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2

Linear Measurement

Linear, or length, measurement consists of determining the length of a line from one point to another. Since the configuration of objects vary, a length might be the straight-line distance between two points on an object, or the curved- or irregular-line distance between two points on an object. An example of the latter is the periphery of the cross-section of a tree stem.

Length measurements can be made directly or indirectly. Direct length measurement is accomplished by placing a prototype standard of a defined unit beside the object to be measured. The number of units between terminals is the length. The application of a foot or meter rule is an example of a direct measurement. Indirect linear measurement is accomplished by employing geometry or trigonometry, or by using knowledge of the speed of sound or light.

The varied task of length measurement in forestry includes determination of the height of a tree, the length of a log, the width of a tree crown's image on an aerial photograph, the length of a tracheid under a microscope, the length of the femur of a mountain lion, the length of the boundary of a tract of land, and so on. Keep in mind, however, that measurement techniques are rarely unique to specific measurements. Hence, rather than describe techniques for many different measurement tasks, only tree diameter and length measurements, the specialized forestry measurements most commonly applied, are discussed in this chapter. Forest land measurements are covered in Chapter 3. Information on measurements peculiar to specialized phases of forestry and allied subjects can be obtained from references given throughout the text. However, the reader should be able to understand the broad applications of linear measurement techniques from the information given in this chapter.

2-1 DIAMETER MEASUREMENT

A *diameter* is a straight line passing through the center of a circle or sphere and meeting at each end the circumference or surface. The most common diameter measurements taken in forestry are of the main stem of standing trees, cut portions of trees, and branches. Diameter measurement is important because it is one of the directly measurable dimensions from which tree cross-sectional area, surface area, and volume can be computed.

The use of the word diameter implies that trees are circular in cross section. In many cases, however, the section is somewhat wider in one direction than another.

or it may be eccentric in other ways. Since for computational purposes tree cross sections are assumed to be circular, the objective of any tree diameter measurement is to obtain the diameter of a circle with the same cross-sectional area as the tree. This will be further discussed in Section 2-1.2.

The point at which diameters are to be measured will vary with circumstances. In the case of standing trees, a standard position has been established. In the United States, the diameter of standing trees is taken at 4.5 feet above the ground level. This is referred to as diameter breast high and is abbreviated to d.b.h. or dbh. In countries that use the metric system the diameter of standing trees is taken at 1.3 meters above the ground level and is abbreviated as d (IUFRO, 1959). On slopes, dbh is commonly taken from the average ground level occupied by the tree, whereas d is commonly taken from the uphill side. Diameters at other points along the stem are often indicated by subscripts: $d_{0.5h}$ = diameter at half total height; $d_{0.1h}$ = diameter at 0.1 total height; d_6 = diameter at 6 meters above ground level. Diameters should be qualified as o.b. (outside bark) or i.b. (inside bark). However, when this designation is omitted from breast height measurements (dbh or d), as it often is, the measurement is assumed to be outside bark.

In the United States, tree diameters are generally recorded in inches. In countries where the metric system is used, diameters are generally recorded in centimeters (occasionally in millimeters).

In measuring at breast height in the field (dbh or d), we recommend the following standard procedures.

- When trees are on slopes, measure 4.5 feet (for dbh) or 1.3 meters (for d) above the ground on the uphill side of the tree.
- When a tree has a limb, a bulge, buttresses, or some other abnormality at breast height, measure diameter above the abnormality; strive to obtain the diameter the tree would have had if the abnormality had not been present.
- When a tree consists of two or more stems forking below breast height, measure each stem separately. When a tree forks at or above breast height, measure it as one tree. If the fork occurs at brea ight, or slightly above, take the diameter measurement below the enlargement that is caused by the fork.
- When a tree has a paint mark to designate the breast height point, assume that the point of measurement is at the top of the paint mark.

2-1.1 Bark Thickness

Whether *inside bark* (i.b.) or *outside bark* (o.b.) diameter measurements are taken depends on the purpose for which the measurements are made.

When a rule, or scale stick, is used on a cut section, it is simple to measure diameter inside bark (d.i.b.) or outside bark (d.o.b.). One simply measures an appropriate line, or lines, on the section along which the bark is intact. If both d.i.b. and d.o.b. are measured, bark thickness is one-half of the difference between them. If one measures d.o.b. and bark thickness, d.i.b. equals d.o.b. minus twice the bark thickness.

Bark thickness on standing trees can be determined with a bark gauge. The Swedish bark gauge is commonly used. The cutting edge of this instrument is a half circle that is dull on one side so that the instrument can be driven through the soft bark, but not through the wood. A sliding cross arm is provided. When this arm is pressed against the bark, one can read the bark thickness on a scale without removing the instrument. Mesavage (1969) found that sizable errors could occur in measuring bark thickness due to the unevenness of the bark surface and failure to reach the wood with the cutting edge. He found that such errors could be practically eliminated by measuring radially from the wood surface to the line of a diameter tape wrapped tautly around the tree (o.b.). The required measurement can be obtained with a Swedish bark gauge or, particularly when the bark is thick and tough, by boring to the wood surface with a brace and bit and obtaining the distance from tape to wood with a small ruler.

2-1.2 Effect of Eccentricity of Tree Cross Sections

Although the cross section of the woody parts of trees approaches circular form, the shape is often not circular. However, for computational purposes, the cross section is assumed to be circular. Consequently, the objective of any tree diameter measurement is to obtain the diameter of a circle with the same cross-sectional area as the tree. (We will refer to this as the "true" diameter.)

When a tree cross section is elliptical, one might measure the major and minor diameters, d_1 and d_2, and obtain the average diameter from the arithmetic mean of d_1 and d_2. This would, however, overestimate the "true" diameter of the cross section. If the periphery of the elliptical cross section was measured with a tape, and the periphery divided by π (this assumes the periphery is a circle), the "true" diameter of the cross section would also be overestimated.

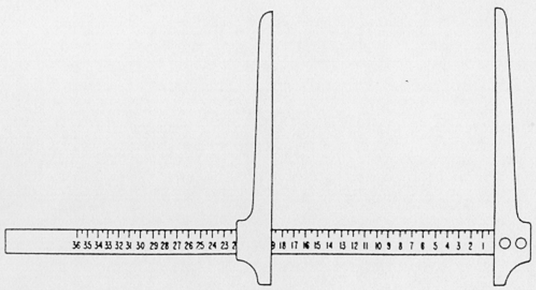
So how does one obtain the "true" diameter of an elliptical cross section? One should use the geometric mean (geometric mean = $\sqrt{d_1 d_2}$).

Unfortunately, tree cross sections often depart from elliptical form as well as from circular form. Consequently, for practical purposes, the best practice is to take the arithmetic average of the long and short "diameters," or axes, or if it is not feasible to secure the long and short "diameters," to take the arithmetic average of two diameters perpendicular to each other.

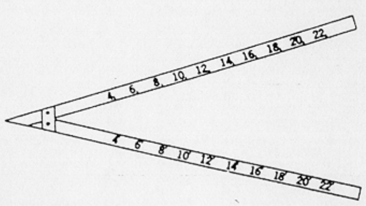
2-1.3 Instruments for Measuring Tree Diameters

In this section, we describe the most commonly used instruments for measuring tree diameter at breast height. These instruments may also be used to measure diameters at other points on the tree, although there are other instruments that are more efficient for this purpose (Section 2-1.4).

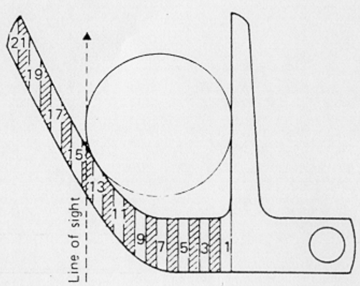
Calipers. Calipers are often used to measure tree diameters when the diameters are not over 24 inches (about 60 centimeters). Calipers of sufficient size to measure large trees, or those with high buttresses (as in tropical regions), are awkward to carry and handle, particularly in dense undergrowth. Figure 2-1 shows three examples of calipers. One form (a), which may be constructed of metal or wood, consists of a graduated beam with two arms perpendicular to it. One arm is fixed at the origin of the scale and the other arm slides. When the beam is pressed against the tree and the arms are closed, the tree diameter can be read on the scale. Another form (b) consists of a set of fixed arms. The graduations are calibrated so that, when the fork is placed on the tree, the points of tangency indicate the tree diameter. The Finnish parabolic caliper (c) consists of a parabolic scale radiating from a common origin (Wiljamaa, 1942; Vuokila, 1955). Numerous other caliper-type instruments have been devised, including ones that provide for the automatic recording of measurements for data processing.



(a) Wooden beam.



(b) Fork.



(c) Finnish parabolic.

Fig. 2-1 *Calipers for tree diameter measurement.*

For an accurate reading, the beam of the caliper shown in Fig. 2-1a must be pressed firmly against the tree with the beam perpendicular to the axis of the tree stem and the arms parallel and perpendicular to the beam. It is, of course, important that all types of calipers be held perpendicular to the axis of the tree stem at the point of measurement.

Rapid measurement of trees is possible with calipers, particularly when only one measurement is taken per tree. Eccentric trees may be treated as explained in Section 2-1.2.

Diameter Tape. The diameter of a tree cross section may be obtained with a flexible tape by measuring the "circumference" of the tree and dividing by π ($D = C/\pi$). The diameter tapes used by foresters, however, are graduated at intervals of π units (inches or centimeters), thus permitting a direct reading of diameter. These tapes are accurate only for trees that are circular in cross section. In all other cases, the tape readings will be slightly too large, because the circumference of a circle is the shortest line that can encompass any given area.

The diameter tape is convenient to carry and, in the case of eccentric trees, requires only one measurement. Although it is slower to use than other diameter measuring instruments, the time element is generally not important. Care must be taken that the tape is correctly positioned at the point of measurement, that it is kept in a plane perpendicular to the axis of the stem, and that it is set firmly around the tree trunk.

Biltmore Stick. The Biltmore stick, which can hardly be classed as a measuring instrument, is an aid in estimating diameter at breast height. It consists of a straight rule, normally 24 to 36 inches long, that is held perpendicular to the axis of the tree stem. By holding the stick (Fig. 2-2) so that the 0-point of the graduation at one end of the stick lies on the line EA that is tangent to the tree cross section at A, the diameter of the tree can be read at the intersection at the other end of the stick on

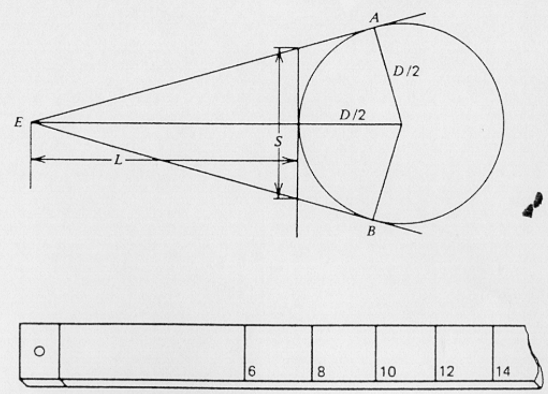


Fig. 2-2 *Biltmore stick.*

the line EB that is tangent to the tree cross section at B. The distance, L, from the eye, E, to the tree is usually 25 inches; however, it may have some other value. The graduations, S, of the stick for different values of D and L are obtained from the following formula.

$$S = \sqrt{\frac{D^2 L}{L + D}} \tag{2-1}$$

Inaccuracies in estimating diameters with the Biltmore stick are due to (1) the difficulty of holding the stick exactly L inches from the eye, (2) the failure to keep the eye at breast height level, (3) the failure to hold the stick at breast height level, and (4) the eccentricity of tree cross sections (the Biltmore stick is correct only for circular cross sections).

Sector Fork. Bitterlich's sector fork (*Visiermesswinkel*), Fig. 2-3, is similar in principle to the Biltmore stick (Bitterlich, 1959). It determines diameter from a sector of a circular cross section. One side of the sector is a metal arm; one side is a line of sight. The line of sight intersects a curved scale on which diameters or cross-sectional areas are printed. It is not necessary to hold the instrument at a fixed distance from the eye because a sighting pin fixes the line of sight for any distance. The instrument is especially suited for measuring trees with diameters of less than 50 centimeters (about 20 inches).

Because many trees are eccentric in cross section, all instruments for measuring tree diameter will in the long run give results that average too large. However, when an accurate determination of cross-sectional area is of prime importance, the calipers will give the best results. But when different people caliper the same irregular tree,

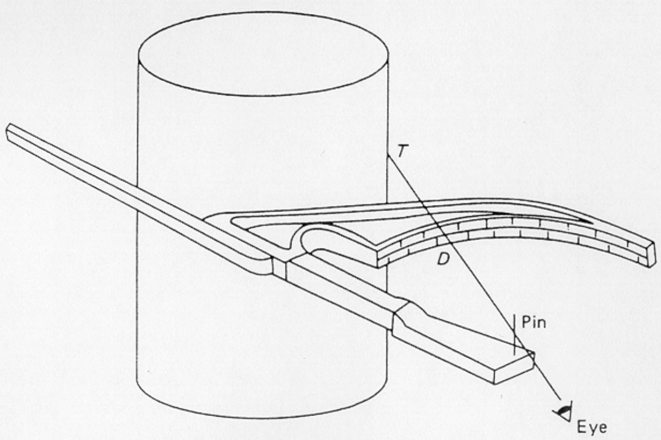


Fig. 2-3 *Sector fork (Visiermesswinkel). (T is the point of tangency of the line of sight; D is the diameter reading on the scale.)*

there will always be some variation in the measurements. A good part of this variation results because the tree is not always calipered in the same direction. Since this element of the variation does not affect measurements with the diameter tape, tape measurements are more consistent. Such consistency is important in growth studies, when the same trees are remeasured at intervals. Then, the actual diameters of the trees are less important than the changes in diameter during the period between measurements. These changes will be accurately determined by the diameter tape. Errors due to eccentricity will appear in both measurements and will not significantly affect the difference between them. In any case, whenever repetitive diameter measurements are made to determine growth, it is desirable to mark the position of measurement with a scribe or paint mark.

When the average of a number of diameter measurements, d, is required, one might use the arithmetic mean. However, if the primary interest is to obtain an average for the calculation of cross-sectional area and volume, then the quadratic mean, Q, is more appropriate.

$$Q = \sqrt{\frac{\sum_{i=1}^{n} d_i^2}{n}} \tag{2-2}$$

The quadratic mean gives the diameter of the tree of arithmetic average cross-sectional area.

2-1.4 Upper-Stem Diameters

Tree stem diameters above breast height are often required to estimate form or taper and to compute the volume of sample trees from the measurement of diameters at several points along the stem. Upper-stem diameters can be obtained by climbing a tree and using the instruments described in Section 2-2.3, or by mounting calipers on a pole (Ferree, 1946), or by mounting a diameter tape on a pole (Godman, 1949). When the caliper or tape is mounted on a pole, diameters are usually taken at the top of the first log (about 17 feet) to determine Girard form class (Section 8-6.2).

The most practical dendrometers[1] for measuring upper-stem diameters employ optical means that allow diameters to be determined from the ground at some distance from a tree. Optical dendrometers may be classified (Grosenbaugh, 1963a; Smith, 1970) as optical forks, optical calipers, fixed-base rangefinders, and fixed-angle rangefinders.

Optical forks. An optical fork employs a fork angle on which the lines are tangent to the tree cross section at the level of the diameter measurement and on which the

[1] There is considerable ambiguity in the use of the term *dendrometer*. In one usage, the term denotes an instrument that takes diameters at points above breast height. The Society of American Foresters (1971), however, collaborating with the International Union of Forest Research Organizations, defines a dendrometer as "Generally, any instrument for measuring the dimensions of trees or logs . . . ." or "More specifically, any instrument for measuring or estimating the diameters of trees or logs."

$$d = 2r = 2R \sin \frac{a}{2}$$

$$\text{Since: } \sin \frac{a}{2} = \tan \frac{a}{2} \cos \frac{a}{2}$$

$$\text{then: } d = 2R \tan \frac{a}{2} \cos \frac{a}{2}$$

$\tan \frac{a}{2}$ can be expressed in terms of angles ϕ and $\frac{\theta}{2}$ since:

$$\tan \frac{a}{2} = \sec \phi \tan \frac{\theta}{2}$$

and:

$$d = 2R \sec \phi \tan \frac{\theta}{2} \cos \frac{a}{2}$$

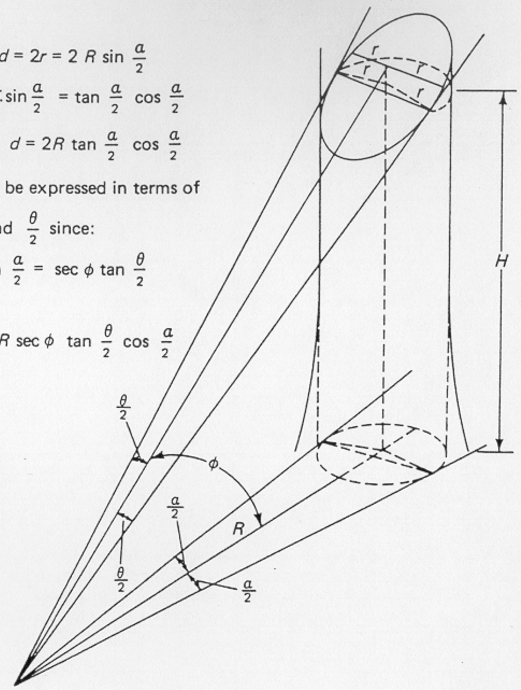


Fig. 2-4 *Principle of the optical fork.*

vertex is at the observer's eye. The basic geometry is shown in Fig. 2-4. Note that the cos (a/2) can be approximated by using cos (θ/2).

Optical forks vary in design and use, depending on whether the fork angle is fixed or variable or whether it is coupled to the vertical angle. If the fork angle is fixed, the distance from the tree must be varied. If it is varied, a fixed distance from the tree may be used. When the fork angle is coupled to the vertical angle, the need to measure it and introduce it in subsequent calculations is eliminated.

There are several instruments and methods based on the optical fork principle. These include the Spiegel relaskop, the Tele-relaskop with 8 power magnification (Bitterlich, 1978), the use of terrestrial photogrammetry (Ashley and Roger, 1969), and the use of a transit (Leary and Beers, 1963). With the transit neither the vertical angle ϕ nor the angle θ/2 need be measured (Fig. 2-4). The horizontal angle is formed on the plates of the transit when one sights to the selected upper-stem diameter.

Upper-stem diameter is then

$$d = 2R \sin \frac{a}{2}$$

or for small angles,

$$d = R \sin a$$

A modification of the engineer's transit, the transit dendrometer, has been developed and placed on the market by an American forestry supply company.

Optical Calipers. Basically, an optical caliper is made up of a graduated scale, two penta prisms, and a sighting tube that may, or may not, have an optical system to produce magnification (Fig. 2-5a). (The sighting tube may be omitted, if magnification is not used.) One of the penta prisms is fixed in place at the zero mark on the scale; the other is mounted so it may be moved along the scale at right angles to a direct line of sight to the tree. The observer sights over the fixed prism to the point of tangency on the left side of the tree cross section and, at the same time, through both prisms to the right side of the tree cross section. The sliding prism is moved along the scale until the line of sight through the prism is tangent to the right side of the tree cross section (i.e., until the "picture" shown in Fig. 2-5b is obtained).

Since the measurements obtained with the optical caliper are based on the assumption that the lines of sight to the tree are parallel, the instrument is, in effect, a "long-armed" caliper. When the instrument is properly oriented and sighted, diameters may be read directly from the scale for any cross section on the tree stem or on a limb. Accuracy is limited by the degree of parallelism of the lines of sight and by the limitations of the human eye. Without magnification, if parallelism obtains, diameters may be obtained by the average individual within a range of ± 0.1 inches at 50 feet, ± 0.2 inches at 100 feet, and so on. With magnification, accuracy will, both theoretically and practically, increase directly as the power of the optical system increases.

For a penta prism the deviation, D, of a light ray will be $D = 2(90° - A)$, where A is the main prism angle (Fig. 2-5). Thus, if A equals 45 degrees, a deviation of 90 degrees will be produced. This deviation will be obtained even if the line of sight is not perpendicular to the prism face which it enters. Consequently, in an optical caliper, collimation and adjustment are simple. If total reflection prisms (two angles of 45 degrees and one of 90 degrees) or mirrors are used, collimation and adjustment are difficult. Several instruments of this type have been devised. The Wheeler penta prism caliper (Wheeler, 1962) is the one most commonly used. This instrument is available with a base line that will permit measurement of diameters up to 34 inches. The instrument may be mounted on a tripod to obtain greater precision.

Fixed-Base Rangefinders. The principle of the fixed-base rangefinder is embodied in the Barr and Stroud dendrometer (Hummel, 1951; Jeffers, 1956; Grosenbaugh, 1963a). From a fixed base the optics of the system are manipulated so that an angle is varied to measure the required distances (Fig. 2-6).

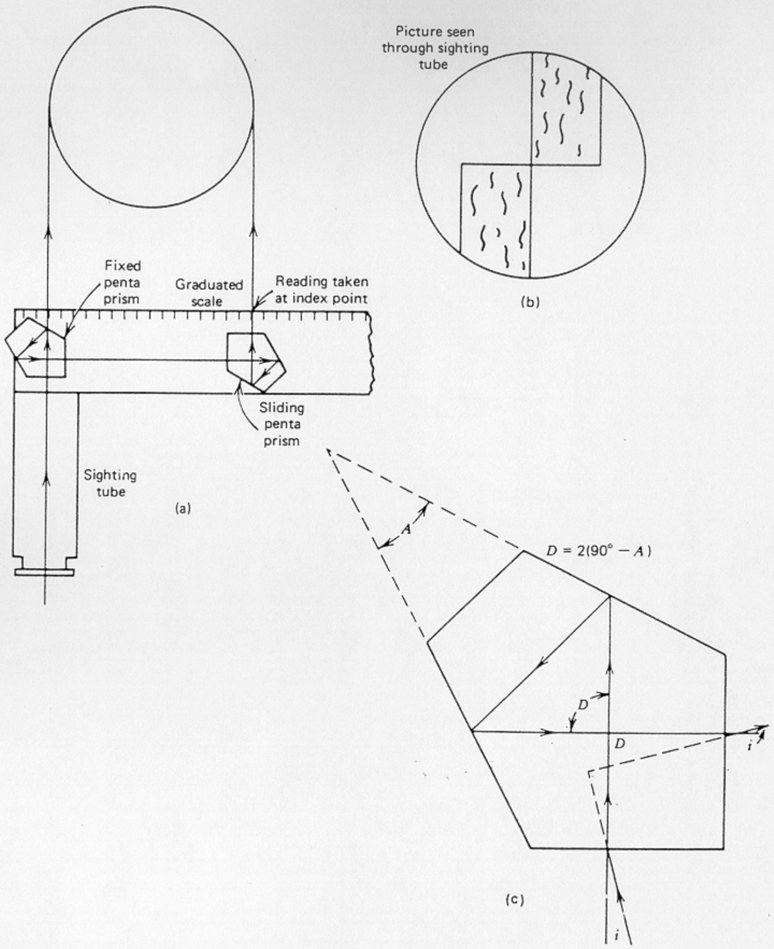
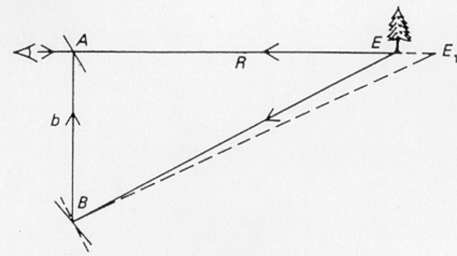


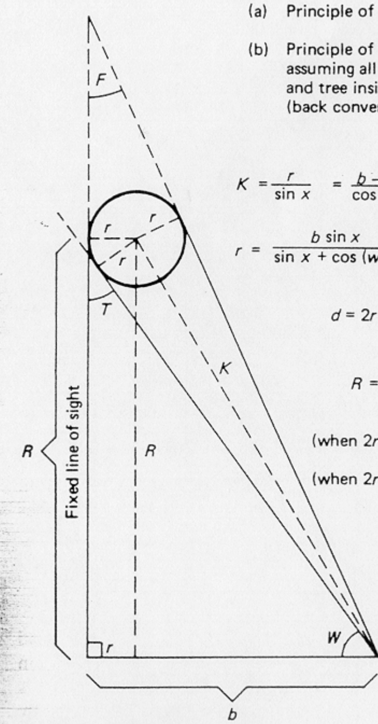
Fig. 2-5 *The optical caliper.*

Fixed-Angle Rangefinders. The principle of the fixed-angle rangefinder is embodied in the Breithaupt Todis dendrometer (Eller and Keister, 1979). From a fixed angle the optics of the system are manipulated, so that the baseline is varied to measure the required distances.



(a) Principle of rangefinder.

(b) Principle of rangefinder dendrometer – assuming all sights in a horizontal plane and tree inside fixed line of sight (back convergence).



$$K = \frac{r}{\sin x} = \frac{b-r}{\cos(w+x)} = \frac{b}{2\sin\frac{1}{2}T\cos\frac{1}{2}F}$$

$$r = \frac{b\sin x}{\sin x + \cos(w+x)} = \frac{b(\sin\frac{1}{2}T\cos\frac{1}{2}F - \cos\frac{1}{2}T\sin\frac{1}{2}F)}{2\sin\frac{1}{2}T\cos\frac{1}{2}F}$$

$$d = 2r = b\left(1 - \frac{\tan\frac{1}{2}F}{\tan\frac{1}{2}T}\right)$$

$$R = \frac{b}{2}(\cot\tfrac{1}{2}T - \tan\tfrac{1}{2}F)$$

(when $2r = b$, $F = 0$ and $\tan\frac{1}{2}F = 0$)

(when $2r > b$, $\tan\frac{1}{2}F$ becomes negative)

Fig. 2-6 *Rangefinder dendrometer.*

In the foregoing discussion the optical caliper has been given the most space because this instrument is one of the best for measuring upper-stem diameters and because it can be constructed by a novice. Optical components are available from optical supply companies at reasonable prices. However, if the student desires to delve deeper into the construction and use of other optical devices, we suggest Smith (1970), one of the best books available on the subject.

2-1.5 Precise Measurement of Diameter Change

Precise measurement of minute changes in diameter may be required in research. Such changes, which may be for periods as short as an hour, cannot be detected with calipers or diameter tapes. Consequently, dendrometer bands, dial-gauge micrometers, recording dendrographs, and transducers are used to measure minute changes. Dendrometer bands, as described by Liming (1957), Bower and Blocker (1966), and Yocom (1970), consist of aluminum or zinc bands with vernier scales. The band is placed around the stem of a tree and held taut with a spring. Changes in diameter as small as 0.01 inches can be read.

A dial-gauge micrometer was first described by Reineke (1932). This instrument utilized a stationary reference point, a hook screwed into the xylem. The distance from this fixed point to a metal contact glued to the bark is measured with the micrometer. Changes in diameter as small as 0.001 inches can be read. Daubenmire (1945) modified this instrument by inserting three screws into the xylem as the fixed reference point.

A precision dendrograph was devised by Fritts and Fritts (1955). The instrument consists of a pen on an arm bearing on a fixed point on the tree stem. The pen records diameter changes to 0.001 inches on a chart mounted on the drum of an eight-day clock. Phipps and Gilbert (1960) designed an electrically operated dendrograph similar in principle to the dial-gauge micrometer. A potentiometer was fixed to a tree by screws anchored in the xylem. A movable shaft was fixed to the outer bark; any displacement in the shaft was measured by a change in electrical resistance and recorded on a continuous strip chart. Imprens and Schalck (1965) used a variable differential transformer rather than a potentiometer in a similar instrument.

A transducer was designed by Kenerson (1973). The transducer uses a linear motion potentiometer fixed to an invar plate. When the device is fixed to a tree stem, changes in the stem diameter move the potentiometer shaft.

2-1.6 Crown Diameter

Crown diameter measurements are useful to estimate diameter at breast height and therefore tree volume. They are best made on vertical aerial photographs (Section 13-5). Field determination is difficult because of the irregularity of a tree crown's outline. The usual field technique is to project the perimeter of the crown vertically to the ground and to make "diameter" measurements on this projection. Most of the instruments used to achieve the vertical projection incorporate a mirror, a right-angle prism, or a penta prism; they may be hand-held or staff-mounted. Instruments that employ the penta prism are recommended because they do not invert or revert images, and because slight movement of the prism will not affect the true right angle of

reflection. Instruments for determining the vertical projection have been described by Husch (1947), Nash (1948), Raspopov (1955), and Shepperd (1973).

Crown diameters measured on vertical aerial photographs are more clearly defined than those measured on the ground, although crown diameters measured on aerial photographs are smaller than those measured on the ground, because parts of the crown are not resolved on photographs. However, the photo measurement is probably a better measure of the functional growing space of a tree and is better correlated with tree and stand volume.

2-2 HEIGHT MEASUREMENT

Height is the linear distance of an object normal to the surface of the earth, or some other horizontal datum plane. Aside from land elevation, tree height is the principal vertical distance measured in forestry.

At the outset one should understand that "tree height" is an ambiguous term unless it is clearly defined. Thus, a logical classification of height measurements that can be applied to standing trees of either excurrent or deliquescent form is shown in Fig. 2-7. Here are the pertinent definitions.

Total height is the distance along the axis of the tree stem between the ground and the tip of the tree. (In determining this height the terminals—i.e., the base and tip of the tree—can be more objectively determined than other points on the stem. It is often difficult, however, to see the tip of a tree in closed stands and to determine the uppermost limit of a large-crowned tree.)

Bole height is the distance along the axis of the tree stem between the ground and the crown point. (As shown in Fig. 2-7, the crown point is the position of the first crown-forming branch. Therefore, bole height is the height of the clear, main stem of a tree.)

Merchantable height is the distance along the axis of the tree stem between the ground and the terminal position of the last usable portion of the tree stem. (The position of the upper terminal is somewhat subjective. It is taken at a minimum top diameter or at a point where branching, irregular form, or defect, limit utilization. The minimum top diameter will vary with the intended use of the timber and with market conditions. For example, it might be 4 inches for pulpwood and 8 inches for sawtimber.)

Stump height is the distance between the ground and the basal position on the main stem where a tree is cut. (A standard stump height, generally about a foot, is established for volume table construction and timber volume estimation.)

Merchantable length is the distance along the axis of the tree stem between the top of the stump and the terminal position of the last usable portion of the tree stem.

Defective length is the sum of the portions of the merchantable length that cannot be utilized because of defect.

Sound merchantable length equals the merchantable length minus the defective length.

Crown length is the distance on the axis of the tree stem between the crown point and the tip of the tree.

2-2.1 Tree Height and Length Measurements

Generally speaking, the techniques and instruments devised for general height measurement may be applied to tree-height measurement. However, instruments must be economical, light, portable, and usable in closed stands.

Total tree heights may be estimated from measurements made on aerial photographs. The use of photo height measurements is discussed in Chapter 13.

The heights of short trees can be measured directly with an engineer's self-reading level rod, a graduated pole, or similar devices. The heights of many taller trees can be measured directly, or accurately estimated, with the aid of sectional or sliding poles made of wood, fiber glass, or lightweight metal. Although measurement with poles is slow, they are often used to measure height on continuous forest inventory plots where high accuracy is desired and merchantable length is less than 70 feet.

Most height measurements of tall trees are taken indirectly with hypsometers (*hypso,* meaning height, plus *meter,* meaning an instrument for measuring). Hypsometers are based on the relation of the legs of similar triangles or on the tangents of angles. A comprehensive summary of hypsometers has been compiled by Hummel (1951). Patrone (1963) and Pardé (1961) describe the more popular European hypsometers. (Note that terms such as altimeter and clinometer are also applied to instruments that are used to measure height.)

2-2.2 Hypsometers Based on Similar Triangles

The Christen, Klausner, Merritt, Chapman, and JAL hypsometers are examples of instruments of this type. Of these, only the Christen and Merritt hypsometers are in general use in the United States. Consequently, only these two instruments will be described in this section.

Christen Hypsometer. This hypsometer consists of a scale about 10 inches long (Fig. 2-8). To use it, a pole (usually 5 or 10 feet long) is held upright against the base of the tree, or a mark is placed on the tree at a height of 5 or 10 feet above the ground. The hypsometer is then held vertically at a distance from the eye such that the two inside edges of the flanges are in line with the top and base of the tree. It may be necessary for the observer to move closer to or farther from the tree to accomplish this, but except for this, the distance from the tree is immaterial. The graduation on the scale that is in line with the top of the pole, or the mark, gives the height of the tree. The following proportion gives the formula for graduating the instrument.

$$\frac{A'C'}{AC} = \frac{A'B'}{AB} \qquad (2\text{-}3)$$

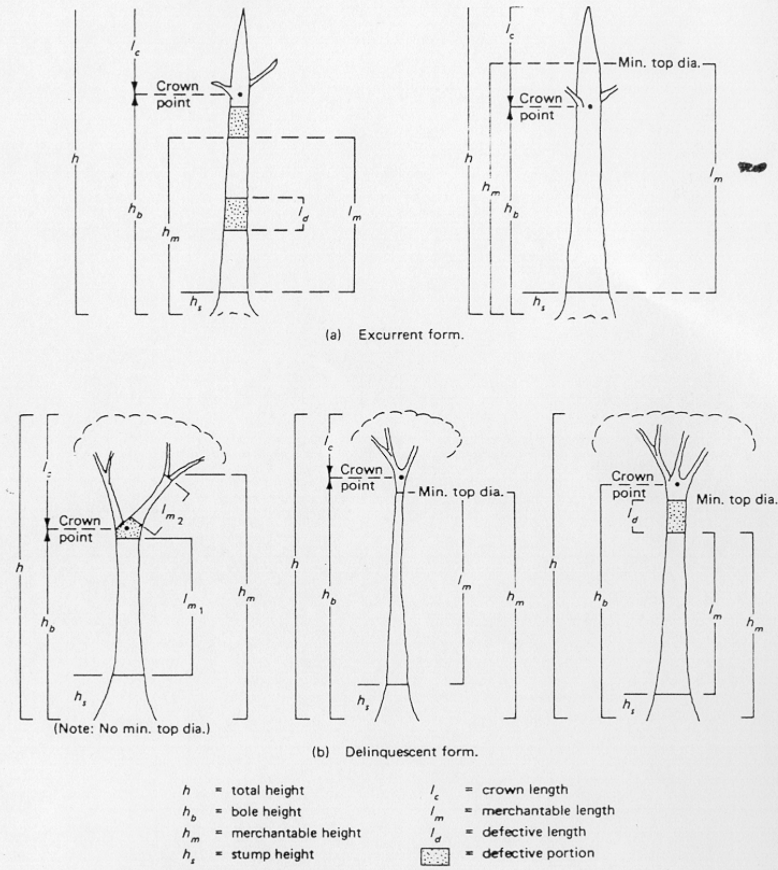


Fig. 2-7 *Tree height and stem length classification.*

For a given length of instrument $A'B'$ and a given pole length or mark height AC, the graduations $A'C'$ can be obtained by substituting different values of height AB in the equation.

Merritt Hypsometer. This simple instrument, which is often combined with the Biltmore stick, is a convenient aid in estimating the number of logs in a tree. It consists of a straight graduated stick that is held vertically at a predetermined distance, usually

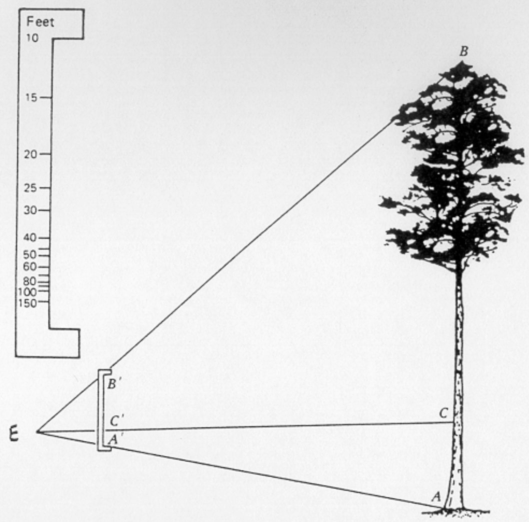


Fig. 2-8 *Christen hypsometer.*

25 inches, from the eye. (If the stick is held 25 inches from the eye along a horizontal line, then the distance to the tree should be measured on the horizontal; if the stick is held 25 inches from the eye along a line to the lower end of the stick, as shown in Fig. 2-9, then the distance to the tree should be measured on the slope.) The observer must stand at a predetermined distance, such as 50 feet, from the tree. Then the height of the tree may be read on the stick at the point where the line of sight to the top terminal on the tree intersects the scale. In Fig. 2-9,

$$\frac{A'B'}{AB} = \frac{EA'}{EA} \tag{2-4}$$

Now, if EA' is 25 inches and EA is 50 feet, for each 16 feet of height in AB the length of $A'B'$ will be 8.00 inches. The stick may thus be graduated in 8-inch intervals with the successive graduations marked 1, 2, 3 (indicating 16-foot logs). When merchantable length is measured, as it often is with the Merritt hypsometer, one should remember that the line of sight EA (Fig. 2-9) must be to the top of the stump.

Although the Christen hypsometer may be used to measure any type of height, it is practical only for total height measurements. Furthermore, an examination of Fig. 2-8 shows a crowding of scale graduations at the bottom of the scale. This makes the instrument unreliable for the determination of the height of tall trees.

In case of the Merritt hypsometer, it is difficult to hold the stick in a vertical position exactly 25 inches from the eye. Small deviations in orientation result in considerable

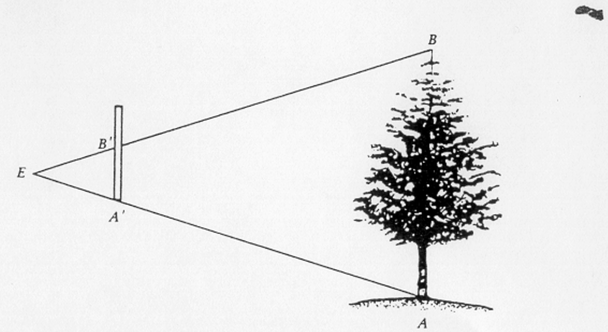


Fig. 2-9 *Merritt hypsometer.*

errors in height readings. Therefore, this instrument should be used only to make rough checks on ocular height estimates.

2-2.3 Hypsometers Based on Tangents of Angles

Although numerous hypsometers of this type have been developed, the basic principle is the same for all of them. One first sights to the upper terminal of the height desired and takes a reading; then one sights to the base of the tree, or the top of the stump, depending on the height desired, and takes a second reading. Figure 2-10a illustrates the situation, with the distance D and the angles α_1 and α_2 known. Then, if the vertical arc is graduated in degrees, the total height of the tree may be calculated as follows.

$$\tan \alpha_1 = \frac{BC}{D}$$

from which we obtain

$$BC = D \tan \alpha_1$$

Similarly,

$$CA = D \tan \alpha_2$$

Since the height of the tree AB is $BC + CA$,

$$AB = D(\tan \alpha_1 + \tan \alpha_2) \tag{2-5}$$

On steep ground a situation as shown in Fig. 2-10b might occur. In this case, the height of the tree AB is $BC - CA$, and

$$AB = D(\tan \alpha_1 - \tan \alpha_2)$$

Hypsometers often are provided with percentage arcs and topographic arcs. The percentage arc is based on an angular unit represented by the ratio of 1 unit vertically to 100 units horizontally. It also means for any vertical angle α

$$\tan \alpha = \frac{\text{percent } \alpha}{100}$$

Thus, when using a percentage arc (Fig. 2-10a),

$$AB = \frac{D}{100} (\text{percent } \alpha_1 + \text{percent } \alpha_2)$$

The percentage arc simplifies height calculations, particularly when readings are taken at such distances as 100, 75, 50, and 25 feet from a tree.

The topographic arc (topo α) is based on an angular unit represented by the ratio of 1 unit vertically to 66 units horizontally. It also means for any vertical angle α

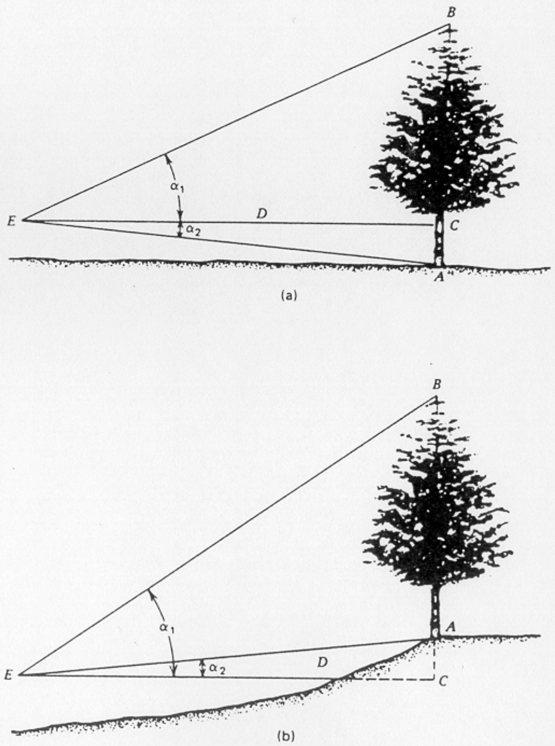


Fig. 2-10 *Measuring tree height with hypsometers based on the tangents of angles.*

$$\tan \alpha = \frac{\text{topo } \alpha}{66}$$

Thus, when using a topographic arc (Fig. 2-10a),

$$AB = \frac{D}{66} (\text{topo } \alpha_1 + \text{topo } \alpha_2)$$

More details on the topographic arc are given in Chapter 3. However, the topographic arc is as convenient to use as the percentage arc if readings are taken at such distances as 99, 66, and 33 feet from a tree.

The relationship between topographic and percentage readings is as follows

$$\text{Percent } \alpha = \frac{100 \text{ topo } \alpha}{66}$$

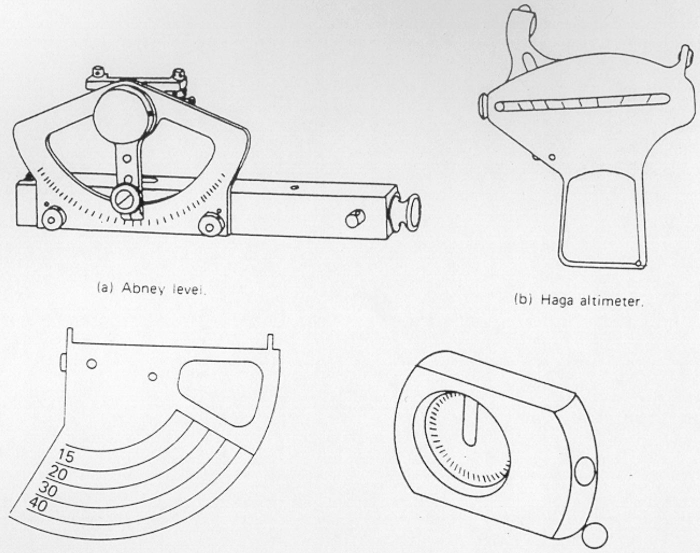
Hypsometers may also be provided with arcs based on angular units represented by the ratio of 1 unit vertically to 15, 20, 25, 30, 45, 60, or other, units horizontally. One should note that scales of this type (and this includes percentage and topographic scales) can be used with any units: feet, yards, meters, and so on. One only has to read the scale in the same unit as one measures the baseline. However, since scale numbers usually represent convenient base distances for tree measurement with a particular unit, the unit to use is generally indicated.

Tree heights might be measured with a transit. However, the transit is too clumsy and too slow for practical forest work. Furthermore, the precision is greater than necessary. Consequently, lighter, simpler, and more economical instruments are used. The most commonly used hypsometers in the United States are the Abney level (Calkins and Yule, 1935), the Haga altimeter (Wesley, 1956), the Blume-Leiss altimeter (Pardé, 1955), and the Suunto clinometer. The Spiegel relaskop (Daniel, 1955) is also used for height measurement.

Abney Level. This instrument consists of a graduated arc mounted on a sighting tube about 6 inches long (Fig. 2-11a). The arc may have a degree, percentage, or topographic scale. When the level bubble, which is attached to the instrument, is rotated while a sight is taken, a small mirror inside the tube makes it possible to observe when the bubble is horizontal. Then the angle between the bubble tube and the sighting tube may be read on the arc.

Haga Altimeter. This instrument consists of a gravity-controlled, damped, pivoted pointer and a series of scales on a rotatable, hexagonal bar in a metal, pistol-shaped case (Fig. 2-11b). The six regular American scales are 15, 20, 25, 30, percentage, and topographic. Sights are taken through a gun-type peep sight; the indicator needle is locked by squeezing a trigger, and the observed reading is taken on the scale. A rangefinder is available with this instrument.

Blume-Leiss Altimeter. This instrument is similar in construction and operation to the Haga altimeter, although its appearance is somewhat different (Fig. 2-11c). The five regular metric scales are 15, 20, 30, and 40. A degree scale is also provided. All scales can be seen at the same time. A rangefinder is incorporated into the instrument.



(a) Abney level. (b) Haga altimeter. (c) Blume-Leiss. (d) Suunto clinometer.

Fig. 2-11 *Examples of hypsometers based on trigonometric principles.*

Suunto Clinometer. This instrument is a hand-held device housed in a corrosion-resistant lightweight alloy body (Fig. 2-11d). The scale is supported by a jewel-bearing assembly, and all moving parts are immersed in a damping liquid inside a hermetically sealed plastic capsule. The liquid dampens undue scale vibrations. The instrument is held to one eye and raised or lowered until the hairline is seen at the point of measurement. At the same time, the position of the hairline on the scale gives the reading. Due to an optical illusion, the hairline seems to continue outside the frame and can be observed at the point of measurement. The instrument has provisions for only two scales. However, in the United States the instrument is available with the following scale combinations: percentage and degree, percentage and topographic, topographic and degree. It is also available with a rangefinder.

Generally speaking, hypsometers based on the tangents of angles are more accurate than hypsometers based on similar triangles. The Abney level, the Haga altimeter, the Blume-Leiss Altimeter, and the Suunto clinometer are similar in accuracy. The Abney level, however, is slower to use, and large vertical angles are difficult to measure, because of the effect of refraction on observations of the bubble through the tube from beneath. This makes the Abney level difficult to use in tall timber that is so dense that the tops cannot be seen from a considerable distance. A choice among the other three instruments is largely a matter of personal preference.

Curtis and Bruce (1968) have shown how a clinometer that measures slope in percent can be used in conjunction with a pole of fixed length to measure height without measuring the distance from the observer to the tree. The principle is similar to that of the Christen hypsometer, an instrument specifically designed to eliminate the need to measure the distance from the observer to the tree. Using the Curtis-Bruce method, a pole of known length (between one-fourth and one-fifth of tree height) is placed against the tree. Then slopes are read from any convenient point to tree top α_t, to pole top α_p, and to tree base α_b. When p is the pole height, tree height h is

$$h = p\,\frac{\alpha_t - \alpha_b}{\alpha_p - \alpha_b}$$

Bell and Gourley (1980) point out that this method simplifies locating a point in a dense stand where the tree tip is visible.

When the distance of the observer from the tree is to be determined, a suitable method of determining distance should be selected. With the Merritt hypsometer, which is not highly accurate, pacing is appropriate; but with the other, more accurate, instruments, a tape or a rangefinder should be used.

2-2.4 Special Considerations in Measuring Heights with Hypsometers

It is difficult to accurately measure the height of large, flat-crowned trees, such as the oaks, elms, maples, and so on. There is a tendency to overestimate their height. Consequently, total height determinations for such trees are of little value and are seldom taken.

In general, the *optimum viewing distance* for any hypsometer is the distance along the slope equal to the height to be measured. This rule of thumb, which is adapted from Beers (1974), should be used with discretion. For example, if one were using a Suunto clinometer with a percentage scale to determine the merchantable height of a tree estimated to be about 56 feet, it would be logical to use a viewing distance of 50 feet.

Since all hypsometers assume trees are vertical, trees leaning away from an observer will be underestimated and trees leaning toward an observer will be overestimated. This error will be minimized if measurements are taken such that the lean is to the left or right of the observer. If one must measure leaning trees, one should determine the point on the ground where a plumb bob would fall if suspended from the tip of the tree. Then height should be measured from this point to the tip. The measured height is multiplied by the secant of the angle of lean ϕ to obtain the correct height (Fig. 2-12). The correction will be small except for abnormally leaning trees.

2-2.5 Ocular Estimates of Height

The measurement of tree height with an accurate hypsometer is slow and expensive. Consequently, it is customary to make ocular estimates whenever precision is not essential, such as for commercial timber inventories where none of the required values

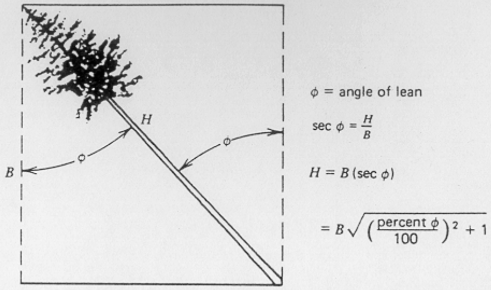


Fig. 2-12 *Measurement of the height of a leaning tree.*

are precisely determined, and where the large number of trees measured makes precision of individual measurements unimportant. Good ocular estimates can be obtained by an experienced person. Ocular estimates are, however, subject to serious errors because of sudden changes in the timber type or the weather. Furthermore, most people do not make reliable estimates at the start of the day or after a rest. Consequently, estimates should be checked frequently by instrumental measurements. Although the Merritt hypsometer is often used for this purpose, we recommend that one use the Haga altimeter, the Blume-Leiss altimeter, the Suunto clinometer, or a similar lightweight hypsometer.

2-3 EXPRESSIONS OF STAND HEIGHT

It is often necessary to use a single value to characterize the height of a forest stand. The principal applications of such values are to determine stand volume from a stand volume equation and to determine site index from a curve or an equation. The following guidelines describe the more important methods that have been proposed to obtain the average height of a stand.

1. Measure and average the heights of all of the trees, or a sample of the trees, regardless of their size or relative position in the stand. (This method is excessively time consuming.)

2. Measure and average the heights of the dominant trees, or of the dominant and codominant trees. (The selection of a dominant or a codominant tree is subjective. Consequently, different observers often obtain different results. This height expression is commonly used in determining the site index in North America.)

3. Measure and average the heights of a fixed number of the largest trees (normally, largest in diameter, although it could be largest in height) per unit area, usually 100 per acre. (This height expression is used in Great Britain and New Zealand.)

4. Determine the average height that will yield the correct volume when using the following equation.

$$V = \bar{h}Gf \tag{2-6}$$

where

V = volume per unit area
G = total basal area per unit area
$\bar{h}$ = average height of the stand
f = form class of the stand

The best known average height of this type is Lorey's mean height h_L.

$$h_L = \frac{n_1g_1h_1 + n_2g_2h_2 + \cdots + n_zg_zh_z}{G} = \frac{\sum_{i=1}^{z} n_ig_ih_i}{\sum_{i=1}^{z} n_ig_i} \tag{2-7}$$

where

n_i = number of trees in a diameter class
g_i = average basal area of a diameter class
h_i = average height of the trees in a diameter class

Lorey's mean height is derived from equation 2-6 by considering that the stand is made up of a series of diameter classes.

$$Gh_Lf = n_1g_1h_1f_1 + n_2g_2h_2f_2 + \cdots + n_zg_zh_zf_z$$

By assuming a constant form factor (i.e., $f_1 = f_2 = \cdots = f_z = f_i$), equation 2-7 is obtained.

It should be noted that the arithmetic average height of the trees selected in horizontal point sampling (Chapter 14) yields Lorey's mean height (Kendall and Sayn-Wittgenstein, 1959).

In addition to the above, several expressions of average stand height have been developed that utilize a height-diameter curve constructed from sample-tree data representative of the stand. After one has prepared the curve, one decides what average diameter is representative of the stand and reads the height of a tree of this diameter from the curve. For example, the diameter of the tree of average basal area might be used (symbolized h_g); the arithmetic mean diameter might be used (symbolized h_d); the median diameter might be used (symbolized h_{dM}); or the diameter of the tree of median basal area might be used (symbolized h_{gM}).