vigas hiperestáticas

ESTRUCTURAS 2

2

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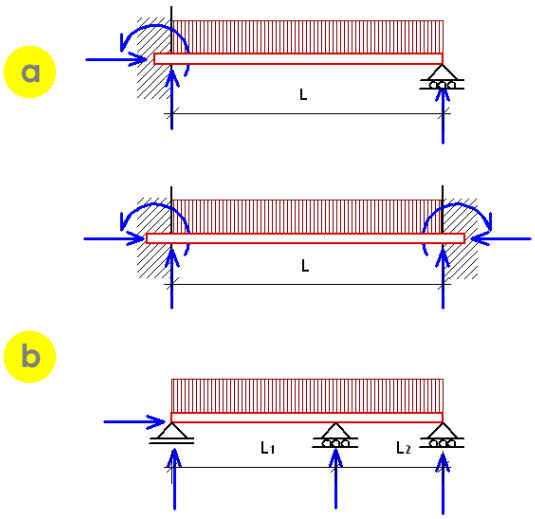
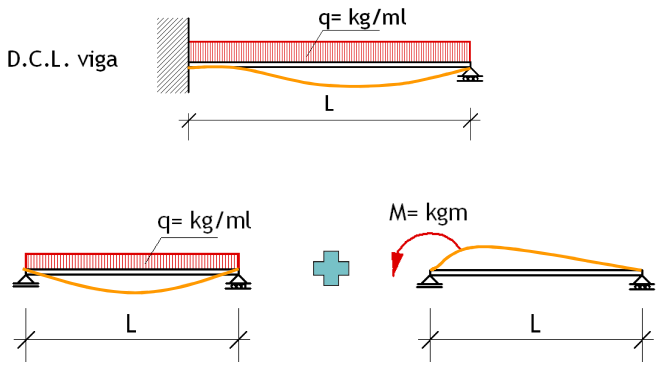
Vigas Hiperestáticas

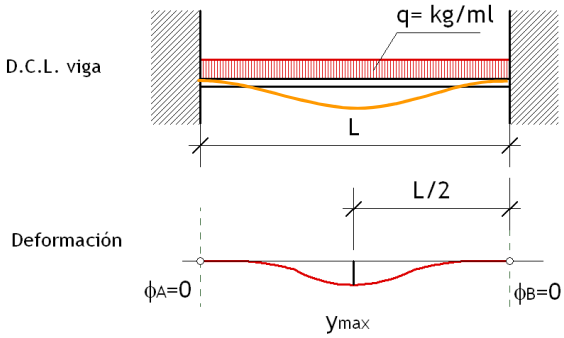


Concepto de vigas hiperestáticas por empotramiento

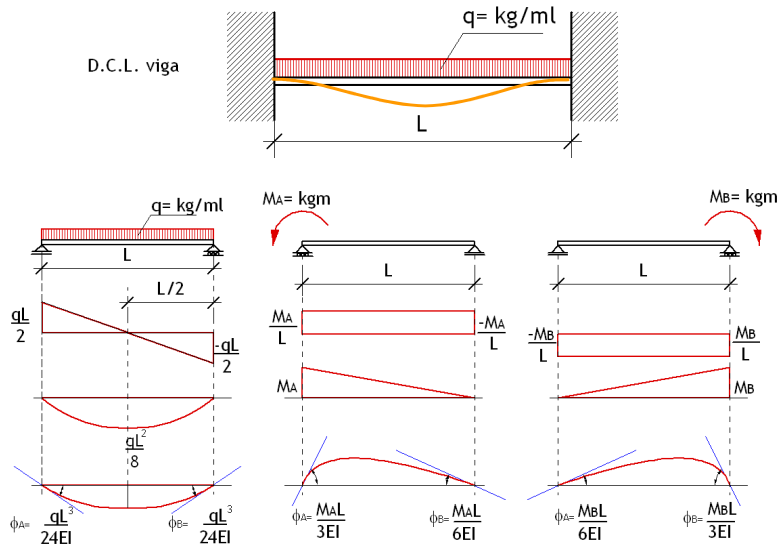


$$\phi_{empotramiento} = 0$$

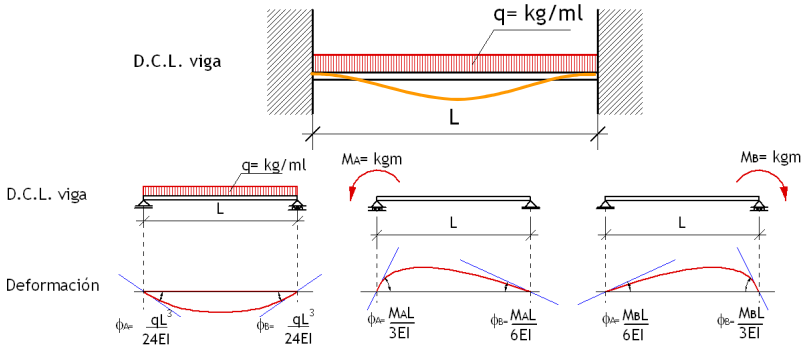
Ejemplo Viga bi-empotrada con carga uniformemente repartida



Ejemplo Viga bi-empotrada con carga uniformemente repartida



Ejemplo Viga bi-empotrada con carga uniformemente repartida



$$\phi_A = 0$$

$$-\frac{qL^3}{24EI} + \frac{M_A L}{3EI} + \frac{M_B L}{6EI} = 0$$

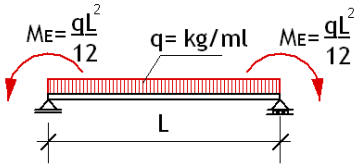
$$\frac{2M_A L + M_B L}{6EI} = \frac{qL^3}{24EI}$$

$$M_A = M_B = M_E$$

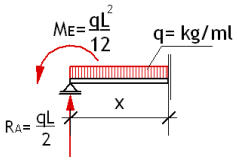
$$\frac{3M_E L}{6EI} = \frac{qL^3}{24EI}$$

$$\boxed{M_E = \frac{qL^2}{12}}$$

Ejemplo Viga bi-empotrada con carga uniformemente repartida



$$Ra = Rb = \frac{qL}{2}$$



$$Mx = \frac{qLx}{2} - \frac{qx^2}{2} - M_E$$

El momento máximo en una viga simétrica se encuentra en X = L/2

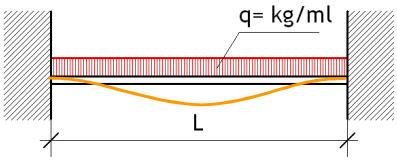
$$M_{(L/2)} = \frac{qL}{2}\frac{L}{2} - \frac{q}{2}\left(\frac{L}{2}\right)^2 - M_E$$

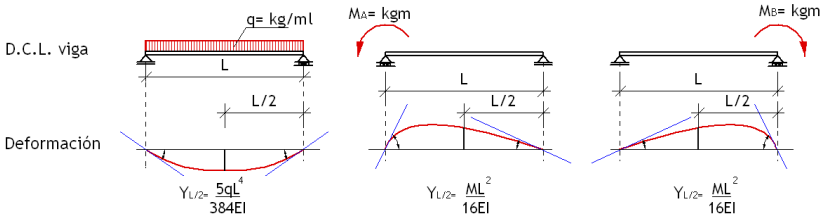
$$M_{(L/2)} = \frac{qL^2}{4} - \frac{qL^2}{8} - \frac{qL^2}{12}$$

$$M_{MAX} = \frac{qL^2}{24}$$

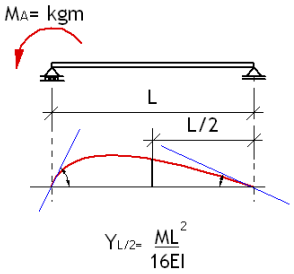
Ejemplo Viga bi-empotrada con carga uniformemente repartida

D.C.L. viga





E S T R U C T U R A S II



$$EI\ y = -\frac{Mx^3}{6L} + \frac{MLx}{6}$$

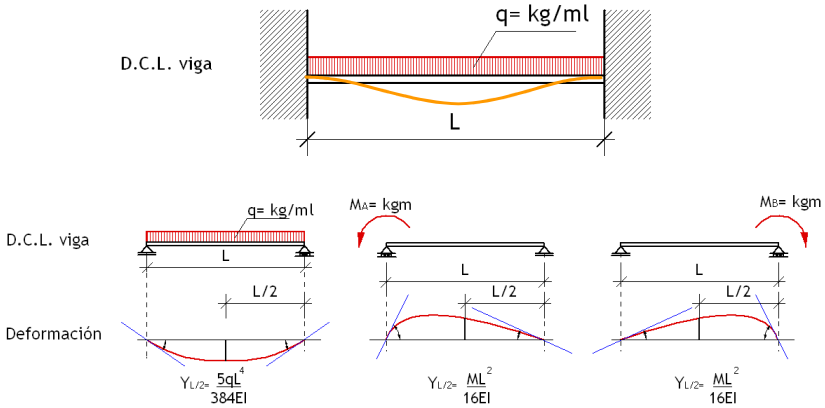
$$EI\ y_{L/2} = -\frac{M}{6L}\left(\frac{L}{2}\right)^3 + \frac{ML}{6}\frac{L}{2}$$

$$EI\ y_{L/2} = -\frac{ML^2}{48} + \frac{ML^2}{12}$$

$$y_{L/2} = \frac{ML^2}{16EI}$$

$$y_{L/2} = \frac{qL^2}{12}\frac{L^2}{16EI}$$

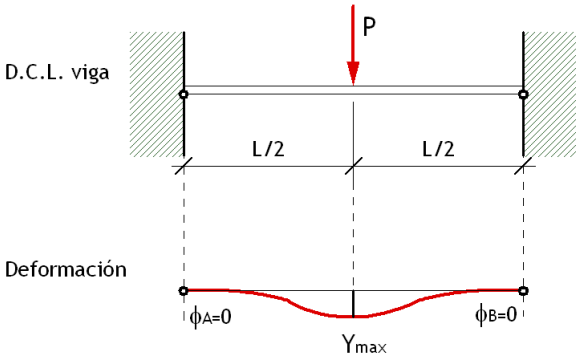
$$y_{L/2} = \frac{qL^4}{192EI}$$

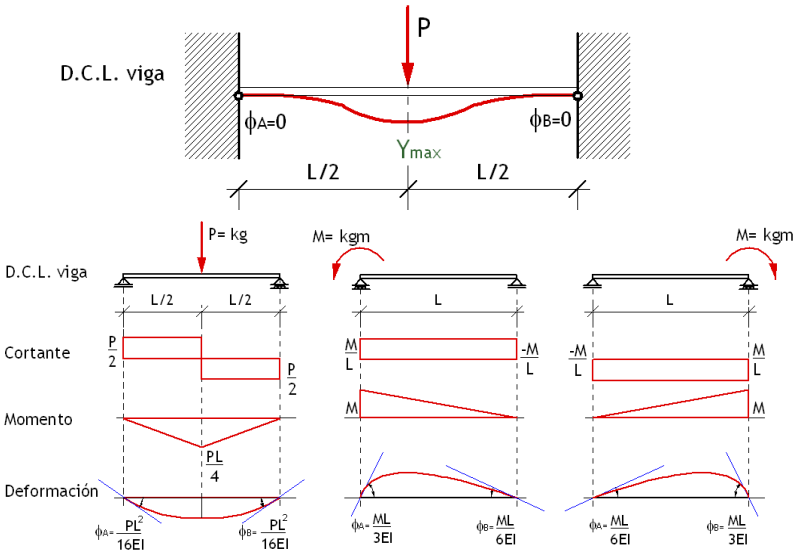


$$Y_{MAX} = -\frac{5qL^4}{384EI} + \frac{qL^4}{192EI} + \frac{qL^4}{192EI}$$

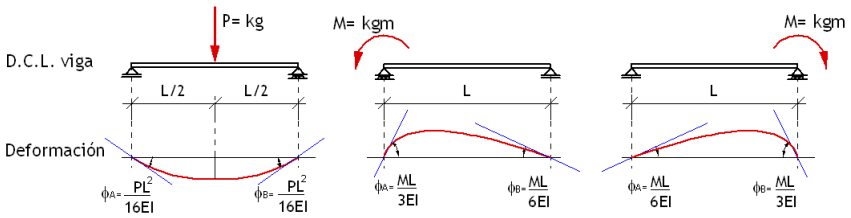
$$\boxed{Y_{MAX} = -\frac{qL^4}{384EI}}$$

Ejemplo Viga bi-empotrada con carga puntual al centro





E S T R U C T U R A S II



$$\phi_A = 0$$

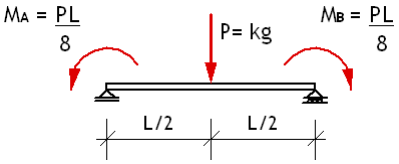
$$-\frac{PL^2}{16EI} + \frac{M_A L}{3EI} + \frac{M_B L}{6EI} = 0$$

$$\frac{2M_A L + M_B L}{6EI} = \frac{PL^2}{16EI}$$

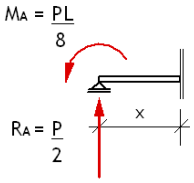
$$M_A = M_B = M_E$$

$$\frac{3M_E L}{6EI} = \frac{PL^2}{16EI}$$

$$\boxed{M_E = \frac{PL}{8}}$$



$$R_A = R_B = \frac{P}{2}$$



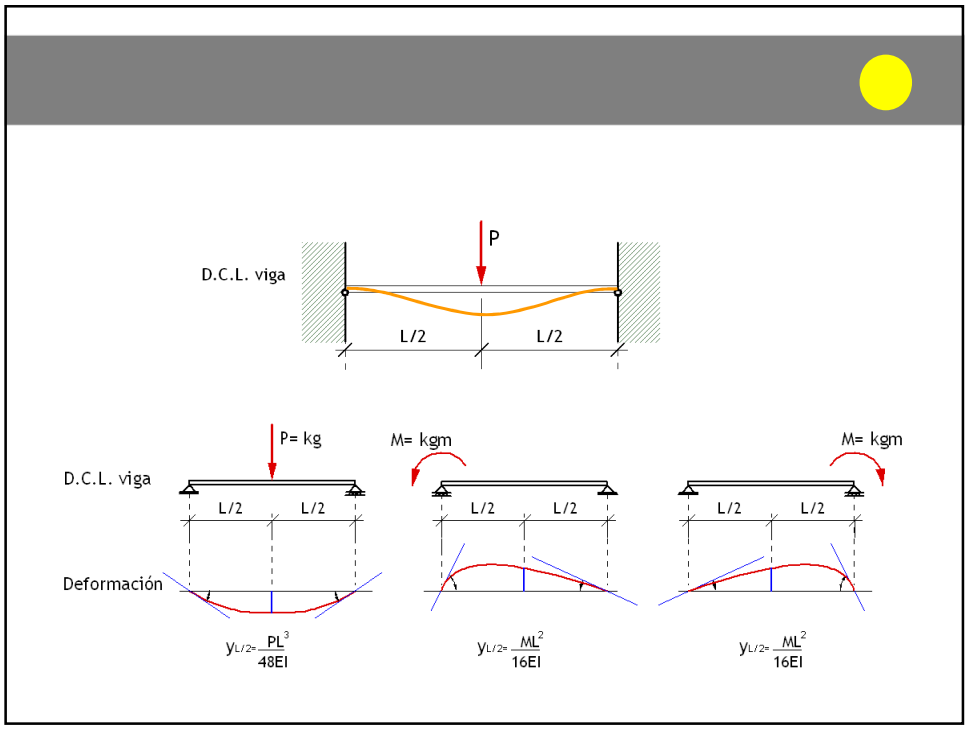
$$Mx = -\frac{PL}{8} + \frac{Px}{2}$$

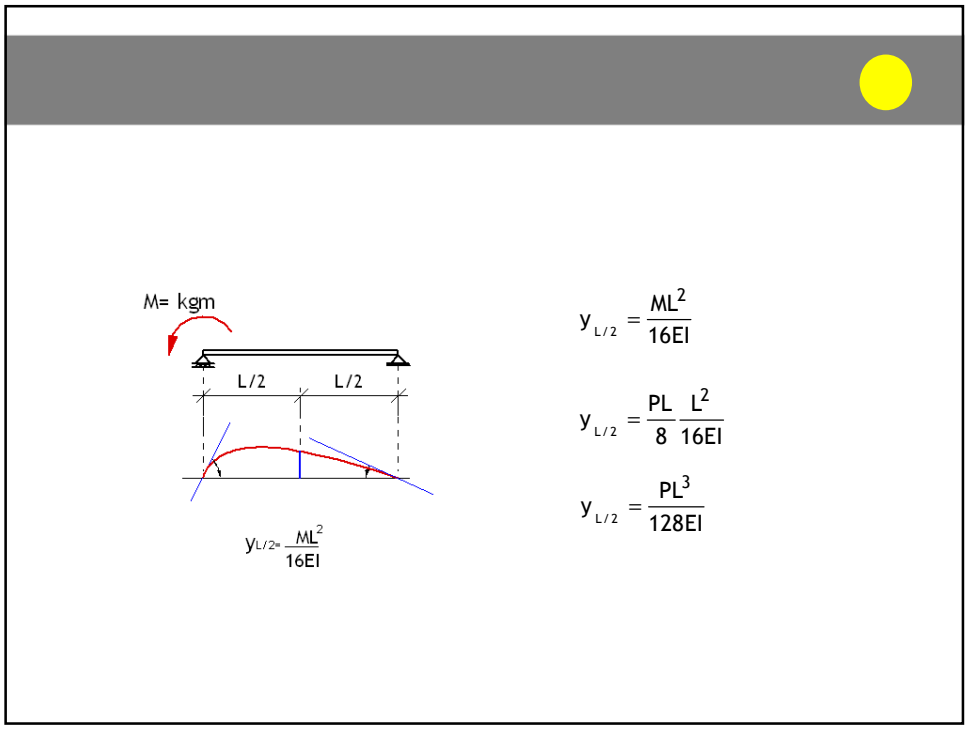
El momento máximo en una viga simétrica se encuentra en X=L/2

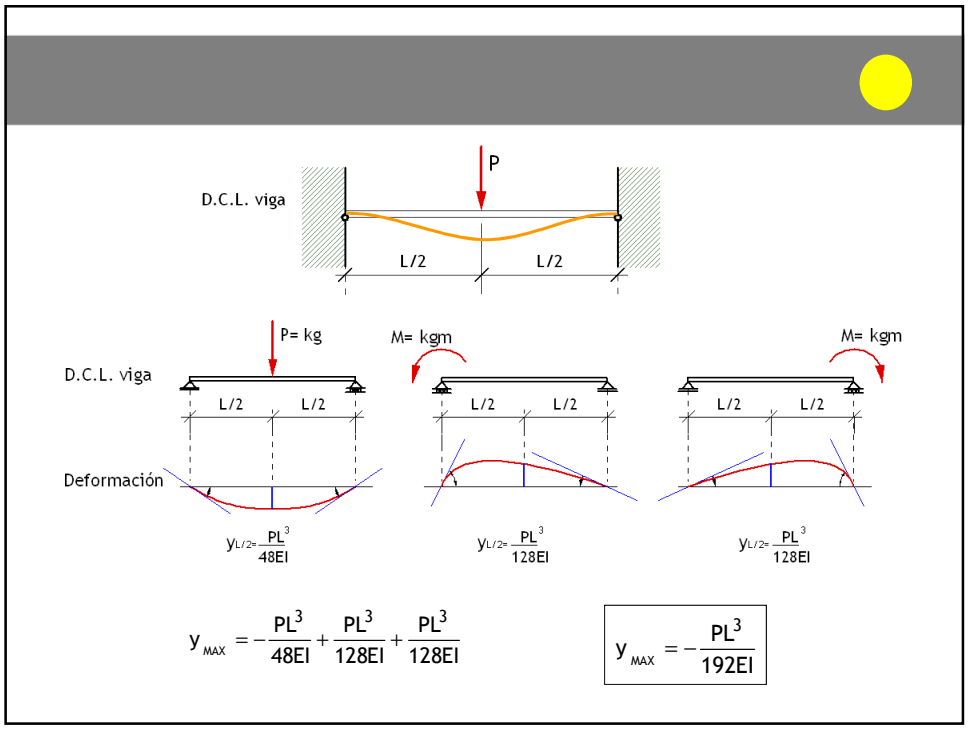
$$M_{L/2} = -\frac{PL}{8} + \frac{P}{2}\frac{L}{2}$$

$$\boxed{M_{MAX} = \frac{PL}{8}}$$

E S T R U C T U R A S II

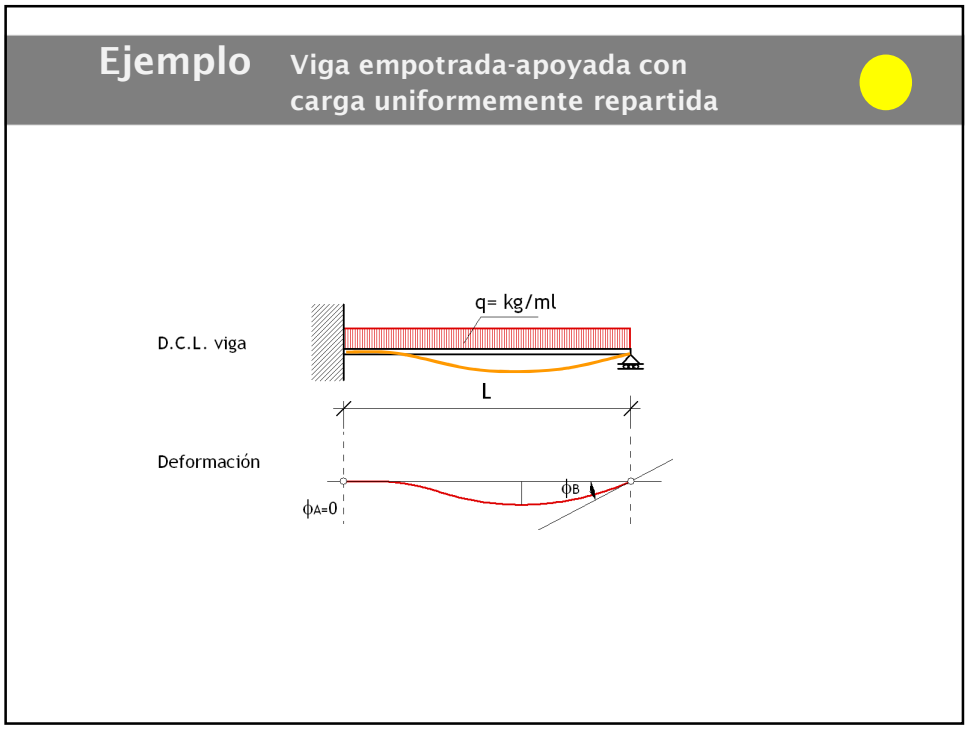






$$y_{MAX} = -\frac{PL^3}{48EI} + \frac{PL^3}{128EI} + \frac{PL^3}{128EI}$$

$$\boxed{y_{MAX} = -\frac{PL^3}{192EI}}$$



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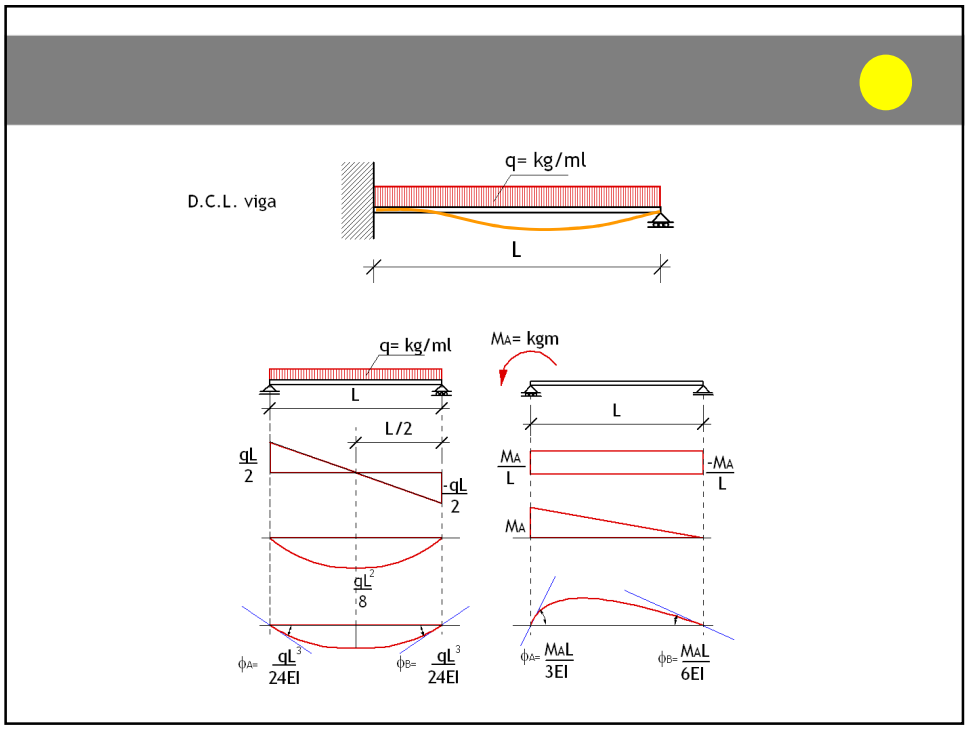
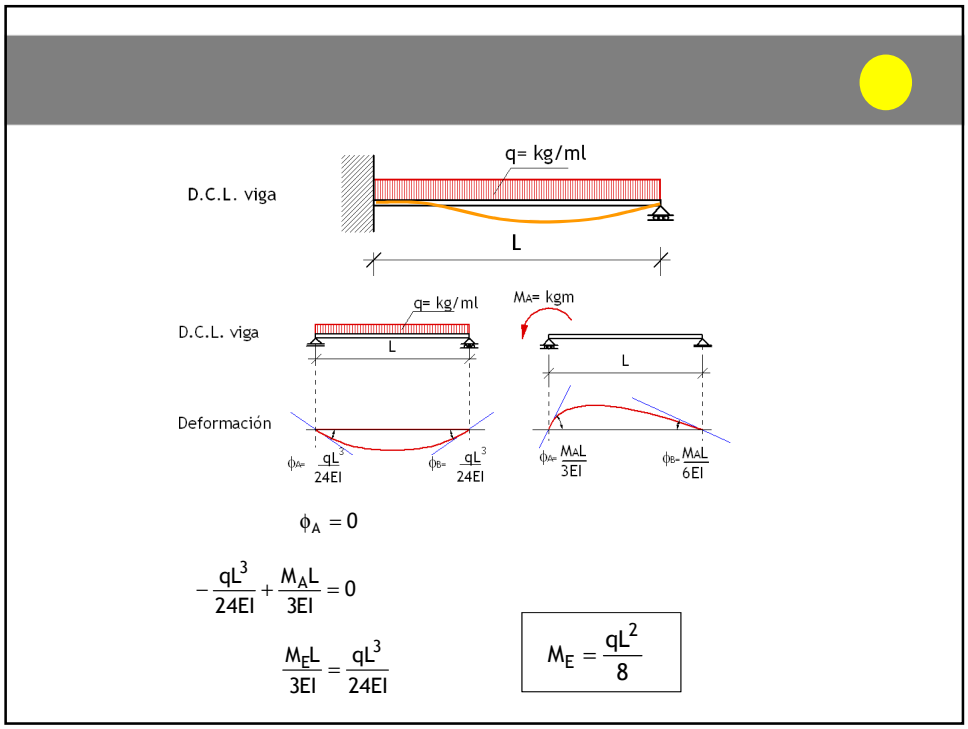




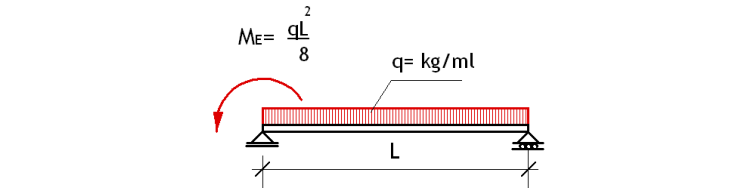
$$\phi_A = 0$$

$$-\frac{qL^3}{24EI} + \frac{M_A L}{3EI} = 0$$

$$\frac{M_E L}{3EI} = \frac{qL^3}{24EI}$$

$$M_E = \frac{qL^2}{8}$$

E S T R U C T U R A S II

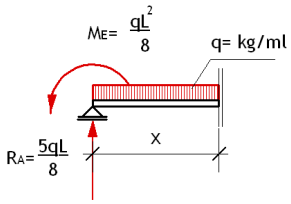


$$Ra = \frac{qL}{2} + \frac{Me}{L} = \frac{qL}{2} + \frac{qL}{8} = \frac{5qL}{8} \qquad Rb = \frac{qL}{2} - \frac{Me}{L} = \frac{qL}{2} - \frac{qL}{8} = \frac{3qL}{8}$$

$$Vx = 0$$

$$\frac{5qL}{8} - q.x = 0$$

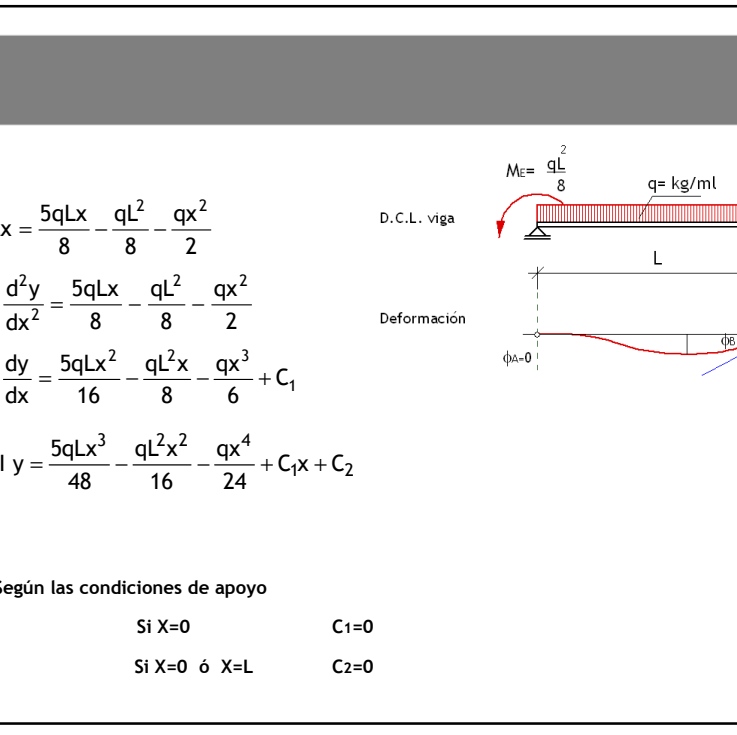
$$X = \frac{5L}{8}$$



$$Mx = \frac{5qLx}{8} - \frac{qx^2}{2} - Me$$

$$M_{MAX} = \frac{25qL^2}{64} - \frac{25qL^2}{128} - \frac{qL^2}{8}$$

$$M_{MAX} = \frac{9qL^2}{128}$$



$$Mx = \frac{5qLx}{8} - \frac{qL^2}{8} - \frac{qx^2}{2}$$

$$EI\frac{d^2y}{dx^2} = \frac{5qLx}{8} - \frac{qL^2}{8} - \frac{qx^2}{2}$$

$$EI\frac{dy}{dx} = \frac{5qLx^2}{16} - \frac{qL^2x}{8} - \frac{qx^3}{6} + C_1$$

$$EI\,y = \frac{5qLx^3}{48} - \frac{qL^2x^2}{16} - \frac{qx^4}{24} + C_1x + C_2$$

D.C.L. viga

Deformación

Según las condiciones de apoyo

Si X=0 $C_1=0$

Si X=0 ó X=L $C_2=0$

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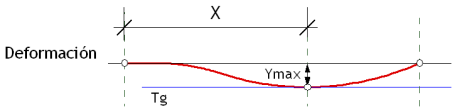
Flecha máx en dy/dx=0



$$0 = \frac{5qLx^2}{16} - \frac{qL^2x}{8} - \frac{qx^3}{6}$$

$$X_1 = 0$$

$$X_2 = \frac{-15L + \sqrt{33L^2}}{-16} = 0{,}58L$$

$$X_3 = \frac{-15L - \sqrt{33L^2}}{-16} = 1{,}3L$$

$$Y = \frac{5qL}{48EI}(0{,}58L)^3 - \frac{qL^2}{16EI}(0{,}58L)^2 - \frac{q}{24EI}(0{,}58L)^4$$

$$Y_{MAX} = -0{,}005\frac{qL^4}{EI} = -\frac{qL^4}{185EI}$$