

ESTRUCTURAS I

CURSO ESTRUCTURAS I

MARCOS ISOSTATICOS ENTRAMADOS Y MAQUINAS

■ Profesor: Jing Chang Lou

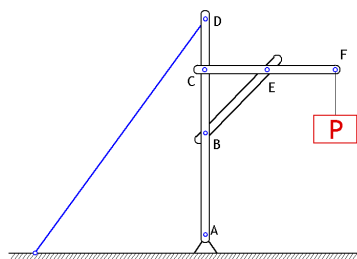
MARCOS ISOSTATICOS ENTRAMADOS Y MAQUINAS

ENTRAMADOS

Son estructuras normalmente fijas y estables.

Están diseñadas para soportar cargas.

Siempre contiene un elemento sometido a varias fuerzas.

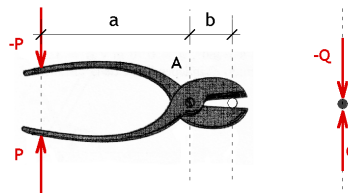


MAQUINAS

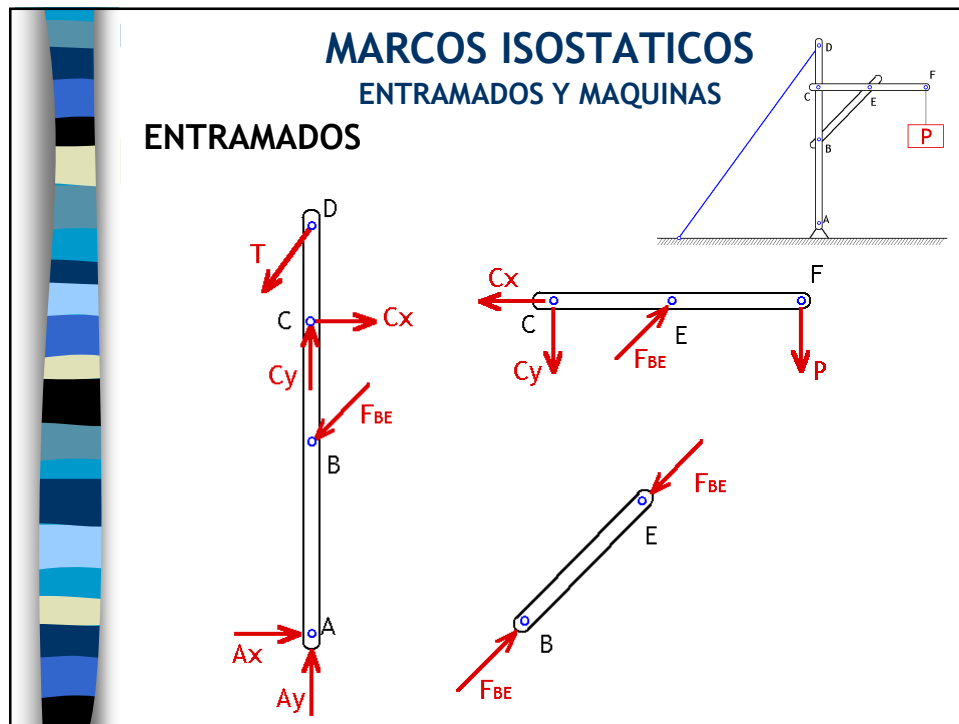
Son estructuras que contienen partes móviles.

Están diseñadas para transformar fuerzas iniciales en fuerzas finales.

Siempre contiene un elemento sometido a varias fuerzas.



ESTRUCTURAS I



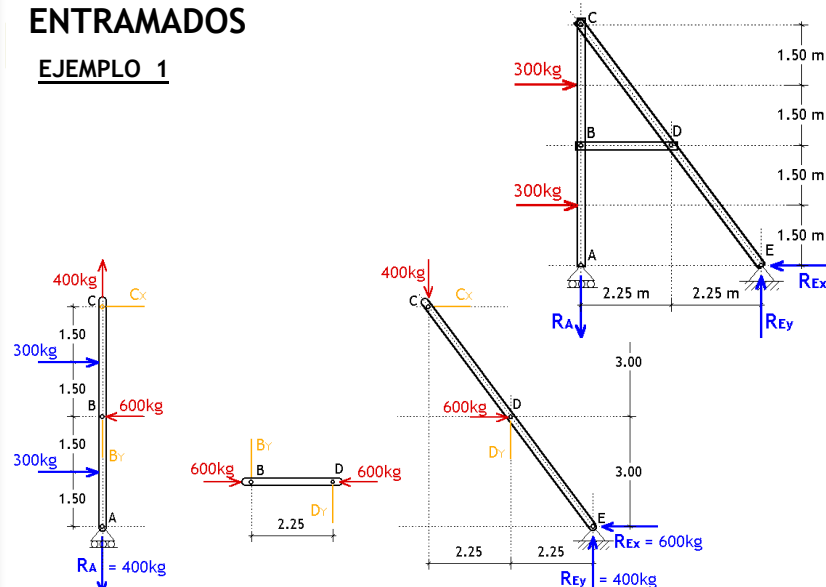
ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

EJEMPLO 1

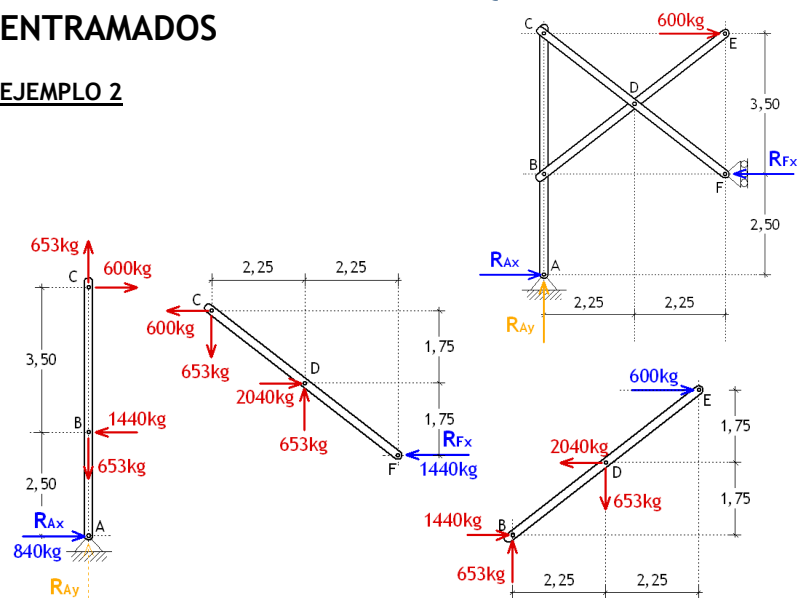


MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

EJEMPLO 2



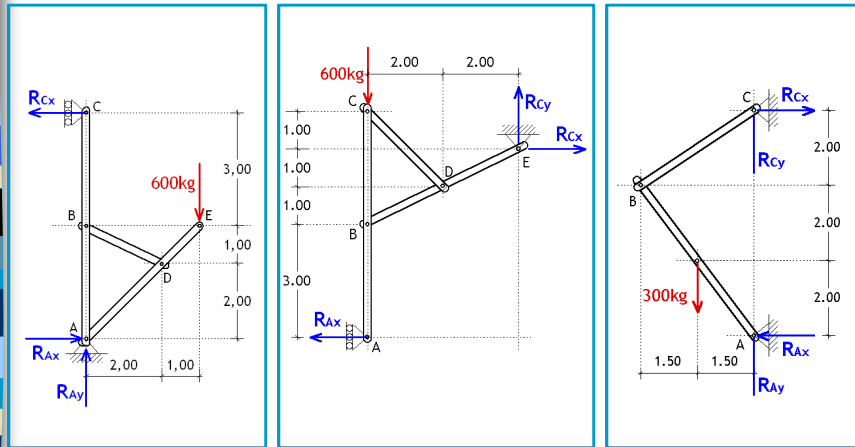
ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

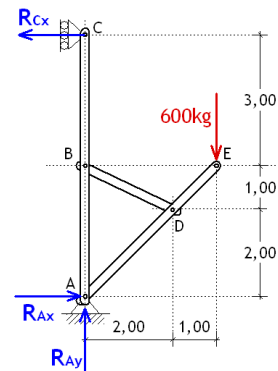
CASOS ESPECIALES 1

EQUILIBRIO EXTERNO

$$\begin{aligned}\sum M_A &= 0 \\ 600 \text{ kg} \cdot 3 \text{ m} - R_{Cx} \cdot 6 \text{ m} &= 0 \\ R_{Cx} &= 300 \text{ kg}\end{aligned}$$

$$\begin{aligned}\sum F_x &= 0 \\ R_{Ax} - R_{Cx} &= 0 \\ R_{Cx} &= 300 \text{ kg}\end{aligned}$$

$$\begin{aligned}\sum F_y &= 0 \\ R_{Ay} - 600 \text{ kg} &= 0 \\ R_{Ay} &= 600 \text{ kg}\end{aligned}$$



ESTRUCTURAS I

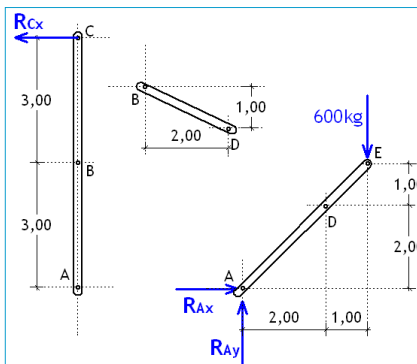
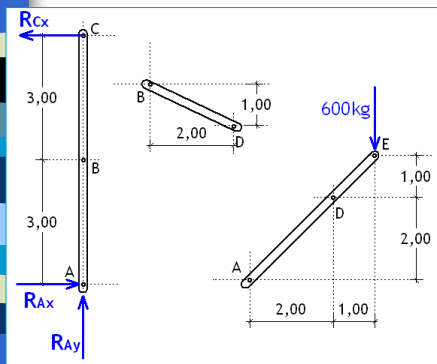
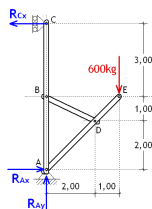
MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

EQUILIBRIO INTERNO

CASOS ESPECIALES 1



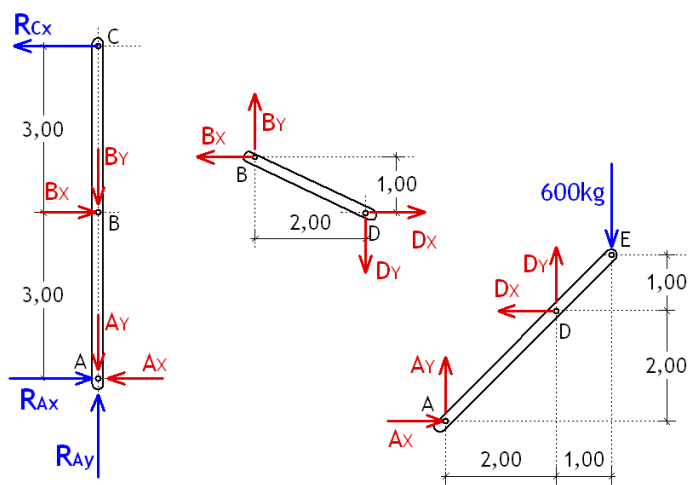
MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

EQUILIBRIO INTERNO

CASOS ESPECIALES 1a



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

ANALISIS INTERNO

BARRA ABC

$$\sum M_A = 0$$

$$-R_{Cx} * 6 \text{ m} - Bx * 3 \text{ m} = 0$$

$$-300 * 6 \text{ m} - Bx * 3 \text{ m} = 0$$

$$Bx = 600 \text{ kg}$$

$$\sum F_x = 0$$

$$R_{Ax} - Ax + Bx - R_{Cx} = 0$$

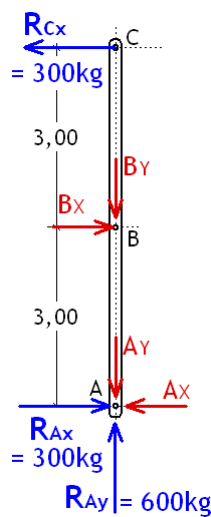
$$300 \text{ kg} - Ax + 600 \text{ kg} - 300 \text{ kg} = 0$$

$$Ax = 600 \text{ kg}$$

$$\sum F_y = 0$$

$$R_{Ay} - Ay - By = 0$$

CASOS ESPECIALES 1a



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

ANALISIS INTERNO

BARRA BD

$$\sum M_D = 0$$

$$By * 2 \text{ m} - Bx * 1 \text{ m} = 0$$

$$By * 2 \text{ m} - 600 \text{ kg} * 1 \text{ m} = 0$$

$$By = 300 \text{ kg}$$

$$\sum F_x = 0$$

$$Dx - Bx = 0$$

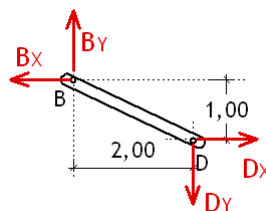
$$Dx = 600 \text{ kg}$$

$$\sum F_y = 0$$

$$By - Dy = 0$$

$$Dy = 300 \text{ kg}$$

CASOS ESPECIALES 1a



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1a

ANALISIS INTERNO

BARRA ADE

$$\sum M_D = 0$$

$$600 \text{ kg} \cdot 1 \text{ m} + A_y \cdot 2 \text{ m} - A_x \cdot 2 \text{ m} = 0$$

$$600 \text{ kg} \cdot 1 \text{ m} + A_y \cdot 2 \text{ m} - 300 \text{ kg} \cdot 2 \text{ m} = 0$$

$$A_y = 300 \text{ kg}$$

$$\sum F_x = 0$$

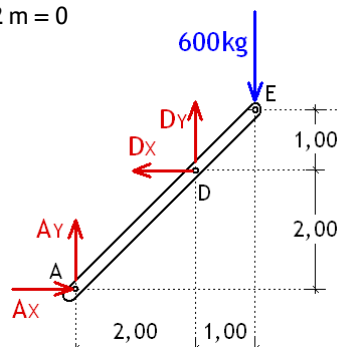
$$A_x - D_x = 0$$

$$600 \text{ kg} - 600 \text{ kg} = 0$$

$$\sum F_y = 0$$

$$A_y + D_y - 600 \text{ kg} = 0$$

$$300 \text{ kg} + 300 \text{ kg} - 600 \text{ kg} = 0$$



MARCOS ISOSTATICOS

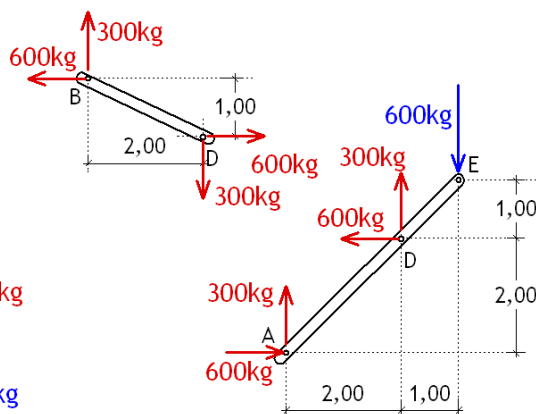
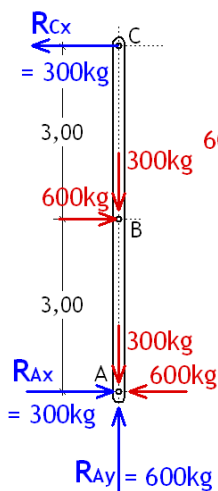
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1a

ANALISIS INTERNO

RESULTADOS



ESTRUCTURAS I

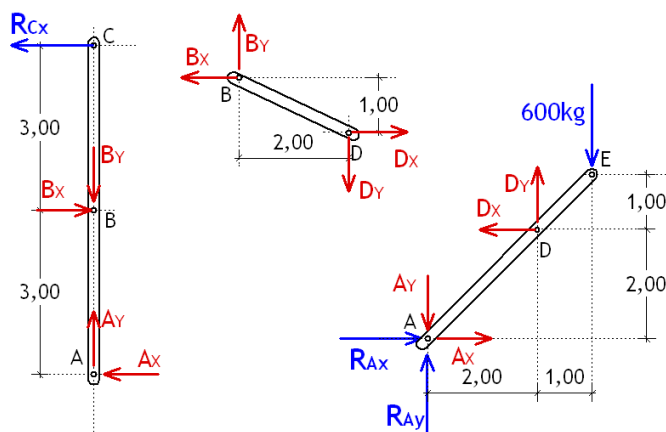
MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1b

EQUILIBRIO INTERNO



MARCOS ISOSTATICOS

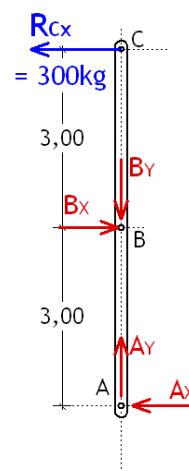
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1b

ANALISIS INTERNO BARRA ABC

$$\begin{aligned}\sum M_A &= 0 \\ -R_{Cx} * 6 \text{ m} - Bx * 3 \text{ m} &= 0 \\ -300 * 6 \text{ m} - Bx * 3 \text{ m} &= 0 \\ Bx &= 600 \text{ kg} \\ \sum F_x &= 0 \\ -Ax + Bx - R_{Cx} &= 0 \\ -Ax + 600 \text{ kg} - 300 \text{ kg} &= 0 \\ Ax &= 300 \text{ kg} \\ \sum F_y &= 0 \\ Ay - By &= 0\end{aligned}$$



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1b

ANALISIS INTERNO

BARRA BD

$$\sum M_D = 0$$

$$B_y * 2 \text{ m} - B_x * 1 \text{ m} = 0$$

$$B_y * 2 \text{ m} - 600 \text{ kg} * 1 \text{ m} = 0$$

$$B_y = 300 \text{ kg}$$

$$\sum F_x = 0$$

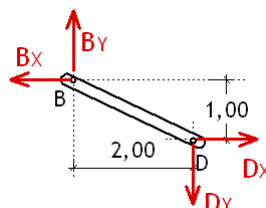
$$D_x - B_x = 0$$

$$D_x = 600 \text{ kg}$$

$$\sum F_y = 0$$

$$B_y - D_y = 0$$

$$D_y = 300 \text{ kg}$$



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1b

ANALISIS INTERNO

BARRA ADE

$$\sum M_A = 0$$

$$600 \text{ kg} * 3 \text{ m} - D_y * 2 \text{ m} - D_x * 2 \text{ m} = 0$$

$$600 \text{ kg} * 3 \text{ m} - D_y * 2 \text{ m} - 300 \text{ kg} * 2 \text{ m} = 0$$

$$D_y = 300 \text{ kg}$$

$$\sum F_x = 0$$

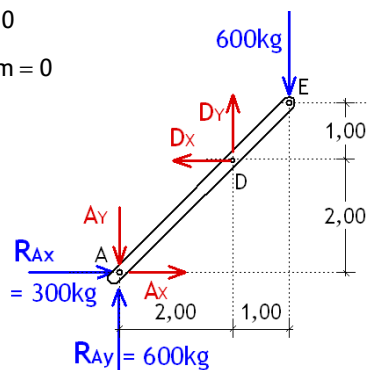
$$R_{Ax} + A_x - D_x = 0$$

$$300 \text{ kg} + 300 \text{ kg} - 600 \text{ kg} = 0$$

$$\sum F_y = 0$$

$$R_{Ay} - A_y + D_y - 600 \text{ kg} = 0$$

$$600 \text{ kg} - 300 \text{ kg} + 300 \text{ kg} - 600 \text{ kg} = 0$$



ESTRUCTURAS I

MARCOS ISOSTATICOS

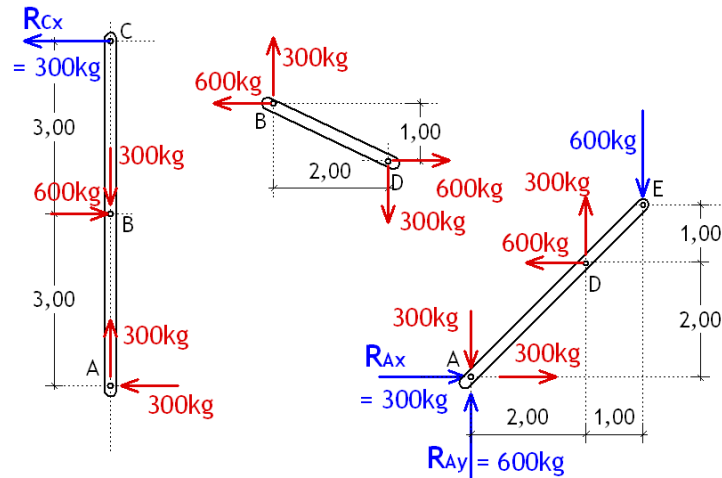
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1b

ANALISIS INTERNO

RESULTADOS



MARCOS ISOSTATICOS

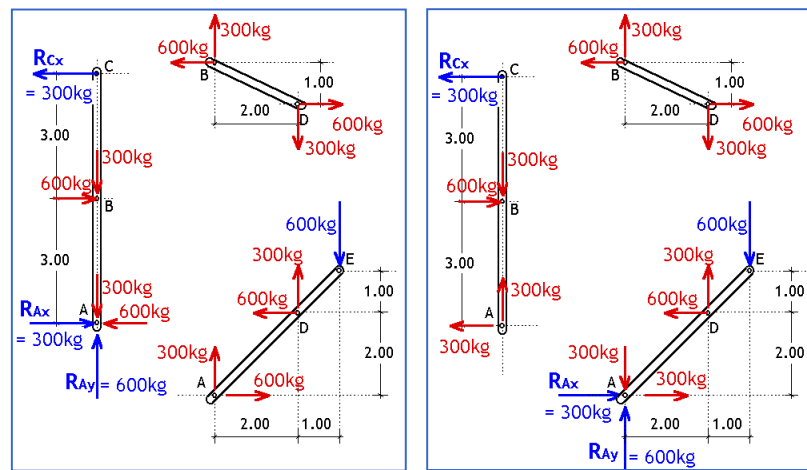
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 1a y 1b

ANALISIS INTERNO

COMPARANDO RESULTADOS



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

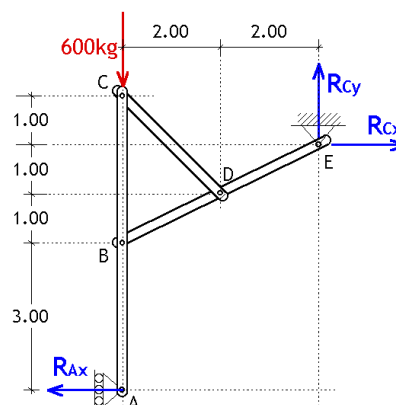
CASOS ESPECIALES 2

EQUILIBRIO EXTERNO

$$\begin{aligned}\sum M_C &= 0 \\ 600 \text{ kg} \cdot 4 \text{ m} - R_{Ax} \cdot 5 \text{ m} &= 0 \\ R_{Ax} &= 480 \text{ kg}\end{aligned}$$

$$\begin{aligned}\sum F_x &= 0 \\ R_{Ax} - R_{Cx} &= 0 \\ R_{Cx} &= 480 \text{ kg}\end{aligned}$$

$$\begin{aligned}\sum F_y &= 0 \\ R_{Cy} - 600 \text{ kg} &= 0 \\ R_{Cy} &= 600 \text{ kg}\end{aligned}$$



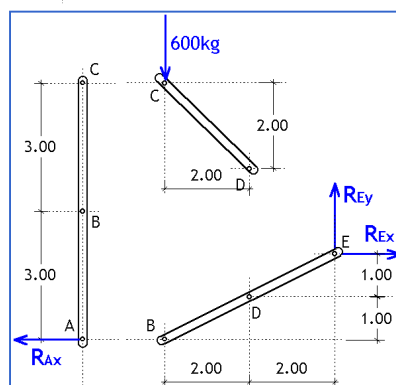
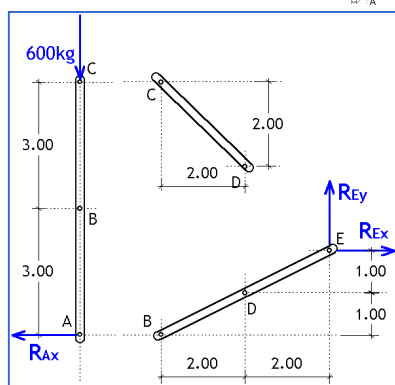
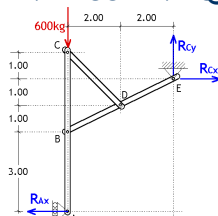
MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2

EQUILIBRIO INTERNO



ESTRUCTURAS I

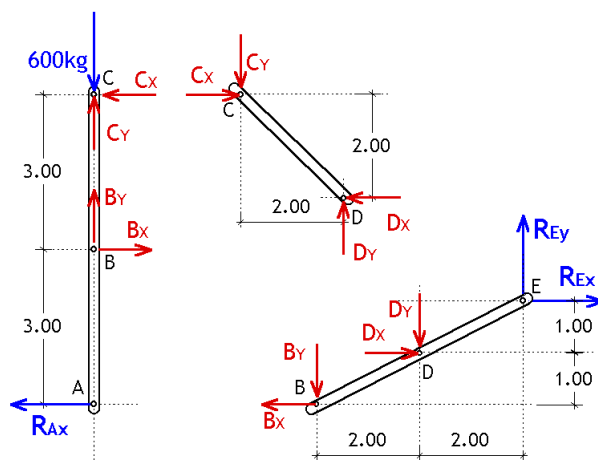
MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2a

EQUILIBRIO INTERNO



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2a

ANALISIS INTERNO BARRA ABC

$$\sum M_B = 0$$

$$R_{Ax} * 3 \text{ m} - Cx * 3 \text{ m} = 0$$

$$480 * 3 \text{ m} - Cx * 3 \text{ m} = 0$$

$$Cx = 480 \text{ kg}$$

$$\sum F_x = 0$$

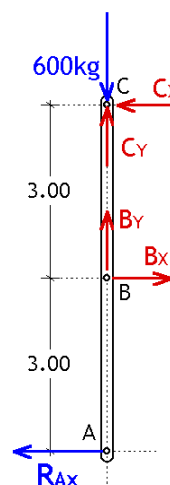
$$-R_{Ax} + Bx - Cx = 0$$

$$-480 \text{ kg} + Bx - 480 \text{ kg} = 0$$

$$Bx = 960 \text{ kg}$$

$$\sum F_y = 0$$

$$-600 \text{ kg} + By + Cy = 0$$



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2a

ANALISIS INTERNO

BARRA BD

$$\sum M_D = 0$$

$$C_x * 2 \text{ m} - C_y * 2 \text{ m} = 0$$

$$480 \text{ kg} * 2 \text{ m} - C_y * 2 \text{ m} = 0$$

$$C_y = 480 \text{ kg}$$

$$\sum F_x = 0$$

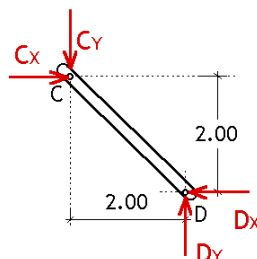
$$C_x - D_x = 0$$

$$D_x = 480 \text{ kg}$$

$$\sum F_y = 0$$

$$D_y - C_y = 0$$

$$D_y = 480 \text{ kg}$$



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2a

ANALISIS INTERNO

BARRA ADE

$$\sum M_D = 0$$

$$B_x * 1 \text{ m} + R_{Ex} * 1 \text{ m} - B_y * 2 \text{ m} - R_{Ey} * 2 \text{ m} = 0$$

$$960 \text{ kg} * 1 \text{ m} + 480 \text{ kg} * 1 \text{ m} - B_y * 2 \text{ m} - 600 \text{ kg} * 2 \text{ m} = 0$$

$$B_y = 120 \text{ kg}$$

$$\sum F_x = 0$$

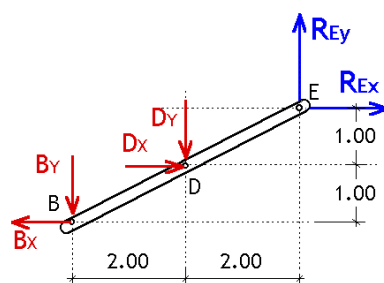
$$R_{Ex} + D_x - B_x = 0$$

$$480 \text{ kg} + 480 \text{ kg} - 960 \text{ kg} = 0$$

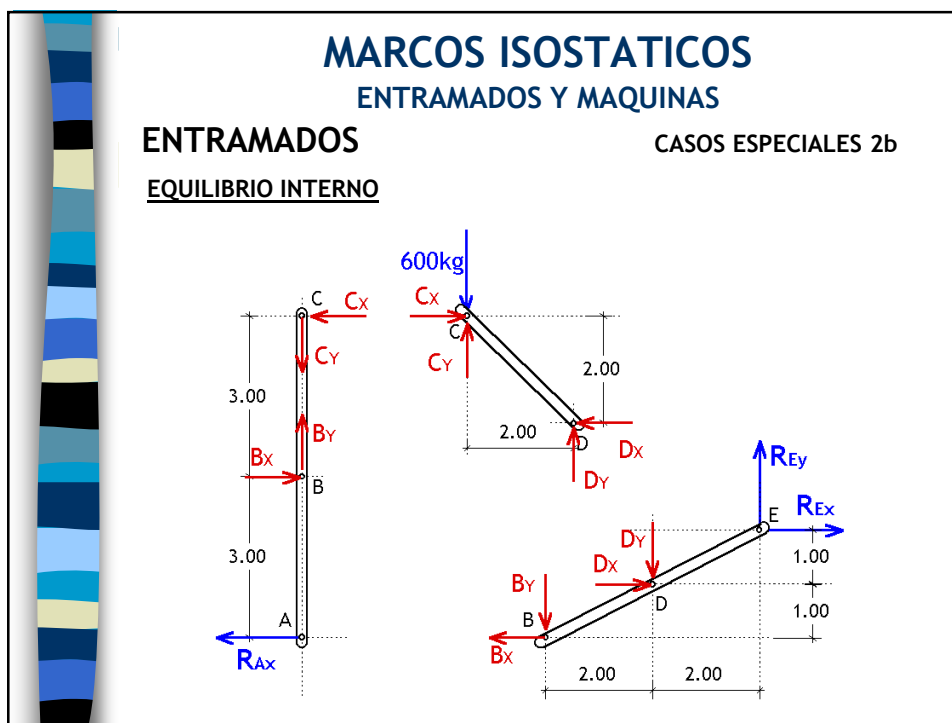
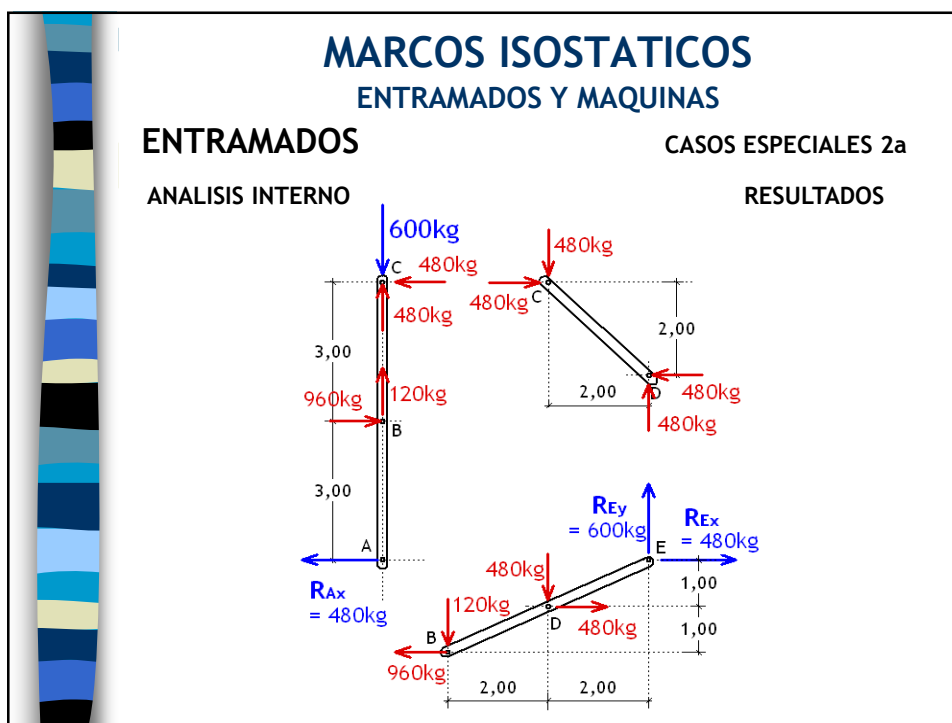
$$\sum F_y = 0$$

$$R_{Ey} - D_y - B_y = 0$$

$$600 \text{ kg} - 120 \text{ kg} - 480 \text{ kg} = 0$$



ESTRUCTURAS I



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2b

ANALISIS INTERNO

BARRA ABC

$$\sum M_B = 0$$

$$R_{Ax} * 3 \text{ m} - Cx * 3 \text{ m} = 0$$

$$480 * 3 \text{ m} - Cx * 3 \text{ m} = 0$$

$$Cx = 480 \text{ kg}$$

$$\sum F_x = 0$$

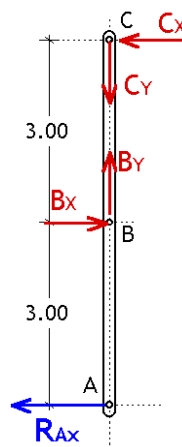
$$-R_{Ax} + Bx - Cx = 0$$

$$-480 \text{ kg} + Bx - 480 \text{ kg} = 0$$

$$Bx = 960 \text{ kg}$$

$$\sum F_y = 0$$

$$By - Cy = 0$$



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2b

ANALISIS INTERNO

BARRA BD

$$\sum M_D = 0$$

$$Cx * 2 \text{ m} - 600 \text{ kg} * 2 \text{ m} + Cy * 2 \text{ m} = 0$$

$$480 \text{ kg} * 2 \text{ m} - 600 \text{ kg} * 2 \text{ m} + Cy * 2 \text{ m} = 0$$

$$Cy = 120 \text{ kg}$$

$$\sum F_x = 0$$

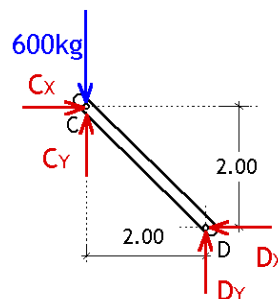
$$Cx - Dx = 0$$

$$Dx = 480 \text{ kg}$$

$$\sum F_y = 0$$

$$Dy + Cy - 600 \text{ kg} = 0$$

$$Dy = 480 \text{ kg}$$



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2b

ANALISIS INTERNO

BARRA ADE

$$\sum M_D = 0$$

$$B_x * 1\text{ m} + R_{Ex} * 1\text{ m} - B_y * 2\text{ m} - R_{Ey} * 2\text{ m} = 0$$

$$960\text{ kg} * 1\text{ m} + 480\text{ kg} * 1\text{ m} - B_y * 2\text{ m} - 600\text{ kg} * 2\text{ m} = 0$$

$$B_y = 120\text{ kg}$$

$$\sum F_x = 0$$

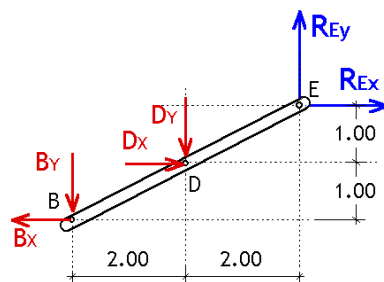
$$R_{Ex} + D_x - B_x = 0$$

$$480\text{ kg} + 480\text{ kg} - 960\text{ kg} = 0$$

$$\sum F_y = 0$$

$$R_{Ey} - D_y - B_y = 0$$

$$600\text{ kg} - 120\text{ kg} - 480\text{ kg} = 0$$



MARCOS ISOSTATICOS

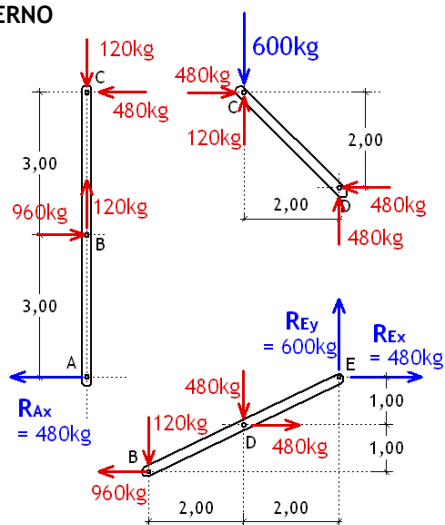
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2b

ANALISIS INTERNO

RESULTADOS



ESTRUCTURAS I

MARCOS ISOSTATICOS

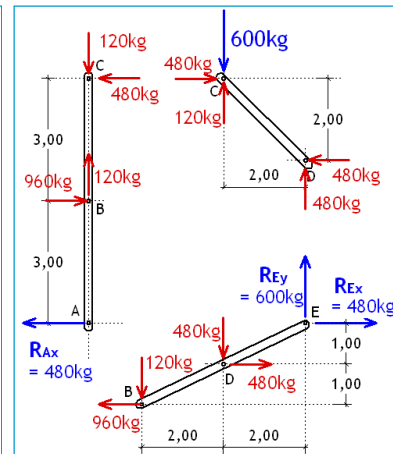
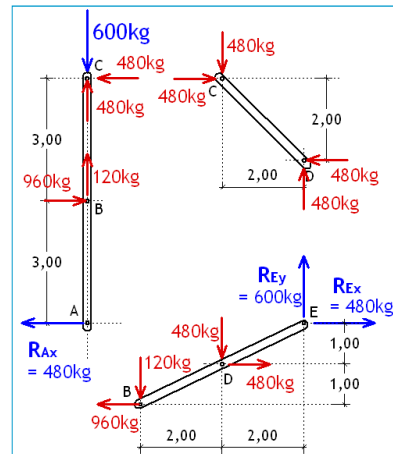
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 2a y 2b

ANALISIS INTERNO

COMPARANDO RESULTADOS



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 3

EQUILIBRIO EXTERNO

$$\sum M_A = 0$$

$$-300 \text{ kg} \cdot 1,5 \text{ m} + R_{Cx} \cdot 6 \text{ m} = 0$$

$$R_{Cx} = 75 \text{ kg}$$

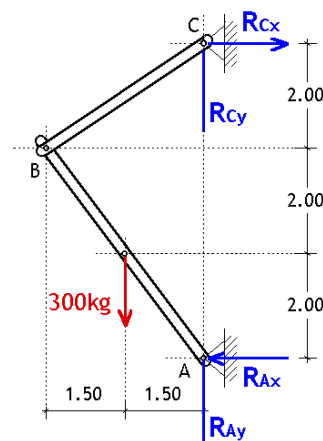
$$\sum F_x = 0$$

$$R_{Ax} + R_{Cx} = 0$$

$$R_{Ax} = 75 \text{ kg}$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{Cy} - 300 \text{ kg} = 0$$



ESTRUCTURAS I

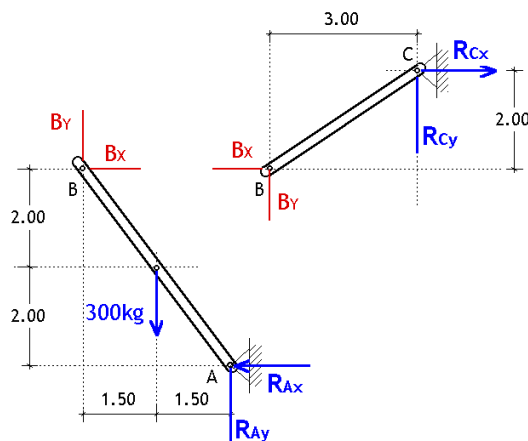
MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 3

ANALISIS INTERNO



MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 3

ANALISIS INTERNO

$$\sum M_B = 0$$

$$75 \text{ kg} \cdot 2 \text{ m} - R_{Cy} \cdot 3 \text{ m} = 0$$

$$R_{Cy} = 50 \text{ kg}$$

$$\sum F_x = 0$$

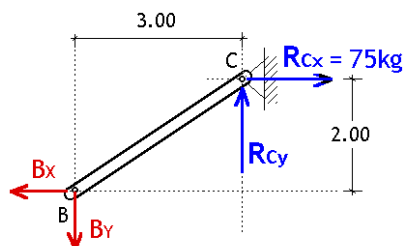
$$R_{Cx} - B_x = 0$$

$$B_x = 75 \text{ kg}$$

$$\sum F_y = 0$$

$$-R_{Cy} + B_y = 0$$

$$B_y = 50 \text{ kg}$$



ESTRUCTURAS I

MARCOS ISOSTATICOS

ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 3

ANALISIS INTERNO

$$\sum F_x = 0$$

$$B_x - R_{Ax} = 0$$

$$B_x = 75 \text{ kg}$$

$$\sum F_y = 0$$

$$R_{Ay} + B_y - 300 \text{ kg} = 0$$

$$R_{Ay} + 50 \text{ kg} - 300 \text{ kg} = 0$$

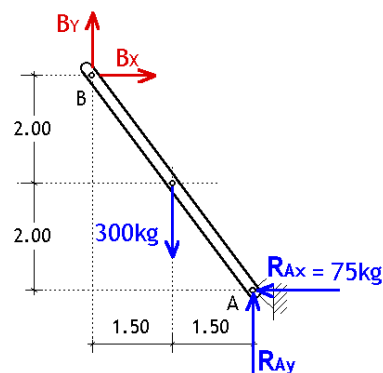
$$R_{Ay} = 250 \text{ kg}$$

COMPROBACION

$$\sum M_B = 0$$

$$R_{Ax} * 4 \text{ m} - R_{Ay} * 3 \text{ m} + 300 \text{ kg} * 1,5 \text{ m} = 0$$

$$75 \text{ kg} * 4 \text{ m} - 250 \text{ kg} * 3 \text{ m} + 300 \text{ kg} * 1,5 \text{ m} = 0$$



MARCOS ISOSTATICOS

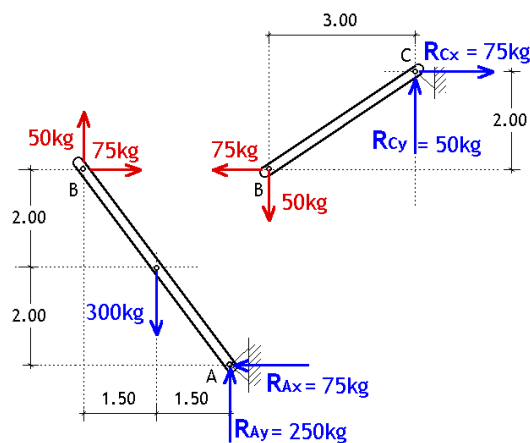
ENTRAMADOS Y MAQUINAS

ENTRAMADOS

CASOS ESPECIALES 3

ANALISIS INTERNO

RESULTADOS



ESTRUCTURAS I

BIBLIOGRAFIA

▪ DISEÑO Y CALCULO DE ESTRUCTURAS

•Bernardo Villasuso (1994) - El Ateneo - Buenos Aires - Argentina.

▪ MECANICA VECTORIAL PARA INGENIEROS - ESTATICA

▪Ferdinand P. Beer, E. Russell Johnston Jr (1990) - Ediciones McGraw-Hill.

▪ DISEÑO ESTRUCTURAL

Rafael Riddell C., Pedro Hidalgo O. (2002) 3° Ed. Ediciones PUC de Chile.

▪ FUNDAMENTOS DE INGENIERIA ESTRUCTURAL PARA ESTUDIANTES DE ARQUITECTURA

Rafael Riddell C., Pedro Hidalgo O. (2000) Ediciones PUC de Chile.