

# ESTRUCTURAS II

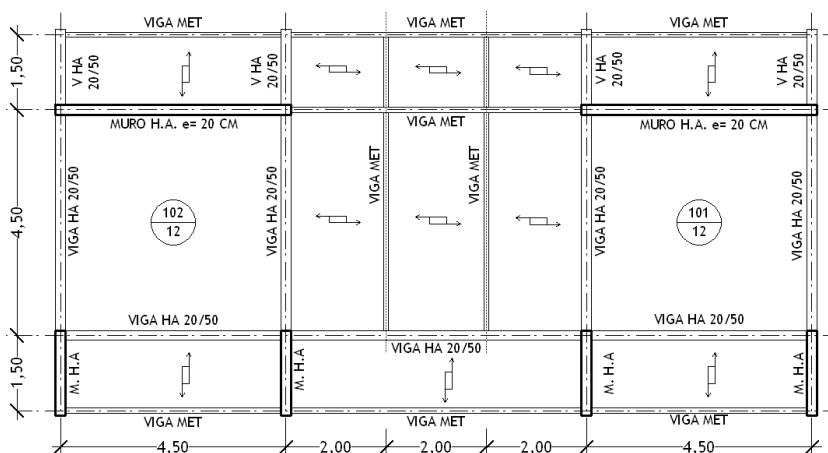
## Deformación en vigas ESTRUCTURAS 2

1

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## Deformación en vigas

### GUIA 4



# ESTRUCTURAS II

## Deformación en vigas

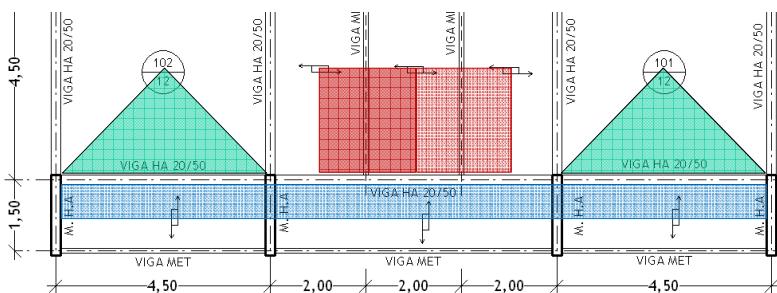
### VIGA DEL EJE 2

#### CARGAS

$$P = 250 \text{ kg/m}^2 * 2 \text{ m} * 2,25 \text{ m} = 1125 \text{ kg}$$

$$q_1 = 250 \text{ kg/m}^2 * 0,75 \text{ m} + 2500 \text{ kg/m}^3 * 0,20 \text{ m} * 0,50 \text{ m} = 437,50 \text{ kg/m}$$

$$q_2 = 550 \text{ kg/m}^2 * 2,25 \text{ m} = 1237,50 \text{ kg/m}$$

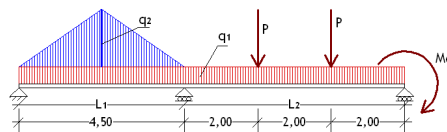
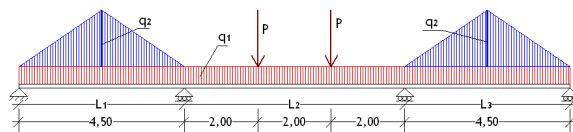


## Deformación en vigas

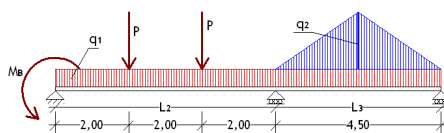
### VIGA DEL EJE 2

#### TEOREMA DE LOS TRES MOMENTOS O CLAPEYRON

$$M_A * L_1 + 2M_B * (L_1 + L_2) + M_C * L_2 = 6 * (T_{C1} + T_{C2})$$



$$2M_B * (L_1 + L_2) + M_C * L_2 = 6 * (T_{C1} + T_{C2})$$



$$M_B * L_2 + 2M_C * (L_2 + L_3) = 6 * (T_{C2} + T_{C3})$$

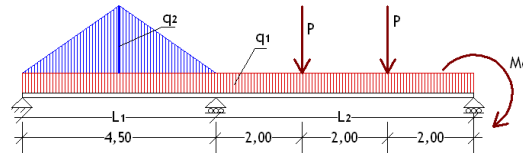
# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE 2

### TEOREMA DE LOS TRES MOMENTOS O CLAPEYRON

$$M_A * L_1 + 2M_B * (L_1 + L_2) + M_C * L_2 = 6 * (T_{C1} + T_{C2})$$



$$0 * L_1 + 2M_B * (L_1 + L_2) + M_C * L_2 = 6 * \left( \frac{q_1 L_1^3}{24} + \frac{5q_2 L_1^3}{192} + \frac{q_1 L_2^3}{24} + \frac{P L_2^2}{9} \right)$$

$$2M_B * (4,5 + 6) + M_C * 6 = 6 * \left( \frac{437,5 * (4,5)^3}{24} + \frac{5 * 1237,5 * (4,5)^3}{192} + \frac{437,5 * (6)^3}{24} + \frac{1125 * (6)^2}{9} \right)$$

$$2M_B * 10,5 + 6M_C = 6 * 13035,27$$

$$21M_B + 6M_C = 78211,62$$

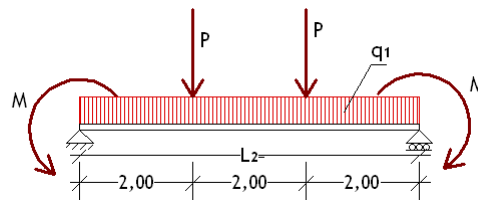
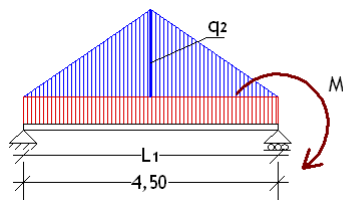
$$M_B = M_C = M$$

$$27M = 78211,62$$

$$M = 2896,72 \text{ kgm}$$

## Deformación en vigas

### VIGA DEL EJE 2



# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE 2

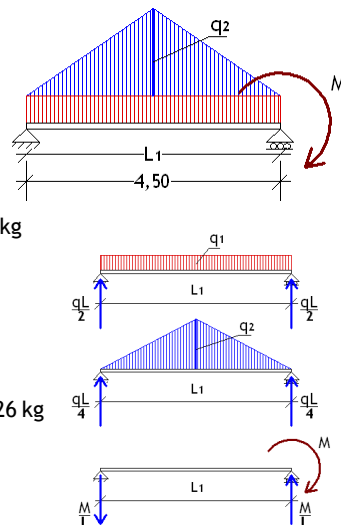
#### REACCIONES - TRAMO 1

$$R_A = \frac{q_1 L_1}{2} + \frac{q_2 L_1}{4} - \frac{M}{L}$$

$$R_A = \frac{437,5 \cdot 4,5}{2} + \frac{1237,5 \cdot 4,5}{4} - \frac{2896,72}{4,5} = 1732,83 \text{ kg}$$

$$R_B = \frac{q_1 L_1}{2} + \frac{q_2 L_1}{4} + \frac{M}{L}$$

$$R_{B_{izq}} = \frac{437,5 \cdot 4,5}{2} + \frac{1237,5 \cdot 4,5}{4} + \frac{2896,72}{4,5} = 3020,26 \text{ kg}$$



## Deformación en vigas

### VIGA DEL EJE 2

#### MOMENTO MAXIMO - TRAMO 1

$$R_A - q_1 \cdot x - \frac{2q_2 x}{L} \cdot \frac{x}{2} = 0$$

$$1732,83 - 437,5 \cdot x - 275 \cdot x^2 = 0$$

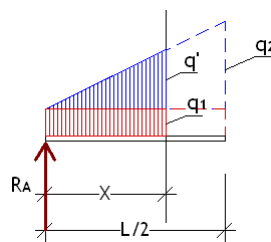
$$x = \frac{-(-437,5) \pm \sqrt{(-437,5)^2 - 4 \cdot (-275) \cdot 1732,83}}{2 \cdot (-275)}$$

$$x_1 = 1,76 \text{ m}$$

$$x_2 = -3,35 \text{ m}$$

$$\frac{q'}{x} = \frac{q_2}{L/2}$$

$$q' = \frac{2q_2 x}{L}$$



# ESTRUCTURAS II

## Deformación en vigas

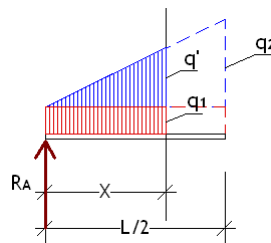
### VIGA DEL EJE 2

MOMENTO MAXIMO - TRAMO 1  $x_1 = 1,76 \text{ m}$

$$M_{MAX} = R_A * x - q_1 * \frac{x^2}{2} - \frac{2q_2x}{L} * \frac{x}{3}$$

$$M_{MAX} = 1732,83 * 1,76 - 437,5 * \frac{1,76^2}{2} - \frac{2 * 1237,5}{4,5} * \frac{1,76^3}{6}$$

$$M_{MAX} = 1872,44 \text{ kgm}$$



## Deformación en vigas

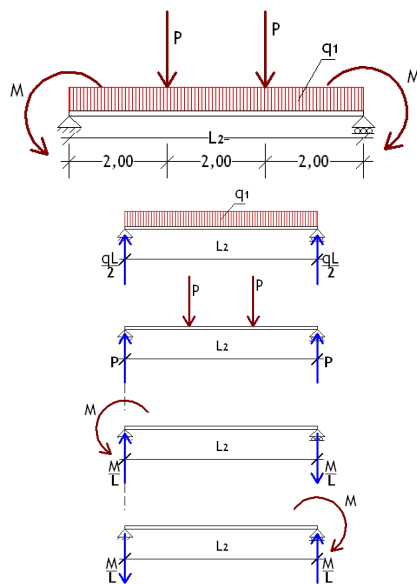
### VIGA DEL EJE 2

REACCIONES - TRAMO 2

$$R_{B_{der}} = \frac{q_1 L_2}{2} + P - \frac{M}{L} + \frac{M}{L}$$

$$R_{B_{der}} = \frac{437,5 * 6}{2} + 1125 = 2437,50 \text{ kg}$$

$$R_{B_{der}} = R_{C_{izq}} = 2437,50 \text{ kg}$$



# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE 2

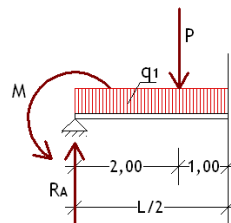
#### MOMENTO MAXIMO - TRAMO 2

$x = 3 \text{ m}$

$$M_{MAX} = R_{B_{der}} * x - q_1 * \frac{x^2}{2} - P * \left(x - \frac{L}{3}\right) - M$$

$$M_{MAX} = 2437,50 * 3 - 437,5 * \frac{3^2}{2} - 1125 * 1 - 2896,72$$

$$M_{MAX} = 1322,02 \text{ kgm}$$



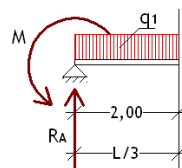
#### MOMENTO FLECTOR

$x = 2 \text{ m}$

$$M_{MAX} = R_{B_{der}} * x - q_1 * \frac{x^2}{2} - M$$

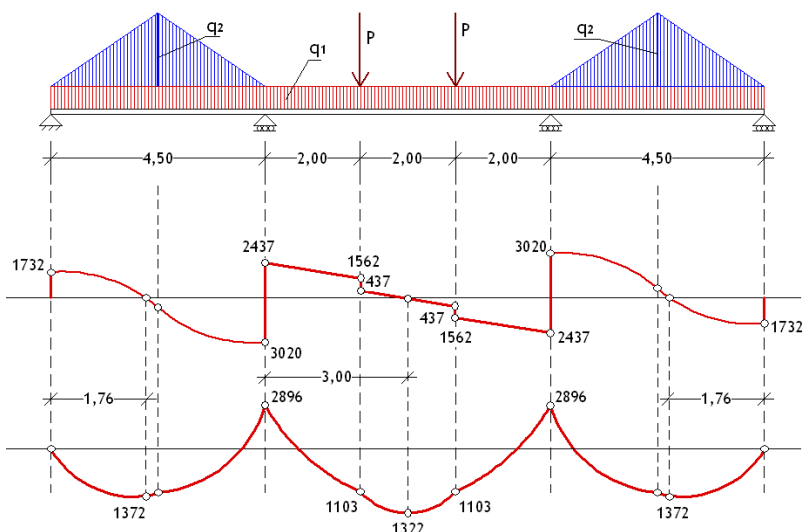
$$M_{MAX} = 2437,50 * 2 - 437,5 * \frac{2^2}{2} - 2896,72$$

$$M_{MAX} = 1103,28 \text{ kgm}$$



## Deformación en vigas

### VIGA DEL EJE 2



# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE 2

#### FLECHA MAXIMA- TRAMO 2

$$y_{\max} \rightarrow x = L/2$$

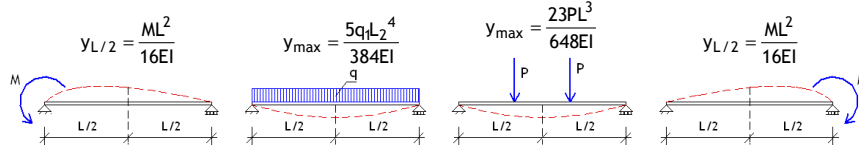
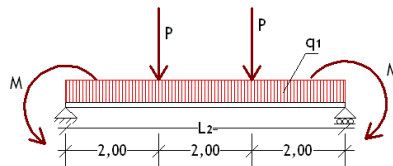
$$y_{\max} = \frac{ML_2^2}{16EI} - \frac{5q_1L_2^4}{384EI} - \frac{23PL_2^3}{648EI} + \frac{ML_2^2}{16EI}$$

$$y_{\max} = \frac{289672 \cdot 600^2}{16EI} - \frac{5 \cdot 4,37 \cdot 600^4}{384EI} - \frac{23 \cdot 1125 \cdot 600^3}{648EI} + \frac{289672 \cdot 600^2}{16EI}$$

$$y_{\max} = 0,07 \text{ cm}$$

$$E_{HA} = 200000 \text{ kg/cm}^2$$

$$I_{HA20/50} = 208333 \text{ cm}^4$$



## Deformación en vigas

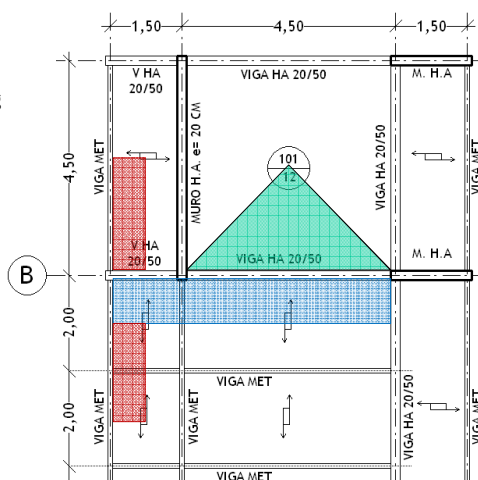
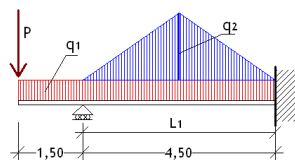
### VIGA DEL EJE B

#### CARGAS

$$P = 250 \text{ kg/m}^2 \cdot 2,25 \text{ m} \cdot 0,75 \text{ m} \\ + 250 \text{ kg/m}^2 \cdot 2 \text{ m} \cdot 0,75 \text{ m} = 796,87 \text{ kg}$$

$$q_1 = 2500 \text{ kg/m}^3 \cdot 0,20 \text{ m} \cdot 0,50 \text{ m} \\ + 250 \text{ kg/m}^2 \cdot 1 \text{ m} = 500 \text{ kg/m}$$

$$q_2 = 550 \text{ kg/m}^2 \cdot 2,25 \text{ m} = 1237,50 \text{ kg/m}$$



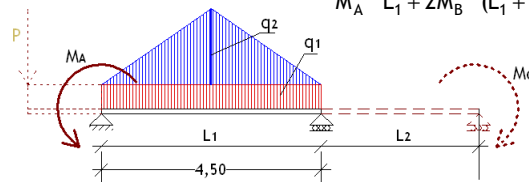
# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE B

### TEOREMA DE LOS TRES MOMENTOS O CLAPEYRON

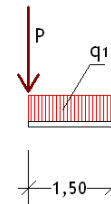
$$M_A * L_1 + 2M_B * (L_1 + L_2) + M_C * L_2 = 6 * (T_{C1} + T_{C2})$$



$$M_A * L_1 + 2M_B * (L_1 + L_2) + 0 * L_2 = 6 * \left( \frac{q_1 L_1^3}{24} + \frac{5q_2 L_1^3}{192} + 0 \right)$$

$$M_A = -796,87 * 1,5 - 500 * 1,5 * 0,75$$

$$M_A = -1757,80 \text{ kgm}$$

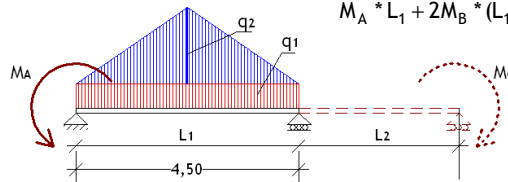


## Deformación en vigas

### VIGA DEL EJE B

### TEOREMA DE LOS TRES MOMENTOS O CLAPEYRON

$$M_A * L_1 + 2M_B * (L_1 + L_2) + M_C * L_2 = 6 * (T_{C1} + T_{C2})$$



$$M_A * L_1 + 2M_B * (L_1 + L_2) + 0 * L_2 = 6 * \left( \frac{q_1 L_1^3}{24} + \frac{5q_2 L_1^3}{192} + 0 \right)$$

$$1757,80 * 4,5 + 2M_B * (4,5 + 0) = 6 * \left( \frac{500 * (4,5)^3}{24} + \frac{5 * 1237,5 * (4,5)^3}{192} \right)$$

$$7910,10 + 9M_B = 6 * 4835,08$$

$$9M_B = 21100,38$$

$$M_B = 2344,48 \text{ kgm}$$

# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE B

#### REACCIONES

$$R_A = \frac{q_1 L_1}{2} + \frac{q_2 L_1}{4} + \frac{M_A}{L} - \frac{M_B}{L}$$

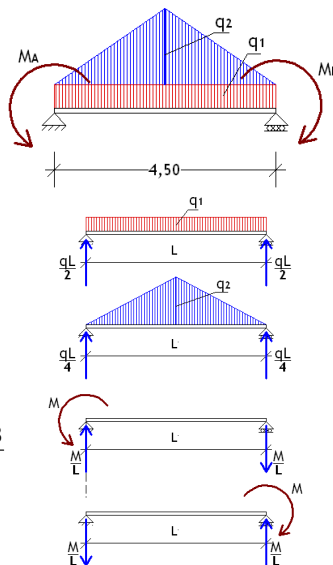
$$R_A = \frac{500 \cdot 4,5}{2} + \frac{1237,5 \cdot 4,5}{4} + \frac{1757,80}{4,5} - \frac{2344,48}{4,5}$$

$$R_A = 2386,82 \text{ kg}$$

$$R_B = \frac{q_1 L_1}{2} + \frac{q_2 L_1}{4} - \frac{M_A}{L} + \frac{M_B}{L}$$

$$R_B = \frac{500 \cdot 4,5}{2} + \frac{1237,5 \cdot 4,5}{4} - \frac{1757,80}{4,5} + \frac{2344,48}{4,5}$$

$$R_B = 2647,56 \text{ kg}$$



## Deformación en vigas

### VIGA DEL EJE B

#### MOMENTO MAXIMO

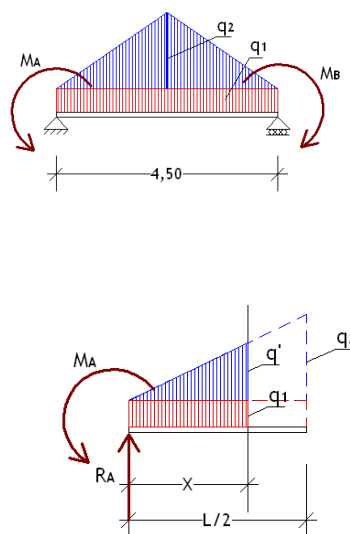
$$R_A - q_1 \cdot x - \frac{2q_2 x}{L} \cdot \frac{x}{2} = 0$$

$$2386,82 - 500 \cdot x - 275 \cdot x^2 = 0$$

$$x = \frac{-(-500) \pm \sqrt{(-500)^2 - 4 \cdot (-275) \cdot 2386,82}}{2 \cdot (-275)}$$

$$x_1 = 2,17 \text{ m}$$

$$x_2 = -3,99 \text{ m}$$



# ESTRUCTURAS II

## Deformación en vigas

### VIGA DEL EJE B

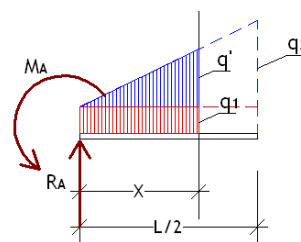
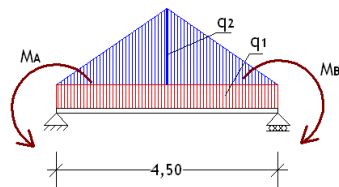
#### MOMENTO MAXIMO

$$x_1 = 2,17 \text{ m}$$

$$M_{MAX} = -M_A + R_A * x - q_1 * \frac{x^2}{2} - \frac{2q_2 x}{L} * \frac{x}{2} * \frac{x}{3}$$

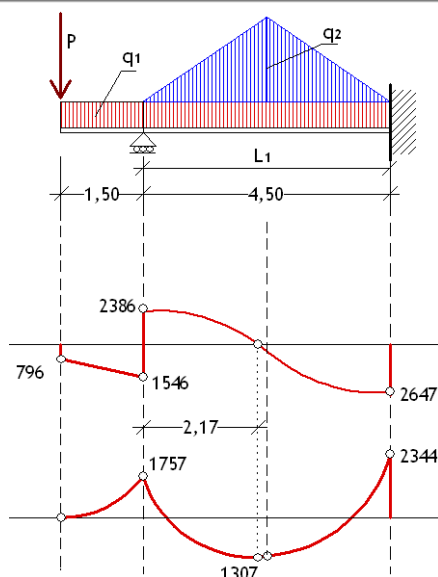
$$M_{MAX} = -1757,80 + 2386,82 * 2,17 - 500 * \frac{2,17^2}{2} - \frac{2 * 1237,5}{4,5} * \frac{2,17^3}{6}$$

$$M_{MAX} = 1307,67 \text{ kgm}$$



## Deformación en vigas

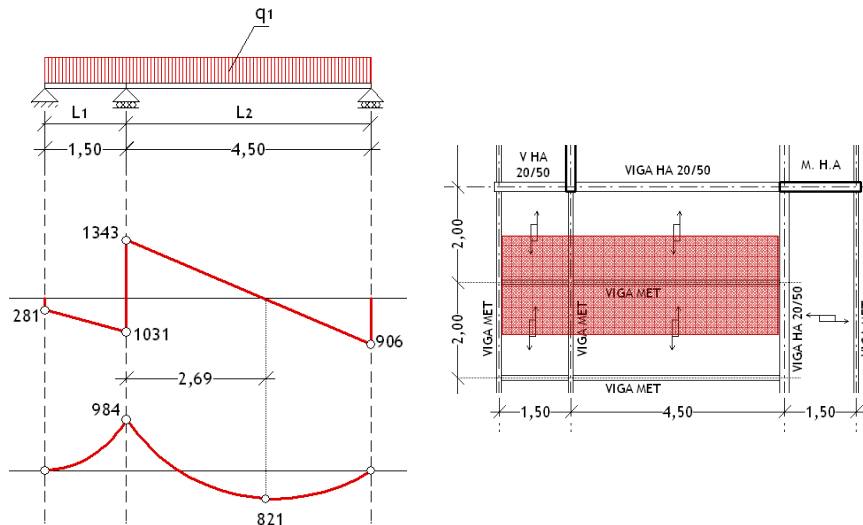
### VIGA DEL EJE B



# ESTRUCTURAS II

## Deformación en vigas

### VIGA METALICA



## Bibliografía

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Rafael Riddell C., Pedro Hidalgo O. (2000) Ediciones PUC de Chile.