independent data set is the preferred method of model validation (Pretzsch et al., 2002; Huang et al., 2003; Kozak and Kozak, 2003). As new data for model validation were not available in this study, observed state variables from the first inventory of the 68 plots measured twice were used to estimate stand basal area and total stand volume at the age of the second inventory, including all the components of the whole stand model. These variables were selected because (i) stand basal area is perhaps the most important variable in planning thinning operations, and (ii) estimation of stand volume involves all the functions included in the whole stand growth model and is closely related to economical assessments.

In order to evaluate whether the model performs acceptably well when used for prediction of stand basal area and stand volume, a critical error, expressed as a percentage of the observed mean, was computed rearranging Freese's (1960) χ_n^2 statistic (Reynolds, 1984; Robinson and Froese, 2004):

$$E_{crit.} = \frac{\sqrt{\tau^2 \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 / \chi^2_{crit.}}}{\bar{y}} \qquad (11)$$

where n is the total number of observations in the data set, y_i the observed value, $\hat{y}_i$ its prediction from the fitted model, $\bar{y}$ the average of the observed values, τ a standard normal deviate at the specified probability level ($\tau = 1.96$ for $\alpha = 0.05$), and $\chi^2_{crit.}$ is obtained for $\alpha = 0.05$ and n degrees of freedom. If the specified allowable error expressed as a percentage of the observed mean is within the limit of the critical error, the χ_n^2 test will indicate that the model does not give satisfactory predictions; otherwise, it will indicate that the predictions are acceptable.

Apart from this statistic, inspection of plots of observed against predicted values of the dependent variables stand basal area and stand volume was carried out.

3. Results and discussion

3.1. Transition function for dominant height growth

The algebraic difference form of the differential function proposed by McDill and Amateis (1992) resulted in the best compromise between biological behaviour and goodness-of-fit statistics, producing the most adequate site index curves (Fig. 4). Therefore, it was selected for height growth prediction and site classification:

$$H_2 = \frac{51.39}{1 - \left(1 - \frac{51.39}{H_1}\right)\left(\frac{A_1}{A_2}\right)^{1.277}} \qquad (12)$$

where H_1 and A_1 represent the predictor dominant height (m) and age (years), and H_2 the predicted dominant height at age A_2.

In selecting the base age, 40 years was best for predicting dominant height at other ages (Diéguez-Aranda et al., 2005a).

To estimate the dominant height (H_2) of a stand for some desired age (A_2), given site index (S) and its associated base age (A_b), S and A_b must be substituted into Eq. (12) for H_1 and A_1, respectively:

$$H_2 = \frac{51.39}{1 - \left(1 - \frac{51.39}{S}\right)\left(\frac{A_b}{A_2}\right)^{1.277}} \qquad (13)$$

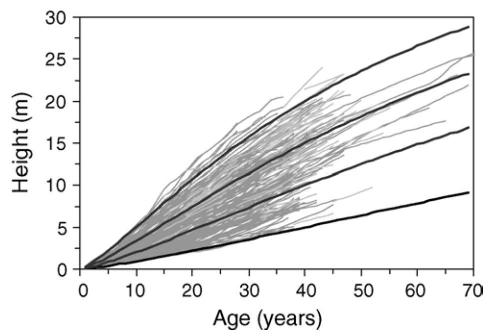

Fig. 4. Site index curves for heights of 5, 10, 15 and 20 m at 40 years overlaid on the trajectories of observed values over time. Same-tree or plot measurements joined by lines.

Similarly, to estimate site index at some chosen base age, given stand height and age, S and A_b must be substituted into Eq. (12) for H_2 and A_2, respectively:

$$S = \frac{51.39}{1 - \left(1 - \frac{51.39}{H_1}\right)\left(\frac{A_1}{A_b}\right)^{1.277}} \tag{14}$$

3.2. Transition function for reduction in tree number

The equation that provided the best results was (Diéguez-Aranda et al., 2005b):

$$N_2 = (N_1^{-1.5896} + 1.138 \times 10^{-9}(S/1000) \times ({A_2}^{3.3079} - {A_1}^{3.3079}))^{-1/1.5896} \tag{15}$$

where N_2 is the predicted number of trees per hectare at age A_2, N_1 the number of trees per hectare at age A_1, and S the site index (m) estimated using Eq. (14) for a reference age of 40 years.

This equation implies that the relative rate of change in the number of trees is proportional to a power function of age. Eq. (15) may be applicable in stands where the initial stand density was similar to those of the plots used in this study, and where no thinnings or only light thinnings have been carried out; otherwise, it is better not to consider reduction in tree number, especially after thinning operations. Fig. 5 shows the trajectories of observed and predicted number of trees over time for different initial densities.

3.3. Transition function for stand basal area growth

The following modification of the Korf (cited in Lundqvist, 1957) function was selected for stand basal area initialization:

$$B = 92.40\,\mathrm{e}^{-(1593/S)A^{-1.369}} \tag{16}$$

where B is the predicted stand basal area ($\mathrm{m^2\,ha^{-1}}$) at age A, and S the site index (m) estimated using Eq. (14) at a reference age of 40 years. Eq. (16) will work well in situations in which the silvicultural practices applied were similar to those carried out in the stands where the experimental data used were collected. It should only be used when no inventory data are available.

The corresponding function for stand basal area projection was:

$$B_2 = 92.40\left(\frac{B_1}{92.40}\right)^{(A_1/A_2)^{1.369}} \tag{17}$$

where B_2 is the predicted stand basal area ($\mathrm{m^2\,ha^{-1}}$) at a given projection age A_2, and B_1 the stand basal area ($\mathrm{m^2\,ha^{-1}}$) at age A_1, which are used as initial values of the projection function expressed in algebraic difference form. These variables provided enough information about the growth trajectory of stand basal area. Similar results were obtained by Bennett (1970), Clutter and Jones (1980), Bailey and Ware (1983), and Murphy and Farrar (1988). Stand basal area predictions overlaid on the trajectories of observed values over time are shown in Fig. 6.

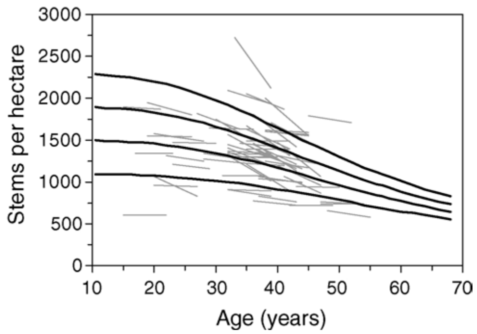

Fig. 5. Trajectories of observed and predicted stem number over time. Model projections for initial stockings of 1100, 1500, 1900 and 2300 stems per hectare. Same-plot measurements joined by lines.

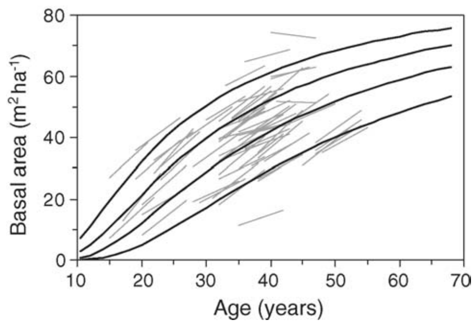

Fig. 6. Basal area predictions overlaid on the trajectories of observed values over time. Initial values obtained for site indexes of 9, 13, 18 and 25 m. Same-plot measurements joined by lines.

3.4. Disaggregation system

3.4.1. Diameter distribution

The equation selected for predicting arithmetic mean diameter and for use in the parameter recovery approach was:

$$\bar{d} = d_g - e^{-1.294+0.000187N+0.0363H} \tag{18}$$

where $\bar{d}$ is the predicted arithmetic mean diameter (cm), d_g the quadratic mean diameter (cm), N the number of trees per hectare, and H the dominant height (m).

3.4.2. Height estimation for diameter classes

The model that provided the best results was a modification of the function proposed by Gaffrey (1988), obtained after (i) substitution of the quadratic mean diameter by dominant diameter, and (ii) removal of one parameter and its associated variable, as it was not significantly different from zero:

$$h = 1.3 + (H - 1.3)\,e^{7.197((1/d_0)-(1/d))} \tag{19}$$

where h is the predicted total height (m) of the subject tree, d its diameter at breast height (cm), and H (m) and d_0 (cm) dominant height and dominant diameter (average values of the 100 thickest trees per hectare) of the stand where the subject tree is included, respectively.

3.4.3. Total and merchantable volume estimation

For total and merchantable volume estimation of the average tree in each diameter class, the compatible system proposed by Fang et al. (2000) was selected. This system assumes that tree bole has three sections, each with a constant form factor that differs between sections. It is constituted by the following components:

- Taper function:

$$d_i = c_1\sqrt{h^{(k-b_1)/b_1}(1-q_i)^{(k-\beta)/\beta}\alpha_1^{I_1+I_2}\alpha_2^{I_2}} \tag{20}$$

where

$I_1 = 1$ if $p_1 \le q_i \le p_2$; 0 otherwise,

$I_2 = 1$ if $p_2 < q_i \le 1$; 0 otherwise

p_1 and p_2 are relative heights from ground level where the two inflection points that assumes the model occur

$$\beta = b_1^{1-(I_1+I_2)}b_2^{I_1}b_3^{I_2} \qquad \alpha_1 = (1-p_1)^{\frac{(b_2-b_1)k}{b_1b_2}}$$

$$\alpha_2 = (1-p_2)^{\frac{(b_3-b_2)k}{b_2b_3}} \qquad r_0 = (1-h_{st}/h)^{k/b_1}$$

$$r_1 = (1-p_1)^{k/b_1} \qquad r_2 = (1-p_2)^{k/b_2}$$

$$c_1 = \sqrt{\frac{a_0 d^{a_1} h^{a_2-k/b_1}}{b_1(r_0-r_1)+b_2(r_1-\alpha_1 r_2)+b_3\alpha_1 r_2}}$$

- Merchantable volume equation:

$$v_i = c_1^2 h^{k/b_1}(b_1 r_0 + (I_1+I_2)(b_2-b_1)r_1 + I_2(b_3-b_2)\alpha_1 r_2 - \beta(1-q_i)^{k/\beta}\alpha_1^{I_1+I_2}\alpha_2^{I_2}) \tag{21}$$

- Volume equation:

$$v = a_0 d^{a_1} h^{a_2} \tag{22}$$

The obtained parameter estimates were:

a_0	7.047×10^{-5}
a_1	1.846
a_2	0.9320
b_1	1.523×10^{-5}
b_2	3.121×10^{-5}
b_3	2.699×10^{-5}
p_1	0.1039
p_2	0.6116

The following notation was used: d = diameter at breast height over bark (cm); d_i = top diameter at height h_i over bark (cm); h = total tree height (m); h_i = height above the ground to top diameter d_i (m); h_{st} = stump height (m); v = total tree volume over bark (m^3) above stump level; v_i = merchantable volume over bark (m^3), the volume from stump level to a specified top diameter d_i; $a_0, a_1, a_2, b_1, b_2, b_3, p_1, p_2$ = regression coefficients to be estimated; $k = \pi/40{,}000$, metric constant to convert from diameter squared in cm^2 to cross-section area in m^2; $q_i = h_i/h$.

3.5. Overall validation of the model

Observed dominant height, number of trees per hectare and stand basal area from the first inventory of the 68 plots measured twice served as initial values for the corresponding transition functions (Eqs. (12), (15) and (17), respectively). They were used to project the stand state at the age of the second inventory. Later, Eq. (18) was used for estimating arithmetic mean diam-

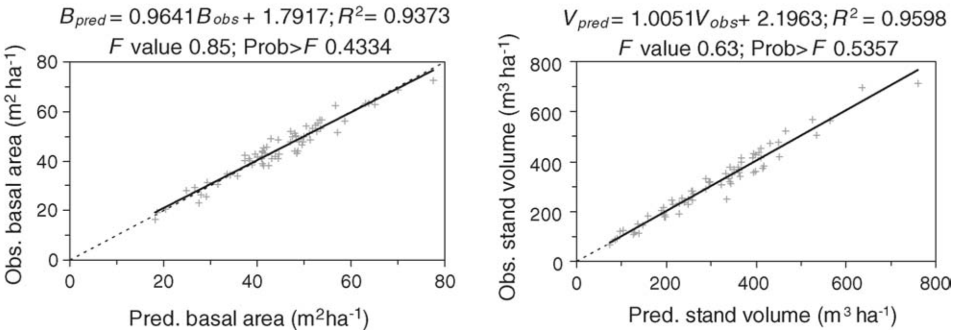

Fig. 7. Plots of observed against predicted values of stand basal area (left) and stand volume (right). The solid line represents the linear model fitted to the scatter plot of data and the dashed line is the diagonal. R^2 is the coefficient of determination of the linear model, and the F value and its associated probability are the results of the simultaneous F-test for intercept = 0 and slope = 1.

eter, which allowed calculation of the variance of the diametric distribution. Eqs. (8) and (9) were used to recover the Weibull parameters, which allowed estimation of the number of trees in each diameter class. Eqs. (19) and (22) were used to estimate the height and the total volume of the average tree in each diameter class, respectively. Aggregation of total tree volume times number of trees in each diameter class provided total stand volume.

Fig. 7 shows observed against predicted values of stand basal area (left) and stand volume (right) obtained following the above procedure. The linear model fitted for each scatter plot and the coefficient of determination showed good behaviour for both variables. The simultaneous F-test for intercept = 0 and slope = 1 showed no bias and provided no reason for rejecting the null hypothesis.

A critical error of 10.9 and 15.3% was also obtained for stand basal area and total stand volume, respectively. Considering the required accuracy in forestry growth modelling, where a mean prediction error of the observed mean at 95% confidence intervals within ±10–20% is generally realistic and reasonable as a limit for the actual choice of the acceptance and rejection levels (Huang et al., 2003), we can state that the model provides satisfactory predictions for the available data. Nevertheless, the model should be tested and modified, if required, as new data become available.

3.6. Application of the model and further studies

This study was based primarily on stands of ages between 10 and 55 years. Predictions for stands less than 10-year-old should not be made, since for younger ages erratic height growth may lead to erroneous classifications. However, the model may be used with caution for ages above 55 years until more data are recorded and used to test or refit the model, because the behaviour of the different components was evaluated and found logical for ages close to the rotation length (for the best results, approximately 70–80 years) (see Figs. 4–6).

The most important limitation of the model is that it does not consider the later effect of thinning and pruning before the trees fully occupy the additional space that has been made available to them. Thus, it is recommended that thinning trials are carried out, and that measurements of dominant height and stand basal area growth are made, and the reduction in tree number per hectare in old pine plantations are monitored to confirm or modify the relationships in a data set corresponding to a longer period.

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