

ESTRUCTURAS II

Deformación en vigas ESTRUCTURAS 2

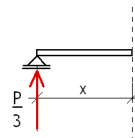
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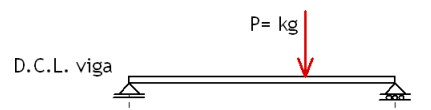
Deformación en vigas

TRAMO 1 (0-2L/3)

$$M_x = \frac{Px}{3}$$



D.C.L. viga



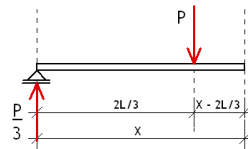
Corte



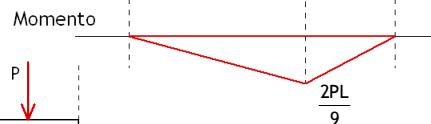
TRAMO 2 (2L/3-L)

$$M_x = \frac{Px}{3} - P \left(x - \frac{L}{3} \right)$$

$$M_x = -\frac{2Px}{3} + \frac{2PL}{3}$$



Momento



ESTRUCTURAS II

Deformación en vigas

TRAMO 1 (0-2L/3)

$$Mx = \frac{Px}{3}$$

$$EI \frac{d^2y}{dx^2} = \frac{Px}{3}$$

$$EI \frac{dy}{dx} = \frac{Px^2}{6} + C_1$$

$$EI y = \frac{Px^3}{18} + C_1x + C_2$$

TRAMO 2 (2L/3-L)

$$Mx = -\frac{2Px}{3} + \frac{2PL}{3}$$

$$EI \frac{d^2y}{dx^2} = -\frac{2Px}{3} + \frac{2PL}{3}$$

$$EI \frac{dy}{dx} = -\frac{Px^2}{3} + \frac{2PLx}{3} + C_3$$

$$EI y = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x + C_4$$

Deformación en vigas

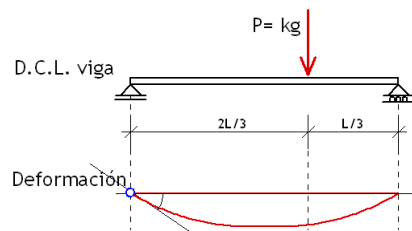
TRAMO 1 (0-2L/3)

$$x = 0 \rightarrow y = 0$$

$$EI y = \frac{Px^3}{18} + C_1x + C_2$$

$$EI 0 = \frac{P}{18} * 0^2 + C_1 * 0 + C_2$$

$$C_2 = 0 \quad (A)$$



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TRAMO 2 ($2L/3-L$)

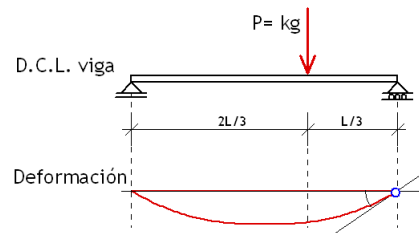
$$x = L \rightarrow y = 0$$

$$EI y = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x + C_4$$

$$EI 0 = -\frac{PL^3}{9} + \frac{PL^3}{3} + C_3L + C_4$$

$$C_4 = +\frac{PL^3}{9} - \frac{PL^3}{3} - C_3L$$

$$C_4 = -\frac{2PL^3}{9} - C_3L \quad (B)$$



Deformación en vigas

TRAMO 1 ($0-2L/3$)

$$EI \frac{dy}{dx} = \frac{Px^2}{6} + C_1$$

$$x = \frac{2L}{3} \rightarrow \frac{dx}{dy} \text{ es el mismo para ambos tramos}$$

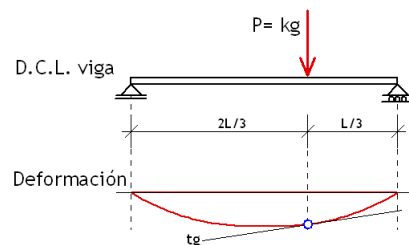
$$\frac{Px^2}{6} + C_1 = -\frac{Px^2}{3} + \frac{2PLx}{3} + C_3$$

$$\frac{P}{6} \frac{4L^2}{9} + C_1 = -\frac{P}{3} \frac{4L^2}{9} + \frac{2PL}{3} \frac{2L}{3} + C_3$$

$$C_1 = \frac{2PL^2}{9} + C_3 \quad (C)$$

TRAMO 2 ($2L/3-L$)

$$EI \frac{dy}{dx} = -\frac{Px^2}{3} + \frac{2PLx}{3} + C_3$$



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TRAMO 1 (0-2L/3)

$$EI y = \frac{Px^3}{18} + C_1x + C_2$$

$$x = \frac{2L}{3} \rightarrow y \text{ es el mismo para ambos tramos}$$

$$\frac{Px^3}{18} + C_1x + 0 = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x + C_4$$

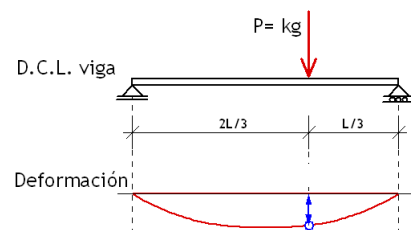
TRAMO 2 (2L/3-L)

$$EI y = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x + C_4$$

$$(A) \quad C_2 = 0$$

$$(B) \quad C_4 = -\frac{2PL^3}{9} - C_3L$$

$$(C) \quad C_1 = \frac{2PL^2}{9} + C_3$$



Deformación en vigas

TRAMO 1 (0-2L/3)

$$EI y = \frac{Px^3}{18} + C_1x + C_2$$

$$x = \frac{2L}{3} \rightarrow y \text{ es el mismo para ambos tramos}$$

$$\frac{Px^3}{18} + C_1x + 0 = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x + C_4$$

$$\frac{Px^3}{18} + \frac{2PL^2x}{9} + C_3x = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x - \frac{2PL^3}{9} - C_3L$$

$$\frac{Px^3}{18} \frac{8L^3}{27} + \frac{2PL^2}{9} \frac{2L}{3} + C_3 \frac{2L}{3} = -\frac{P}{9} \frac{8L^3}{27} + \frac{PLx^2}{3} + C_3 \frac{2L}{3} - \frac{2PL^3}{9} - C_3L$$

$$C_3 = -\frac{22PL^2}{81} \quad (D)$$

TRAMO 2 (2L/3-L)

$$EI y = -\frac{Px^3}{9} + \frac{PLx^2}{3} + C_3x + C_4$$

$$(A) \quad C_2 = 0$$

$$(B) \quad C_4 = -\frac{2PL^3}{9} - C_3L$$

$$(C) \quad C_1 = \frac{2PL^2}{9} + C_3$$

ESTRUCTURAS II

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$$C_1 = \frac{2PL^2}{9} + C_3$$

$$C_1 = \frac{2PL^2}{9} - \frac{22PL^2}{81}$$

$$C_1 = -\frac{4PL^2}{81}$$

$$C_4 = -\frac{2PL^3}{9} - C_3 L$$

$$C_4 = -\frac{2PL^3}{9} - \frac{22PL^3}{81}$$

$$C_4 = -\frac{40PL^3}{81}$$

(A) $C_2 = 0$

(B) $C_4 = -\frac{2PL^3}{9} - C_3 L$

(C) $C_1 = \frac{2PL^2}{9} + C_3$

(D) $C_3 = -\frac{22PL^2}{81}$

Deformación en vigas



TRAMO 1 (0-2L/3)

$$M_x = \frac{Px}{3}$$

Ecuación de Pendiente

$$EI \frac{dy}{dx} = \frac{Px^2}{6} - \frac{4PL^2}{81}$$

Ecuación de Flecha

$$EI y = \frac{Px^3}{18} - \frac{4PL^2}{81} x$$

TRAMO 2 (2L/3-L)

$$M_x = -\frac{2Px}{3} + \frac{2PL}{3}$$

Ecuación de Pendiente

$$EI \frac{dy}{dx} = -\frac{Px^2}{3} + \frac{2PLx}{3} - \frac{22PL^2}{81}$$

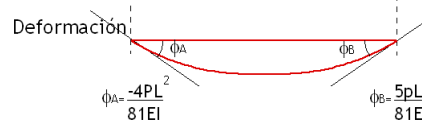
Ecuación de Flecha

$$EI y = -\frac{Px^3}{9} + \frac{PLx^2}{3} - \frac{22PL^2}{81} x - \frac{40PL^3}{81}$$

ESTRUCTURAS II

Deformación en vigas

ANGULOS EN LOS APOYOS



TRAMO 1 (0-2L/3)

$$EI \frac{dy}{dx} = \frac{Px^2}{6} - \frac{4PL^2}{81}$$

$$x = 0$$

$$EI \phi_A = \frac{P}{6} \cdot 0^2 - \frac{4PL^2}{81}$$

$$\phi_A = -\frac{4PL^2}{81EI}$$

TRAMO 2 (2L/3-L)

$$EI \frac{dy}{dx} = -\frac{Px^2}{3} + \frac{2PLx}{3} - \frac{22PL^2}{81}$$

$$x = L$$

$$EI \phi_B = -\frac{P}{3}L^2 + \frac{2PL}{3}L - \frac{22PL^2}{81}$$

$$\phi_B = \frac{5PL^2}{81EI}$$

Deformación en vigas

FLECHA MÁXIMA DE LA VIGA

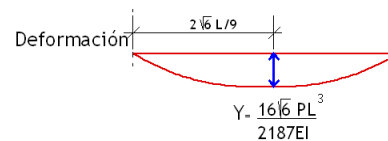
$$\frac{dy}{dx} = 0 \rightarrow y = \text{máximo}$$

$$EI \frac{dy}{dx} = \frac{Px^2}{6} - \frac{4PL^2}{81}$$

$$EI \cdot 0 = \frac{Px^2}{6} - \frac{4PL^2}{81}$$

$$x^2 = \frac{24L^2}{81}$$

$$x = \frac{2\sqrt{6}L}{9}$$



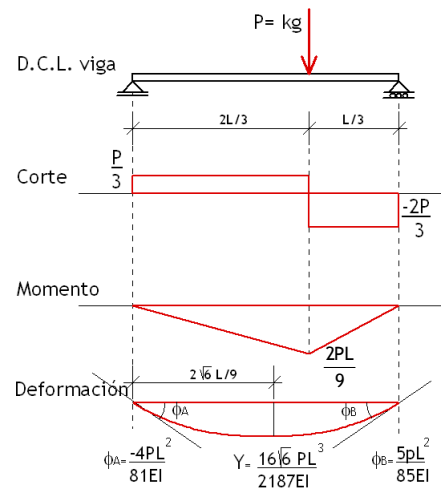
$$EI y_{\max} = \frac{Px^3}{18} - \frac{4PL^2}{81}x$$

$$EI y_{\max} = \frac{P}{18} \left(\frac{2\sqrt{6}L}{9} \right)^3 - \frac{4PL^2}{81} \frac{2\sqrt{6}L}{9}$$

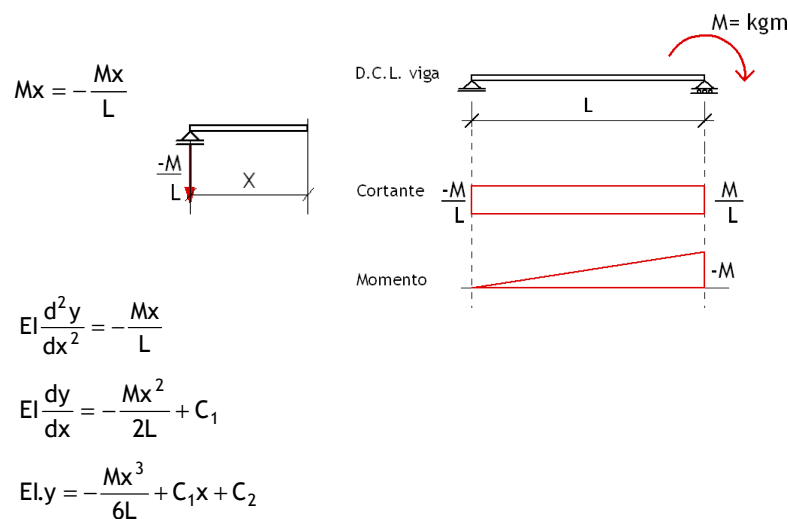
$$y_{\max} = -\frac{16\sqrt{6}PL^3}{2187EI}$$

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Deformación en vigas



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$$EI.y = -\frac{Mx^3}{6L} + C_1x + C_2$$

$$x = 0 \rightarrow y = 0$$

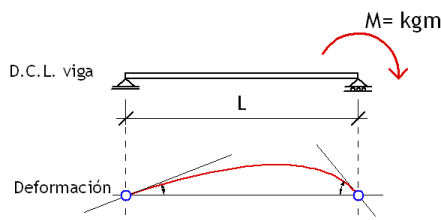
$$0 = -\frac{M}{6} * 0^2 + C_1 * 0 + C_2$$

$$C_2 = 0$$

$$x = L \rightarrow y = 0$$

$$0 = -\frac{ML^2}{6} + C_1 L + 0$$

$$C_1 = \frac{ML}{6}$$



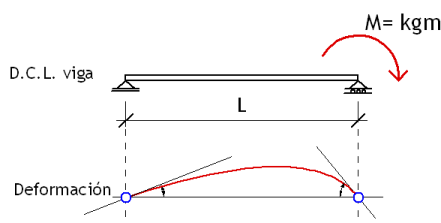
Deformación en vigas

Ecuación de Pendiente

$$EI \frac{dy}{dx} = -\frac{Mx^2}{2L} + \frac{ML}{6}$$

Ecuación de Flecha

$$EI.y = -\frac{Mx^3}{6L} + \frac{MLx}{6}$$



ESTRUCTURAS II

Deformación en vigas

ANGULOS EN LOS APOYOS

$$EI \frac{dy}{dx} = -\frac{Mx^2}{2L} + \frac{ML}{6}$$

$$x = 0$$

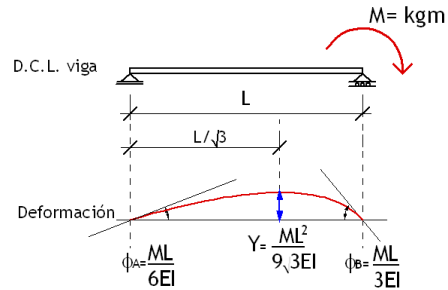
$$EI \phi_A = -\frac{M}{2L} 0^2 + \frac{ML}{6}$$

$$\phi_A = \frac{ML}{6EI}$$

$$x = L$$

$$EI \phi_B = -\frac{M}{2L} L^2 + \frac{ML}{6}$$

$$\phi_B = -\frac{ML}{3EI}$$



Deformación en vigas

FLECHA MÁXIMA DE LA VIGA

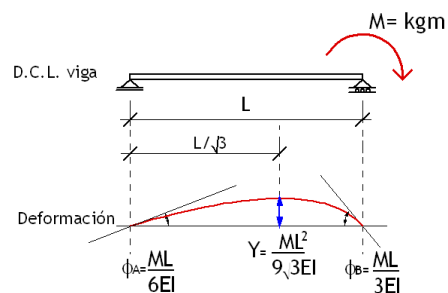
$$\frac{dy}{dx} = 0 \rightarrow y = \text{máximo}$$

$$EI \frac{dy}{dx} = -\frac{Mx^2}{2L} + \frac{ML}{6}$$

$$EI * 0 = -\frac{Mx^2}{2L} + \frac{ML}{6}$$

$$x^2 = \frac{L^2}{3}$$

$$x = \frac{L}{\sqrt{3}}$$



$$EI.y = -\frac{Mx^3}{6L} + \frac{MLx}{6}$$

$$EI.y = -\frac{M}{6L} \left(\frac{L}{\sqrt{3}} \right)^3 + \frac{ML}{6} \frac{L}{\sqrt{3}}$$

$$y = \frac{ML^2}{9\sqrt{3}.EI}$$

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