CURSO
ESTRUCTURAS I

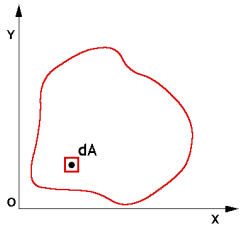
CLASE 8 :
PROPIEDADES DE LA SECCION
AREA - CENTROIDE - INERCIA - RADIO DE GIRO

- Profesor: Jing Chang Lou

PROPIEDADES DE LA SECCION

AREA

$$A = \int_0^A dA$$

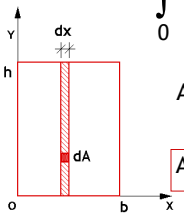


dA = porción insignificante pequeña de una superficie

A = área

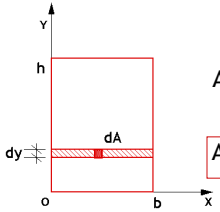
PROPIEDADES DE LA SECCION

AREA $A = \int_0^A dA$



$$A = \int_0^b da = \int_0^b h\,dx = h\int_0^b dx = h\,[x]_0^b$$

$$A = h\,b$$

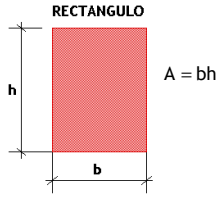


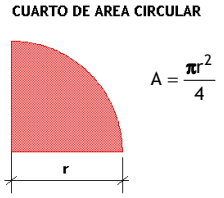
$$A = \int_0^h da = \int_0^h b\,dy = b\int_0^h dy = b\,[y]_0^h$$

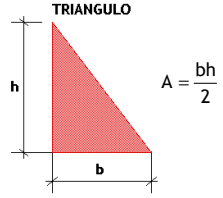
$$A = b\,h$$

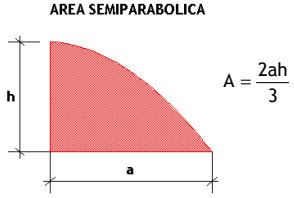
PROPIEDADES DE LA SECCION

AREA DE FIGURAS SIMPLES



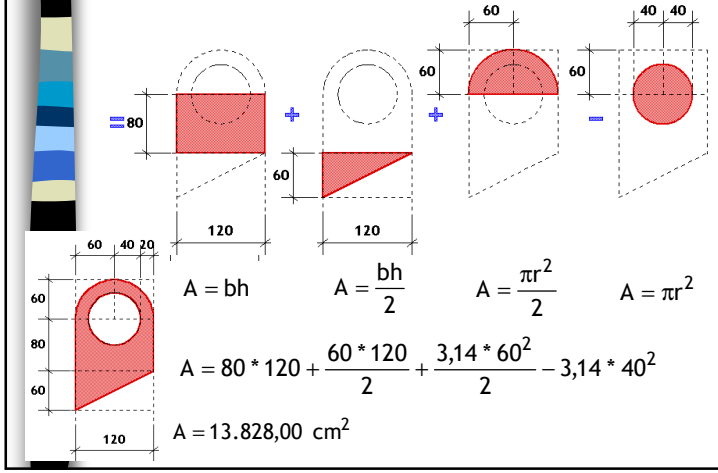






PROPIEDADES DE LA SECCION

AREA DE FIGURAS COMPUESTAS



$A = bh \qquad A = \frac{bh}{2} \qquad A = \frac{\pi r^2}{2} \qquad A = \pi r^2$

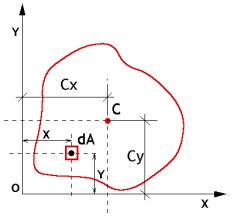
$$A = 80 * 120 + \frac{60 * 120}{2} + \frac{3{,}14 * 60^2}{2} - 3{,}14 * 40^2$$

$A = 13.828{,}00 \ cm^2$

PROPIEDADES DE LA SECCION

CENTROIDES DE AREAS PLANAS

MOMENTO ESTATICO



$M_X = A \ c_y$

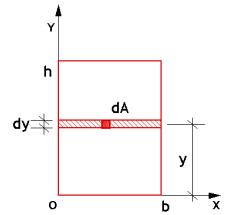
$$M_X = \int_0^A y \ dA \qquad \int_0^A y \ dA = A \ c_y \qquad \frac{\int_0^A y \ dA}{A} = c_y$$

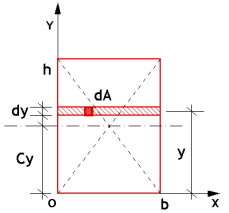
$M_Y = A \ c_X$

$$M_Y = \int_0^A x \ dA \qquad \int_0^A x \ dA = A \ c_x \qquad \frac{\int_0^A x \ dA}{A} = c_x$$

PROPIEDADES DE LA SECCION

CENTROIDES DE AREAS PLANAS





$$\int_0^h y \ dA = A \ c_y$$

$$\int_0^h y \ b \ dy = A \ c_y$$

$$b \int_0^h y \ dy = A \ c_y$$

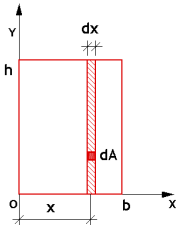
$$b \left[\frac{y^2}{2} \right]_0^h = bh \ c_y$$

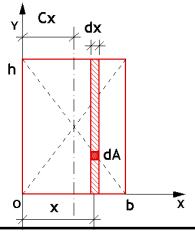
$$\frac{bh^2}{2 \ bh} = c_y$$

$$\boxed{\frac{h}{2} = c_y}$$

PROPIEDADES DE LA SECCION

CENTROIDES DE AREAS PLANAS





$$\int_0^b x \ dA = A \ c_x$$

$$\int_0^b x \ h \ dx = A \ c_x$$

$$h \int_0^b x \ dx = A \ c_x$$

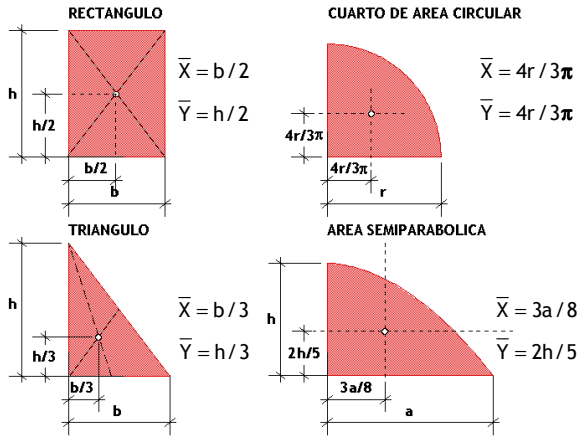
$$h \left[\frac{x^2}{2} \right]_0^b = bh \ c_x$$

$$\frac{hb^2}{2 \ bh} = c_x$$

$$\boxed{\frac{b}{2} = c_x}$$

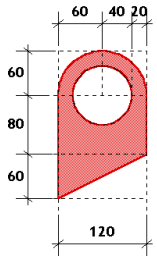
PROPIEDADES DE LA SECCION

CENTROIDES DE FIGURAS SIMPLES



PROPIEDADES DE LA SECCION

CENTROIDE DE FIGURAS COMPUESTAS

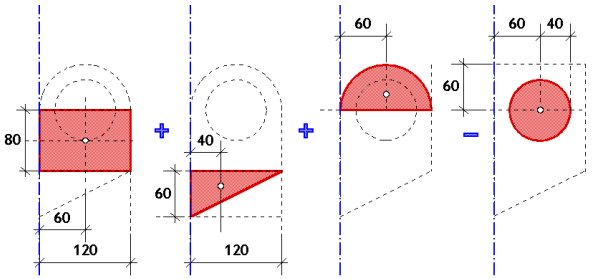


$$\frac{\int_0^A x\,dA}{A} = c_x$$

$$\frac{\int_0^A y\,dA}{A} = c_y$$

PROPIEDADES DE LA SECCION

CENTROIDE DE FIGURAS COMPUESTAS

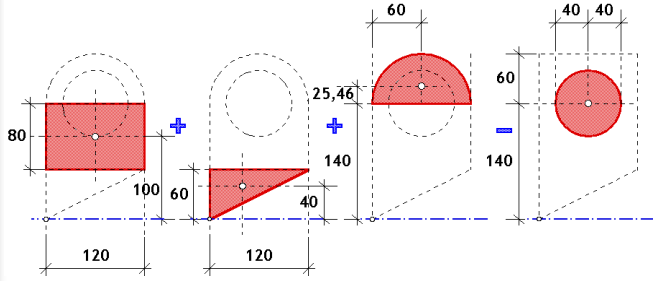


$$\bar{X} = \frac{(80*120)\,60 + \left(\frac{60*120}{2}\right)40 + \left(\frac{3{,}14*60^2}{2}\right)60 - (3{,}14*40^2)\,60}{13.828{,}00}$$

$\bar{X} = 54{,}79$ cm

PROPIEDADES DE LA SECCION

CENTROIDE DE FIGURAS COMPUESTAS

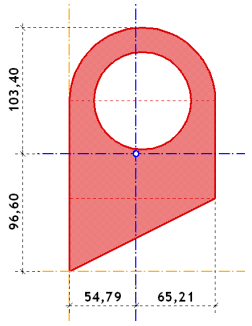


$$\bar{Y} = \frac{(80*120)\,100 + \left(\frac{60*120}{2}\right)40 + \left(\frac{3{,}14*60^2}{2}\right)165{,}46 - (3{,}14*40^2)\,140}{13.828{,}00}$$

$\bar{Y} = 96{,}60$ cm

PROPIEDADES DE LA SECCION

CENTROIDE DE FIGURAS COMPUESTAS

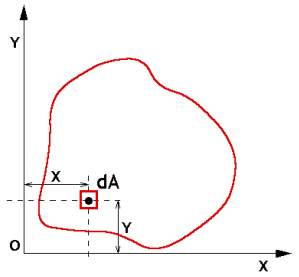


$\overline{X} = 54{,}79$ cm

$\overline{Y} = 96{,}60$ cm

PROPIEDADES DE LA SECCION

MOMENTO DE INERCIA CON RESPECTO A UN EJE



$$I_X = \int_0^A y^2 \, dA$$

$$I_Y = \int_0^A x^2 \, dA$$

PROPIEDADES DE LA SECCION

MOMENTO DE INERCIA CON RESPECTO A UN EJE QUE PASA POR EL CENTROIDE DE LA FIGURA

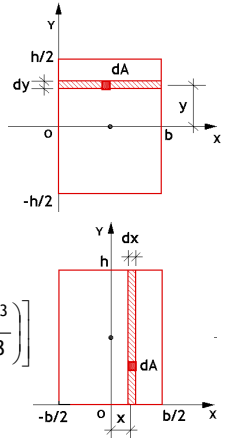
$$I_{GX} = \int_{-h/2}^{h/2} y^2 \, dA$$

$$I_{GX} = \int_{-h/2}^{h/2} y^2 \, b \, dy$$

$$I_{GX} = b \left[\frac{y^3}{3} \right]_{-h/2}^{h/2}$$

$$I_{GX} = \frac{b}{3} \left[\frac{h^3}{8} - \left(-\frac{h^3}{8} \right) \right]$$

$$I_{GX} = \frac{bh^3}{12}$$



$$I_{GY} = \int_{-b/2}^{b/2} x^2 \, dA$$

$$I_{GY} = \int_{-b/2}^{b/2} x^2 \, b \, dx$$

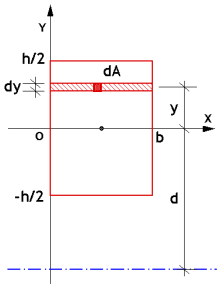
$$I_{GY} = h \left[\frac{x^3}{3} \right]_{-b/2}^{b/2}$$

$$I_{GY} = \frac{h}{3} \left[\frac{b^3}{8} - \left(-\frac{b^3}{8} \right) \right]$$

$$I_{GY} = \frac{hb^3}{12}$$

PROPIEDADES DE LA SECCION

MOMENTO DE INERCIA CON RESPECTO A UN EJE CUALQUIERA



$$I = \int_0^A (y + d)^2 \, dA$$

$$I = \int_0^A \left(y^2 + 2yd + d^2 \right) dA$$

$$I = \int_0^A y^2 \, dA + 2d \int_0^A y \, dA + d^2 \int_0^A dA$$

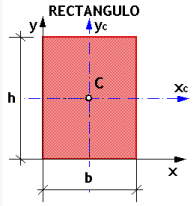
$$I = I_G + 0 + d^2 A$$

$$I = \frac{bh^3}{12} + 0 + d^2 A$$

PROPIEDADES DE LA SECCION

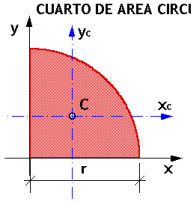
MOMENTOS DE INERCIA DE FIGURAS SIMPLES

RECTANGULO



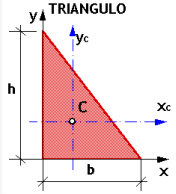
$$I_{X_C} = \frac{bh^3}{12} \qquad I_{Y_C} = \frac{b^3h}{12}$$

CUARTO DE AREA CIRCULAR



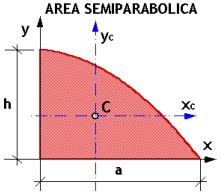
$$I_{X_C} = \frac{(9\pi^2 - 64)\ r^4}{144\pi} \qquad I_{Y_C} = \frac{(9\pi^2 - 64)\ r^4}{144\pi}$$

TRIANGULO



$$I_{X_C} = \frac{bh^3}{36} \qquad I_{Y_C} = \frac{b^3h}{36}$$

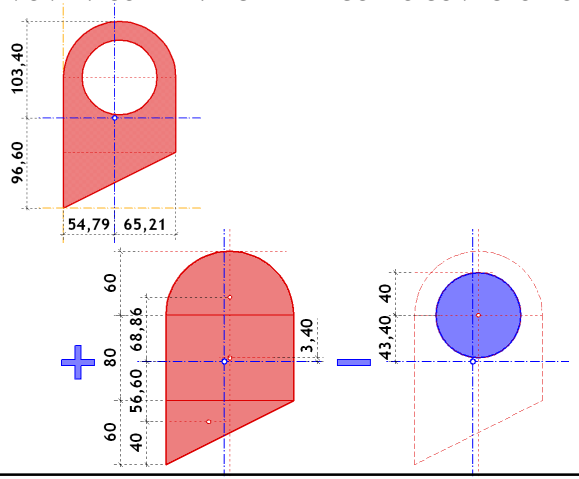
AREA SEMIPARABOLICA



$$I_{X_C} = \frac{8ah^3}{175} \qquad I_{Y_C} = \frac{19a^3h}{480}$$

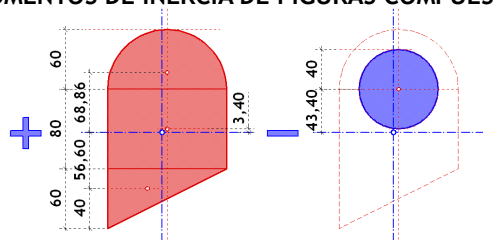
PROPIEDADES DE LA SECCION

MOMENTOS DE INERCIA DE FIGURAS COMPUESTAS



PROPIEDADES DE LA SECCION

MOMENTOS DE INERCIA DE FIGURAS COMPUESTAS



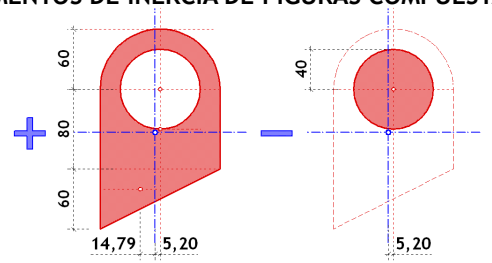
$$I_{X_C} = \left(\frac{120^*80^3}{12}\right) + (120*80)\ 3{,}40^2 + \left(\frac{\left(9^*3{,}14^2 - 64\right)60^4}{72^*3{,}14}\right) + \left(\frac{3{,}14^*60^2}{2}\right) 68{,}86^2$$

$$+\left(\frac{120^*60^3}{36}\right) + \left(\frac{120^*60}{2}\right) 56{,}60^2 - \left[\left(\frac{3{,}14^*40^4}{4}\right) + \left(3{,}14^*40^2\right) 43{,}40^2\right]$$

$$I_{X_C} = 34.229.282{,}89\ cm^4$$

PROPIEDADES DE LA SECCION

MOMENTOS DE INERCIA DE FIGURAS COMPUESTAS



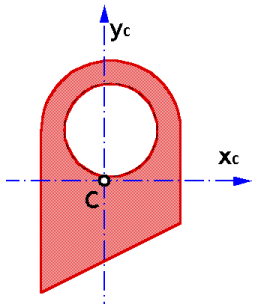
$$I_{Y_C} = \left(\frac{3{,}14^*60^4}{8}\right) + \left(\frac{3{,}14^*60^2}{2}\right) 5{,}20^2 + \left(\frac{80^*120^3}{12}\right) + \left(80^*120\right) 5{,}20^2$$

$$+\left(\frac{60^*120^3}{36}\right) + \left(\frac{60^*120}{2}\right) 14{,}80^2 - \left[\left(\frac{3{,}14^*40^4}{4}\right) + \left(3{,}14^*40^2\right) 5{,}20^2\right]$$

$$I_{Y_C} = 18.541.243{,}88\ cm^4$$

PROPIEDADES DE LA SECCION

MOMENTOS DE INERCIA DE FIGURAS COMPUESTAS

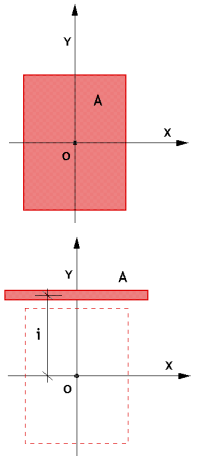


$I_{X_C} = 34.229.282,89 \ cm^4$

$I_{Y_C} = 18.541.243,88 \ cm^4$

PROPIEDADES DE LA SECCION

RADIO DE GIRO



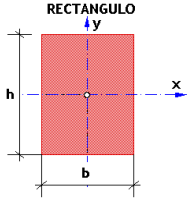
$I = A \ i^2$

$i = \sqrt{\frac{I}{A}}$

PROPIEDADES DE LA SECCION

RADIO DE GIRO DE FIGURAS SIMPLES

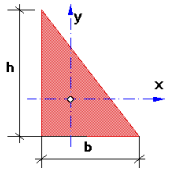
RECTANGULO



$i_X = \frac{h}{\sqrt{12}}$

$i_Y = \frac{b}{\sqrt{12}}$

TRIANGULO

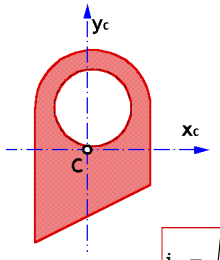


$i_X = \frac{h}{\sqrt{18}}$

$i_Y = \frac{b}{\sqrt{18}}$

PROPIEDADES DE LA SECCION

RADIO DE GIRO DE FIGURAS COMPUESTAS



$A = 13.828 \ cm^2$

$I_{X_C} = 34.229.282,89 \ cm^4$

$I_{Y_C} = 18.541.243,88 \ cm^4$

$i_X = \sqrt{\frac{I_X}{A}} = \sqrt{\frac{34.229.282,89}{13.828}} = 49,75 \ cm$

$i_Y = \sqrt{\frac{I_Y}{A}} = \sqrt{\frac{18.541.243,88}{13.828}} = 36,62 \ cm$

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