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8

Cubic Volume and Measures of Form

It is convenient to think of the tree as consisting of four parts: *roots, stump, stem,* and *crown*.

The *roots* are the underground part of the tree that supply it with nourishment. The *stump* is the lower end of the tree that is left above the ground after the tree has been felled. The *stem* is the main ascending axis of the tree above the stump. (Trees that have the axis prolonged to form an undivided main stem, as exemplified by many conifers, are termed *excurrent;* trees that have the axis interrupted in the upper portion due to branching, as exemplified by many broadleaved species, are termed *deliquescent.*) The *crown* consists of the primary and secondary branches growing out of the main stem, together with twigs and foliage.

The most important portion of a tree, in terms of usable wood, is the stem. In the past, since the roots and stumps of trees were less often utilized, little attention was given to the determination of their volume. The picture is changing, however, because of the increasing interest in complete tree utilization. There has been, and continues to be, a need to determine the volume of crowns because crown-volume data may be used to describe fuel hazards, to estimate volume of material left from line-clearing operations, and to determine volume of pulpwood and fuelwood in crowns. It is interesting to note that the cubic volume of the merchantable stem (to a 4-inch top diameter) of the average tree is about 55 to 65 percent of the cubic volume of the complete tree, excluding foliage, and that the cubic volume of the crown of the average tree, excluding foliage, is about 15 to 20 percent of the cubic volume of the complete tree, excluding foliage, for softwoods and 20 to 25 percent for hardwoods.

Since the stump, stem, and branches are all covered with bark, they are, when utilized, peeled in the woods or hauled, bark and all, to the mill where the bark is removed before the manufacturing process begins. Thus, it is necessary to determine, at one time or another, unpeeled volume and bark volume.

8-1 DIRECT DETERMINATION OF THE CUBIC VOLUME OF TREE PARTS

Direct volume determinations of parts of trees are usually made on sample trees to obtain basic data for the development of relationships between the various dimensions of a tree and its volume (Chapter 9). Relationships of this type are used to estimate the volumes of other standing trees. In the past sample-tree measurements were often taken on trees cut in harvesting operations. But volume relationships developed from such measurements may lead to bias because they may not be representative of all the trees in a stand. Thus, there is a growing tendency to take measurements on a representative sample of standing trees.

The direct determination of the volume of any part of a tree involves clearly defining the part of the tree for which volume is to be determined and carefully taking measurements in accordance with the constraints imposed by the definition. For example, for purposes of measurement we might include the portion of the stem above a fixed-height stump to a minimum upper-stem diameter outside bark, or on stems that do not have a central tendency, to the point where the last merchantable cut can be made. For roots we might include roots larger than some minimum diameter; for tops we might include the branches and the tip of the stem to some minimum diameter outside the bark.

Generally speaking, the tree must be felled and the limbs cut into sections before one can directly determine crown volume. To directly determine root volume, the roots must be lifted from the ground and the soil removed. But of course, stem and stump volume may be directly determined on either standing or felled trees.

The stems and stumps of trees closely resemble certain geometrical solids. Thus, their cubic volume may be determined by formulas (Section 8-2). It is not feasible, however, to determine the volume of roots by formulas because they do not, either in their entirety or in portions, closely resemble any known geometrical solids. Practically speaking, this is also true of crowns.

Thanks to the reasonably regular form of stems, graphical techniques may be used to obtain the cubic volume of the stem and stump of a standing or felled tree (Section 7-1). But again this technique is unsuitable for determining the volume of roots or crowns, because for each set of roots, or for each crown, an inordinate number of diameter measurements and an excessive number of graphs are required to obtain satisfactory results. The displacement method (Section 7-1), on the other hand, can be used to obtain accurately the cubic volume of any part of the tree. It is applicable only to cut trees, however, and it is relatively slow and expensive.

To use formulas or graphical techniques to determine the volume of a stem, it is necessary to have diameter measurements at various intervals along the stem. Both inside and outside bark measurements are desirable. Measurements may be made on down or standing trees (Chapter 2). On standing trees, when it is convenient to obtain only outside bark measurements, inside bark diameters, and thus inside bark volumes, may be calculated by use of bark factors (Section 8-3).

8-2 DIRECT DETERMINATION OF CUBIC VOLUME BY FORMULAS

The stems of excurrent trees are often assumed to resemble neiloids, cones, or paraboloids—solids that are obtained (Fig. 8-1) by rotating a curve of the general form

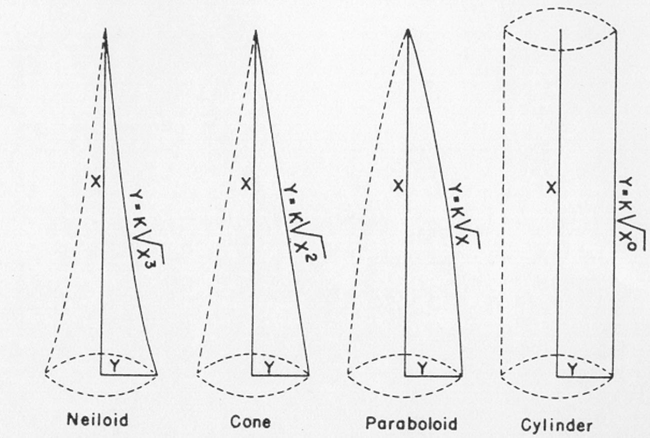

Fig. 8-1 *Solids of revolution descriptive of tree form.*

$Y = K\sqrt{X^r}$ around the X axis. As the form exponent r changes in this equation, different solids are produced. When r is 1, a paraboloid is obtained; when r is 2, a cone; when r is 3, a neiloid; and when r is 0, a cylinder. But the stems of excurrent trees are seldom cones, paraboloids, or neiloids; they generally fall between the cone and the paraboloid. The merchantable portion of stems of deliquescent trees, on the other hand, are assumed to resemble frustums of neiloids, cones, or paraboloids (occasionally cylinders). But they generally fall between the frustum of a cone and the frustum of a paraboloid.

It is more realistic, however, to consider the stem of any tree to be a composite of geometrical solids (Fig. 8-2). For example, when the stem is cut into logs or bolts, the tip approaches a cone or paraboloid in form, the central sections approach frustums of paraboloids, or in a few cases frustums of cones or cylinders, and the butt log approaches the frustum of a neiloid. Although the stump approaches the frustum of a neiloid in form, for practical purposes it is considered to be a cylinder.

Formulas to compute the cubic volume of the solids that have been of particular interest to mensurationists are given in Table 8-1. Newton's formula is exact for all the frustums we have considered. Smalian's and Huber's formulas are exact only when the solid is the frustum of a paraboloid.[1] For example, if the surface lines of a tree section are more convex than the paraboloid frustum, Huber's formula will overestimate the volume while Smalian's formula will underestimate the volume. But if the surface lines of a tree section are less convex than the paraboloid frustum, as they often are, Smalian's formula will overestimate the volume and Huber's formula will

[1] Newton's formula is attributed to Sir Isaac Newton. Prodan (1965) claims that Huber's formula came into use in 1785 and Smalian's formula in 1804.

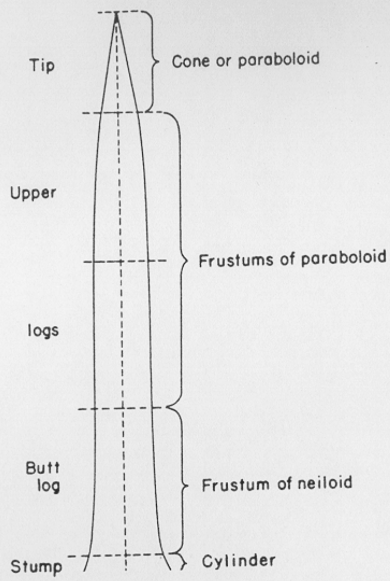

Fig. 8-2 *Geometric forms assumed by portions of a tree stem.*

underestimate the volume. Assuming Newton's formula gives correct volume values, it can be shown by subtracting Newton's formula first from Smalian's formula and then from Huber's formula that the error incurred by Smalian's formula is twice that incurred by Huber's formula and opposite in sign. In a study by Young, Robbins, and Wilson (1967) on 8- and 16-foot softwood logs of 4- to 12-inch diameters, volumes calculated by Newton's, Smalian's, and Huber's formulas were compared with volumes determined by displacement. Average percent errors of about 0 percent were obtained for Newton's formula, +9 percent for Smalian's formula, and −3.5 percent for Huber's formula. They also found that there were no significant errors for any of the three formulas on 4-foot bolts. In a study by Miller (1959) on 16-foot hardwood logs of 8- to 22-inch diameters, volumes calculated by the three formulas were compared with volumes determined by graphical techniques. Average percent errors of about +2 percent were obtained for Newton's formula, +12 percent for Smalian's formula, and −5 percent for Huber's formula.

It should now be apparent that in calculating cubic volume of trees and logs, mensurationists should select their methods carefully. Unless one is willing to accept a rather large error, Smalian's formula should not be used unless it is possible to measure the sections of the tree in 4-foot lengths. For 8- or 16-foot long, Newton's or Huber's formula will give more accurate results.

Table 8-1
Equations to Compute Cubic Volume of Important Solids

Geometrical Solid	Equation for Volume V (cubic units)		Equation Number
Cylinder	$V = A_b h$		(8-1)
Paraboloid	$V = \frac{1}{2}(A_b h)$		(8-2)
Cone	$V = \frac{1}{3}(A_b h)$		(8-3)
Neiloid	$V = \frac{1}{4}(A_b h)$		(8-4)
Paraboloid frustum	$V = \frac{h}{2}(A_b + A_u)$	(Smalian's formula)	(8-5)
	$V = h(A_m)$	(Huber's formula)	(8-6)
Cone frustum	$V = \frac{h}{3}(A_b + \sqrt{A_b A_u} + A_u)$		(8-7)
Neiloid frustum	$V = \frac{h}{4}(A_b + \sqrt[3]{A_b^2 A_u} + \sqrt[3]{A_u^2 A_b} + A_u)$		(8-8)
Neiloid, cone, or paraboloid frustum	$V = \frac{h}{6}(A_b + 4A_m + A_u)$	(Newton's formula)	(8-9)

h = height.
A_b = cross-sectional area at base.
A_m = cross-sectional area at middle.
A_u = cross-sectional area at top.

Newton's formula will give accurate results for all sections of the tree except for butt logs with excessive butt swell. For such butt logs, Huber's formula will generally give better results. Either the paraboloidal formula (equation 8-2, Table 8-1) or the conic formula (equation 8-3, Table 8-1) is appropriate to determine the volume of the tip. The cylindrical formula (equation 8-1, Table 8-1) is normally used to compute the volume of the stump, although the stump actually approaches the neiloid frustum in form.

Newton's and Huber's formulas cannot, of course, be applied to stacked logs, because it is not possible to measure middle diameters. However, these two formulas are as well suited as Smalian's formula to determine the volume of unstacked logs or standing trees.

Newton's formula may be used to compute the volume of the merchantable stem or of the total stem. If the sections are of the same length, the procedure can be summarized in a single formula. To illustrate, consider a stem with diameters in inches from top of stump to a point where the last merchantable cut will be made, d_0, d_1, d_2, d_3, d_4, d_5, and d_6, located at intervals of h feet. To give each section three diameters, the volume is computed by sections of $2h$ length. Thus, with Newton's formula the volume (in cubic feet) V is

$$V = \frac{2h}{6}(0.005454)(d_0^2 + 4d_1^2 + d_2^2) + \frac{2h}{6}(0.005454)(d_2^2 + 4d_3^2 + d_4^2)$$

$$+ \frac{2h}{6}(0.005454)(d_4^2 + 4d_5^2 + d_6^2)$$

$$= h(0.001818)(d_0^2 + 4d_1^2 + 2d_2^2 + 4d_3^2 + 2d_4^2 + 4d_5^2 + d_6^2)$$

$$= 0.003636h\left(\frac{d_0^2}{2} + 2d_1^2 + d_2^2 + 2d_3^2 + d_4^2 + 2d_5^2 + \frac{d_6^2}{2}\right)$$

(The constant 0.005454 comes from the expression: cross-sectional area in square feet $= [\pi/4(144)]d_i^2 = 0.005454d_i^2$. If the metric system is used, d_i will be in centimeters and h in meters, and to obtain volume in cubic meters the constant will come from the expression: cross-sectional area in square meters $= [\pi/4(10{,}000)]d_i^2 = 0.00007854d_i^2$.) This formula may be extended for as many sections as desired, provided there is an odd number of diameters, that is, an even number of sections of h length.

If the number of diameters measured is even, the last interval of h cannot be computed by Newton's formula, because it will have only two end diameters. Thus, its volume must be found by Smalian's formula and added to the previous formula. For eight diameters, or seven intervals of h length, this yields

$$V = 0.003636h\left(\frac{d_0^2}{2} + 2d_1^2 + d_2^2 + 2d_3^2 + d_4^2 + 2d_5^2 + \frac{5d_6^2}{4} + \frac{3d_7^2}{4}\right)$$

Grosenbaugh (1948) described a systematic procedure using this method.

The volume of the merchantable stem can also be calculated with good accuracy using Smalian's formula if the stem is divided into short sections. To illustrate, consider a stem with diameters in inches, from top of stump to a point where the last merchantable cut will be made, $d_0, d_1, d_2, \ldots, d_n$, located at intervals of h feet along the stem. Then, according to Smalian's formula,

$$V = 0.005454h\left(\frac{d_0^2}{2} + d_1^2 + d_2^2 + \cdots + d_{n-1}^2 + \frac{d_n^2}{2}\right)$$

To adapt Huber's formula to the computation of merchantable stem volumes, the diameter measurements are taken at the midpoints of the sections. Thus, when diameter measurements $d_{m_1}, d_{m_2}, \ldots, d_{m_n}$ are taken at the midpoints of sections of h length, Huber's formula yields

$$V = 0.005454h(d_{m_1}^2 + d_{m_2}^2 + \cdots + d_{m_n}^2)$$

Table 8-2
Sample Tree Data for Computation of Volume by Height Accumulation by 2-Inch Taper Steps and 4-Foot Unit Heights

	dob Taper Steps					
dbh	10	8	6	4	2	Sum
9.5	1	3	4	2	0	10
7.7		1	5	2	0	8
10.5	1	5	3	2	0	11
L =	2	9	12	6	0	29
H =	2	11	23	29	29	94
H' =	2	13	36	65	94	210

8-2.1 Determination of Volume by Height Accumulation

The height accumulation concept was conceived and developed by Grosenbaugh (1948, 1954), who stated that the system can be applied by selecting tree diameters above breast height in diminishing arithmetic progression, say 1- or 2-inch taper intervals, and estimating, recording, and accumulating tree height to each successive diameter. The system uses diameter as the independent variable instead of height, is well adapted to use with electronic computers, and permits segregation of volume by classes of material, log size, or grade. But since optimum log lengths for top log grades depend on factors other than diameter, the best grades may not be secured.

To apply the system one must know the number of sections L in some unit height between taper steps, the cumulative total H of L values, the cumulative total H' of H values, and, if inside bark volume is desired, the mean bark factor k (Section 8-3). This requires that one collect the following sample tree data: dbh to nearest 0.1 inch, length of merchantable stem between successive taper steps to nearest unit height, and measurements to compute the mean bark factor. For example, if one used 2-inch taper steps and 4-foot unit heights (a feasible practice), dbh would be rounded to the nearest even inch, and the first unit height L—the 4-foot section between stump height and breast height—would be 1. The unit heights to each taper step might be estimated, but to obtain acceptable accuracy, an instrument, such as the Spiegel relaskop, should be used.

Table 8-2 gives sample tree data needed to compute the volume V of a number of trees, or of individual trees, by height accumulation. Very simply

$$V = A(\Sigma H') + B(\Sigma H) + C(\Sigma L)$$

Volume coefficients A, B, and C for cubic feet are given in Table 8-3 for 2-inch taper steps, 4-foot unit heights, and mean dib/dob ratios. Grosenbaugh (1954) also gives coefficients for 1-inch taper steps and 1-foot unit heights, coefficients for determination of board-foot volume, formulas to calculate coefficients for other cases, and the theory of height accumulation.

Table 8-3
Height-Accumulation Coefficients, *A*, *B*, and *C*, to Compute Cubic-Foot Volume by 2-Inch Taper Steps, 4-Foot Unit Heights, and Various Mean dib/dob Ratios

Mean Ratio dib/dob	Volume Coefficients for Cubic Feet		
	A	*B*	*C*
1.00*	0.175	0	0.0291
0.95	0.158	0	0.0263
0.90	0.141	0	0.0236
0.85	0.126	0	0.0210

[1] When computing volume outside bark, use coefficients for ratio of 1.00.
SOURCE: Grosenbaugh, 1954.

For the trees given in Table 8-2, the total cubic foot volume, inside bark, for a mean dib/dob ratio of 0.90 is

$$V = 0.141(210) + 0(94) + 0.0236(29) = 30.3 \text{ ft}^3$$

Similarly, individual-tree cubic-foot volume is

$$V_{9.5} = 0.141(75) + 0(33) + 0.0236(10) = 10.8 \text{ ft}^3$$

$$V_{7.7} = 0.141(46) + 0(23) + 0.0236(8) = 6.7 \text{ ft}^3$$

$$V_{10.5} = 0.141(89) + 0(38) + 0.0236(11) = \underline{12.8} \text{ ft}^3$$

$$\text{Total} \quad 30.3 \text{ ft}^3$$

Enghardt and Derr (1963) found that computation of cubic volume of young even-aged stands of southern pine by height accumulation required less time and effort than conventional methods and gave satisfactory accuracy for research purposes. Indeed, the potentialities of this unique system have been generally overlooked.

8-3 THE BARK FACTOR AND DETERMINATION OF BARK VOLUME

Average bark volume will run from 10 to 20 percent of unpeeled volume for most species. But we often need to know bark volume more accurately than this to determine peeled stem or log volume from unpeeled stem or log volume, the quantity of bark residue that will be left after the manufacturing process has been completed, and, in cases where the bark has value, the quantity of bark available. We can, of course, compute unpeeled stem or log volume from outside bark diameter, peeled stem or log volume from inside bark diameter, and take the difference to secure a good estimate of bark volume. But the bark-factor method, which deserves more

consideration than most foresters choose to give it, is easier to apply and gives sufficiently accurate results for most purposes. Let us look at this method more closely.

Bark thickness, which must be accurately determined to obtain reliable bark factors, may be determined as described in Section 2-1.1. The accuracy of bark measurements is increased if single-bark thickness is measured at two or more different points on a given cross section of the stem and if the average single-bark thickness b computed. Then, diameter inside bark d_u may be computed from diameter outside bark d from equation: $d_u = d - 2b$. When d_u is plotted as a function of d, the relationship will be linear, or close to linear, with a Y intercept of 0, or close to 0. (Figure 8-3 shows this relationship for white oak for the cross section at breast height.) Consequently, it is reasonable to assume that the prediction equation for this relationship may be written in the general form $d_u = kd$. Since the regression coefficient k is normally determined at stump or breast height, we will call it the *lower-stem bark factor*. Such bark factors range from 0.87 to 0.93, varying with species, age, and site. But since the major portion of the variation can be accounted for by species, it is reasonable and convenient to assume that the ratio will be constant for a given species. It is also convenient to assume that this bark factor will remain the same, for a given species, at all heights on the stem. But for many species, upper-stem bark factors are often not the same as lower-stem bark factors. To account for this one might develop multiple regression equations to predict upper-stem bark factors from such variables as tree age, tree dbh, height above ground of cross section for which bark factor is desired, bark factor at breast height, and diameter outside bark at cross section for which bark factor is desired. In this day of computers and programmable calculators, although these equations are fairly complex, their use is not limited because of computation time, but because of the time required to measure the independent variables. Consequently, the common practice is to assume that the bark factor is the same at all heights on the stem. However, whether we assume that the bark factor is the same at all heights on the stem, or that the bark factor will be different at different heights on the stem, once the bark factor has been obtained, the method of using it to obtain bark volume is the same. We can illustrate the method by assuming that k is the same at all heights on the stem.

An average value of k, to be reliable, should be based on 20 to 50 bark-thickness measurements and corresponding diameter-outside-bark measurements. By the method of least squares, k is determined so that the sum of squared deviations of the individual d_u's (Y's) about the fitted regression line is a minimum. In this case, where we assume that when $X = 0$ then $Y = 0$, it is appropriate to use the following equation to determine k.

$$k = \frac{\Sigma\, d d_u}{\Sigma\, d^2}$$

When the variation of the dependent variable is proportional to the independent variable, as it generally is when d_u is plotted over d, Meyer (1953) has shown that the following formula will give the same results.

$$k = \frac{\Sigma\, d_u}{\Sigma\, d}$$

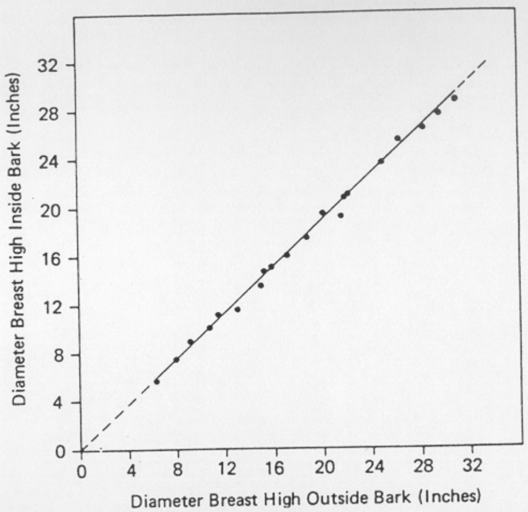

Fig. 8-3 *Relationship between corresponding diameters inside and outside bark of white oak (see Table 8-4).*

For the trees listed in Table 8-4 and plotted in Fig. 8-3, the first formula gives

$$k = \frac{7179.62}{7628.93} = 0.941$$

The second formula gives

$$k = \frac{341.7}{363.3} = 0.941$$

Agreement of these two formulas will not always be this good. But, for calculating k, the second formula will, for practical uses, always give satisfactory results.

The bark thickness b, corresponding to an average value of k, can be determined as follows for any diameter d.

$$b = \frac{1}{2}(d - d_u)$$

And since $d_u = kd$, then

$$b = \frac{1}{2}(d - kd) \tag{8-10}$$

$$= \frac{1}{2}d(1 - k)$$

Thus, for a white oak cross-section of 14.0 inches in diameter, we may estimate the bark thickness to be

$$b = \frac{1}{2}(14.0)(1 - 0.941)$$

$$= 0.413 \text{ inches}$$

The average value of k can also be used to obtain cubic bark volume V_b and cubic volume inside bark V_u from cubic volume outside bark V for a given stem section. If diameter outside bark at the middle of the section d_m, diameter inside bark at the middle of the section d_{mu}, and section length L are all in the same units, we have

$$V = \frac{\pi d_m^2}{4}(L) \quad \text{and} \quad V_u = \frac{\pi d_{mu}^2}{4}(L)$$

And since

$$d_{mu} = kd_m$$

Table 8-4
Diameter and Bark Measurements of 20 White Oak Trees

	d dbh Outside Bark (inches)	$2b$ Double Bark Thickness (inches)	d_u dbh Inside Bark (inches)
	30.8	2.2	28.6
	14.7	1.1	13.6
	24.8	1.2	23.6
	20.0	0.8	19.2
	21.7	1.1	20.6
	7.9	0.4	7.5
	15.8	1.1	14.7
	12.7	1.3	11.4
	18.6	1.0	17.6
	17.0	1.0	16.0
	26.1	0.8	25.3
	28.1	1.7	26.4
	10.6	0.6	10.0
	6.1	0.4	5.7
	29.4	1.6	27.8
	15.1	0.8	14.3
	21.5	2.3	19.2
	22.0	1.2	20.8
	11.4	0.7	10.7
	9.0	0.3	8.7
Sum	363.3	21.6	341.7

$\Sigma d \cdot d_u = 7179.62.$

$\Sigma d^2 = 7628.93.$

then

$$V_u = \frac{\pi}{4}(kd_m)^2(L) = k^2V \quad (8\text{-}11)$$

Finally, since

$$V_b = V - V_u$$

$$V_b = V(1 - k^2) \quad (8\text{-}12)$$

$$V_b\,(\%) = (1 - k^2)100 \quad (8\text{-}13)$$

When V_u is determined by equation 8-11, it will be theoretically correct. However, V_b determined by equation 8-12 will be greater than the actual value. This is because V_b includes air spaces between the ridges of the bark. Indeed, a study by Chamberlain and Meyer (1950) shows that the difference in volume between stacked-peeled and stacked-unpeeled cordwood is, on the average, 80 percent of the volume given by equation 8-12. One might not expect this result. But it comes about because in a stack of wood the ridges of the bark of one log will mesh with the ridges of another log, and because the weight of the logs will compress the bark. Thus, for practical purposes, we can rewrite equation 8-12 to give the bark volume of cordwood in stacks V_{b_s}.

$$V_{b_s} = 0.8V(1 - k^2) \quad (8\text{-}14)$$

8-4 METHODS OF STUDYING STEM FORM

There are wide variations in the form of the main stem of trees due to variations in the rates of diminution in diameter from the base to the tip. This diminution in diameter, known as taper, which is a fundamental reason for variation in volume, varies with species, diameter breast high, age, and site.

In a definitive study, Larson (1963) discussed the biological concept of stem form by a comprehensive review of the literature. In their studies of stem form, mensurationists have looked for pure expressions of stem form that are independent of diameter and height. But pure expressions have not been discovered; they are probably fictions. Nevertheless, the methods that have been developed for studying stem form have been useful. They may be considered under four headings: *form factors, form quotients, form point,* and *taper tables, curves,* and *formulas*.

8-4.1 Form Factors

A form factor is the ratio of tree volume to the volume of a geometrical solid, such as a cylinder, a cone, or a cone frustum, that has the same diameter and height as the tree. (The diameter of the geometrical solid is taken at its base; the diameter of the tree is taken at breast height.) A form factor is different from other measures of form in that it can be calculated only after the volume of the tree is known. In formula form, the form factor f is

$$f = \frac{\text{Volume of tree}}{\text{Volume of geometrical solid of same diameter and height}} \quad (8\text{-}15)$$

Early in the nineteenth century it was recognized that the form of tree stems approached that of the solids discussed in Section 8-2. But it was also recognized that there were many variations in form, and that a tree rarely was of the exact form of one of these solids. Thus, the form factor was conceived as a method of coordinating form and volume. That is to say, the main objective of the early work was to derive factors that would be independent of diameter and height, and by which the volume of standard geometrical solids could be multiplied to obtain the tree volume. For example, the ratio of the volumes of a paraboloid to a cylinder is 0.5 when the base diameter and the height of the two solids are equal; the volume of the paraboloid is obtained by multiplying the volume of the cylinder by 0.5.

The *cylindrical form factor*, which has been most commonly used, may be expressed by the equation

$$f = \frac{V}{gh} \quad (8\text{-}16)$$

where

V = volume of tree in cubic units
g = cross-sectional area of cylinder whose diameter equals tree dbh
h = height of cylinder whose height equals tree height

The usefulness of form factors to estimate the volume of trees of variable form is limited. However, there are some uses for the rapid approximation of volume as well as the volume of trees of little form variation. Belyea (1931) discussed form factors and their uses at some length.

8-4.2 Form Quotients

A form quotient is the ratio of a diameter measured at some height above breast height, such as one-half tree height, to diameter at breast height. In formula form the form quotient q is

$$q = \frac{\text{Diameter above breast height}}{\text{Diameter at breast height}} \quad (8\text{-}17)$$

Next to diameter breast high and height, form quotient is the most important variable that can be used to predict the volume of a tree stem. Thus, it may be used as the third independent variable in the construction of volume tables (Chapter 9).

The original form quotient (Schiffel, 1899) took diameter at one-half total tree height $d_{0.5h}$ as the numerator, and diameter breast high d as the denominator. This was termed the *normal form quotient* q

$$q = \frac{d_{0.5h}}{d} \tag{8-18}$$

For this form quotient, as tree height decreases, the position of the upper diameter comes closer to the breast height point until, for a tree whose height is double breast height, they coincide. To eliminate this anomaly, Jonson (1910) changed the position of the upper diameter to a point halfway between the tip of the tree and breast height, $d_{\frac{1}{2}(h+4.5)}$, and called the ratio the *absolute form quotient* q_a.

$$q_a = \frac{d_{\frac{1}{2}(h+4.5)}}{d} \tag{8-19}$$

The absolute form quotient is a better measure of stem form than the normal form quotient. However, it is not a pure expression of stem form. It is not independent of diameter and height and it varies within a given diameter-height class for a given species. For most species absolute form quotients diminish with increasing diameters and heights, varying between 0.60 and 0.80. (Absolute form quotient is 0.707 for a paraboloid, 0.500 for a cone, and 0.354 for a neiloid when diameter d is taken at the base. These values hold irrespective of the diameter and height of the solid.)

In determining the normal or absolute form quotient, the two diameters may be taken either outside or inside bark. Although it is difficult to obtain an accurate upper-stem diameter inside bark, the ratio of the inside bark measurements is a better index of form than the outside bark measurements, because variable bark thicknesses do not then distort the ratio.

At this point it would be well to discuss the term *form class*. Originally the term was applied to a class, as in a frequency table of absolute form quotients. For example, form classes with class intervals of 0.05 have often been laid out as follows: 0.575–0.625, 0.625–0.675, 0.675–0.725, and 0.725–0.775. The midpoints of these classes, 0.60, 0.65, 0.70, and 0.75, were used to name the classes, and a tree falling into a particular class was said to have the form quotient of the midpoint of the class. Tree-volume computations were classified by absolute form class as well as by diameter and height. Form class may be related to the density of the stand (Jonson, 1911) or to *form point* (Fogelberg, 1953), so it may be determined indirectly.

In North America the term form class has been used in a different sense. Girard (1933), in the course of work in the U.S. Forest Service, developed a form quotient for use as an independent variable in volume table construction. This measure, termed Girard form class, q_G, is the percentage ratio of diameter inside bark at the top of the first standard log d_u to diameter breast high, outside bark. When 16-foot logs are taken as the standard, the upper diameter $d_{u17.3}$ is taken at the height of a standard 1-foot stump plus 16.3 feet. Thus,

$$q_G = \frac{d_{u17.3}}{d}(100) \tag{8-20}$$

This measure of form has three advantages over normal and absolute form quotients:

1. The top of the first log is close enough to the ground so that the diameter at that point may be accurately estimated or measured.
2. The reference diameter is near enough to the ground to give a measure of butt swell.
3. Bark thickness is taken into account and its effect on taper partially eliminated.

As mentioned in Chapter 9, Girard form class is a useful form quotient that has been widely employed in U.S. forest practice.

Efforts to develop form quotients that can be computed from more accessible diameters have led to a number of form quotients of the type advocated by Maass (1939). Maass's form quotient is the ratio of the diameter at 2.3 meters above the ground to diameter at 1.3 meters above the ground. Unpublished studies by Miller (1952), however, of similar form quotients indicated that this quotient, whether measurements are inside or outside bark, is too variable for trees of the same species, diameter, and height to be of practical use.

8-4.3 Form Point

Form point is the percentage ratio of the height to the center of wind resistance on the tree, approximately at the center of gravity of the crown, to the total tree height.

It was hypothesized by Jonson (1912) that the development of the form of a tree stem, as exemplified by the absolute form quotient, depends on the mechanical stresses to which the tree is subjected. These stresses come from the dead weight of the stem and crown, and the wind force. The wind force, it was concluded, is the most important stress, and "causes" the tree to "construct" its stem in such a way that the relative resistance to fracture or shearing will be the same at all points on the longitudinal axis of the stem. Thus, the main determinative of stem form is the focal point of wind force, or the point of greatest resistance to wind bending. Since the crown offers most of the resistance, it will be the location of the focal point of the wind force. Thus, the point where the wind resistance is the greatest is approximately at the center of gravity of the crown; it will vary with the size and shape of the crown. Except as they affect the size and shape of the crown, such tree characteristics as diameter, height, species, and age, and such things as the site factors, do not, so it is claimed, affect stem form. The greater the form point, the more nearly cylindrical will be the form of the tree.

In cases where the form point has been used, the average form points for the various diameter classes of a stand are obtained by sampling and, with these values as the independent variables, form classes are read from curves or tables (Fogelberg, 1953). Also, with the form point of an individual tree as the independent variable, the form class for an individual tree may be read from a curve or table.

It should be emphasized that the main value of the form point is to predict the absolute form quotient; there appears to be a good correlation between form quotient and form point. There are, however, some serious limitations to the use of form point. The focal point of wind resistance within the crown of any tree varies with the density of the crown and the position of the crown in the stand. Thus, the point is difficult to locate; two estimators will differ considerably on their selection.

8-4.4 Taper Tables, Curves, and Formulas

If sufficient measurements of diameters are taken at successive points along the stems of trees, one can prepare average taper tables that give a good picture of stem form. The ultimate purpose of all taper tables is to portray stem form in such a way that the data can be used in the calculation of stem volume or in the construction of volume tables. There are several ways of preparing taper tables (Chapman and Meyer, 1949), but the method of greatest utility is to average upper-log taper rates inside bark for standard log lengths according to diameter breast high and merchantable height in standard logs (Table 8-5).

Since taper tables may be expressed in curve form, it is logical to express a taper curve by a mathematical function. And it is logical to use the formula to obtain the volume of the solid of revolution. Many formulas have been developed, but as Grosenbaugh (1966) said in his comprehensive study of stem form:

> *Many mensurationists have sought to discover a single simple two-variable function involving only a few parameters which could be used to specify the entire tree profile. Unfortunately, trees seem capable of assuming an infinite variety of shapes, and polynomials (or quotients of polynomials) with degree at least two greater than the observed number of inflections are needed to specify*

Table 8-5
Average Upper-Log Taper Inside Bark (inches) in 16-Foot Logs

dbh (inches)	2-Log Tree	3-Log Tree		4-Log Tree		
	2nd Log	2nd Log	3rd Log	2nd Log	3rd Log	4th Log
10	1.4	1.2	1.4			
12	1.6	1.3	1.5	1.1	1.4	1.9
14	1.7	1.4	1.6	1.2	1.5	2.0
16	1.9	1.5	1.7	1.2	1.6	2.1
18	2.0	1.6	1.8	1.3	1.7	2.2
20	2.1	1.7	1.9	1.4	1.8	2.4
22	2.2	1.8	2.0	1.4	2.0	2.5
24	2.3	1.8	2.2	1.5	2.2	2.6
26	2.4	1.9	2.3	1.5	2.3	2.7
28	2.5	1.9	2.5	1.6	2.4	2.8
30	2.6	2.0	2.6	1.7	2.5	3.0

SOURCE: Mesavage and Girard, 1946.

> *variously inflected forms. Furthermore, coefficients would vary from tree to tree in ways that could only be known after each tree has been completely measured. Thus, explicit analytic definition of tree form requires considerable computational effort, yet lacks generality. . . . Each tree must be regarded as an individual that must be completely measured, or else as a member of a definite population whose average form can only be estimated by complete measurement of other members of the population selected according to a valid sampling plan. . . . Hence, polynomial analysis may rationalize observed variation in form after measurement, but it does not promise more efficient estimation procedures.*

Thus, it appears unwise to derive complicated relationships to characterize tree form. This notion is corroborated by Kozak and Smith (1966) who, after studying the multivariate techniques of Fries (1965) and Fries and Matern (1965), concluded that the use of simple functions, sorting, and graphical methods is adequate for many uses in operations and research.