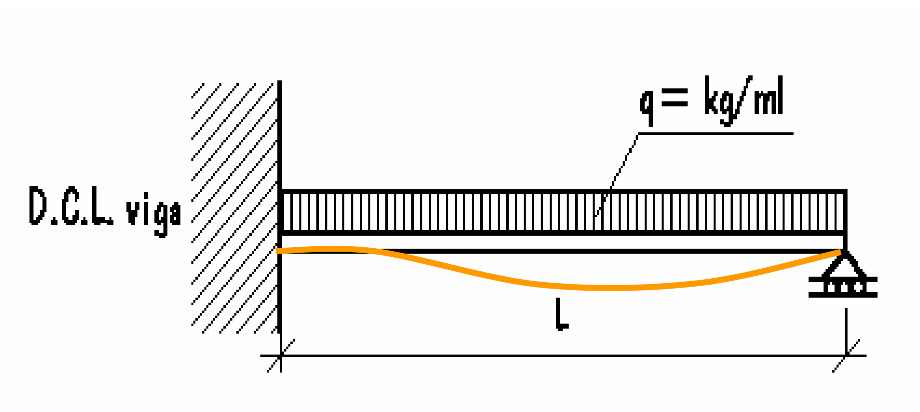
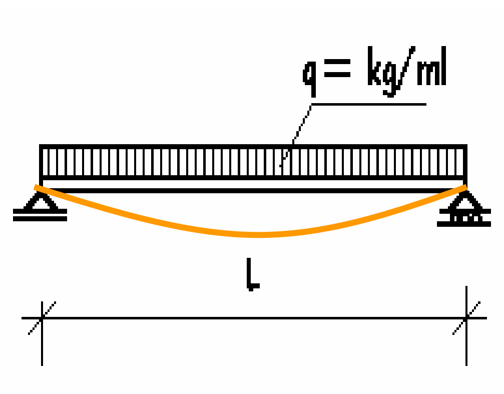
004

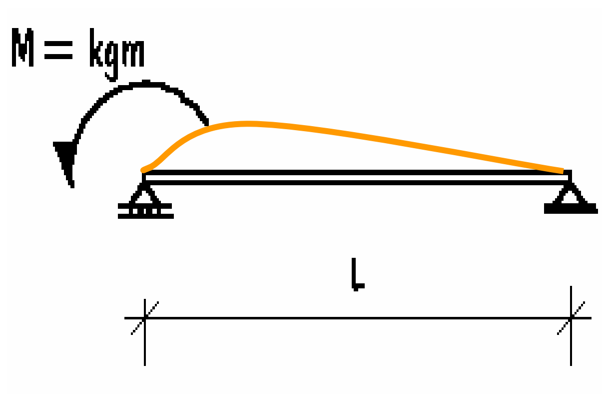
VIGAS HIPERESTÁTICAS

Verónica Veas B. – Gabriela Muñoz S.

a) VIGAS HIPERESTATICAS POR EMPOTRAMIENTO

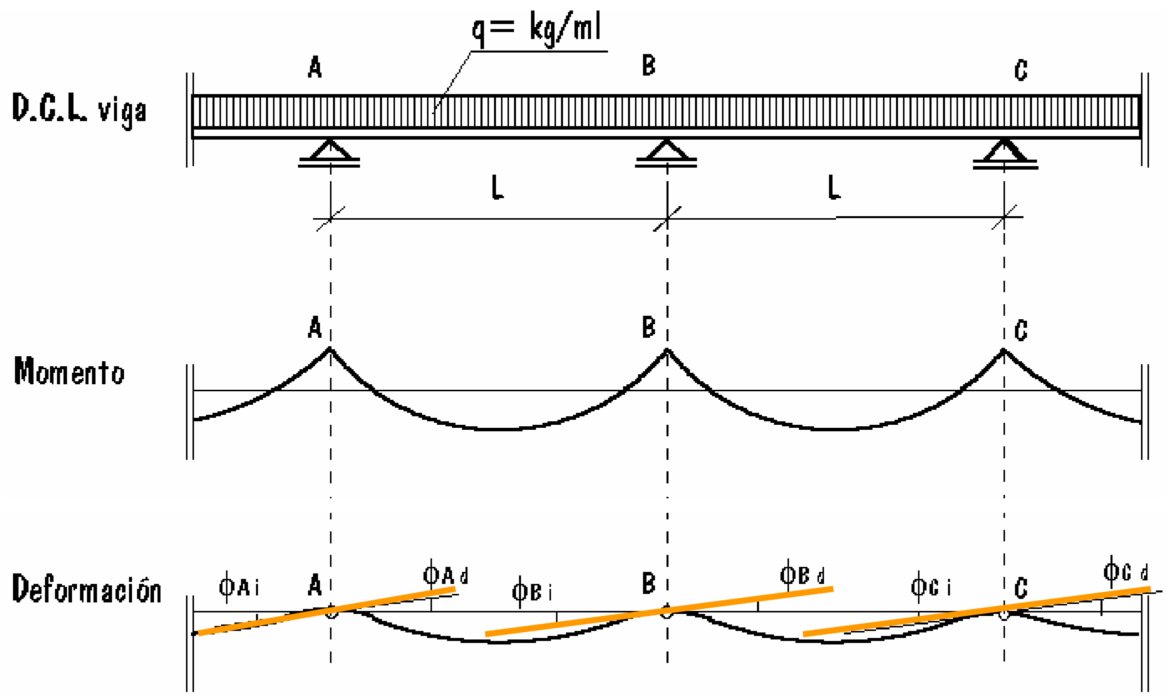






$\varnothing_{\text{empotramiento}} = 0$

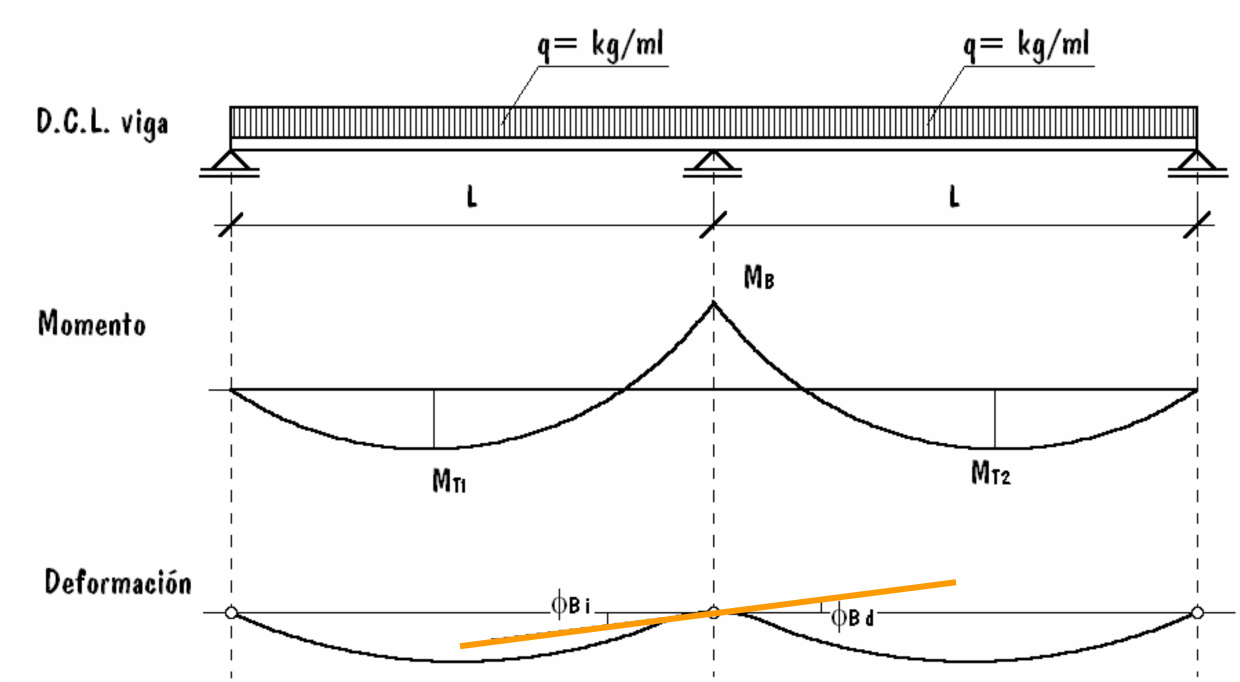
b) VIGAS HIPERESTATICAS POR CONTINUIDAD



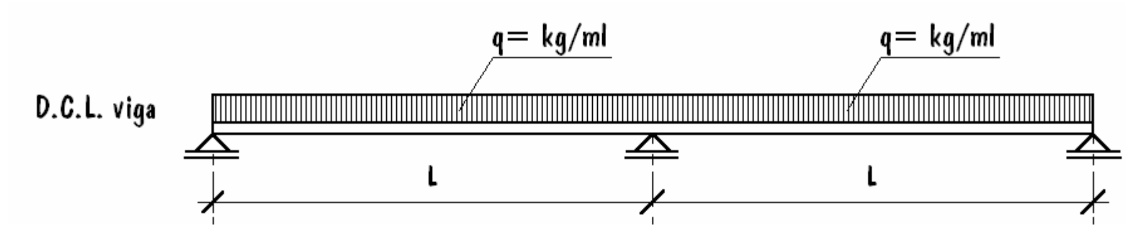
$$\phi B_{izquierdo} = - \phi B_{derecho}$$

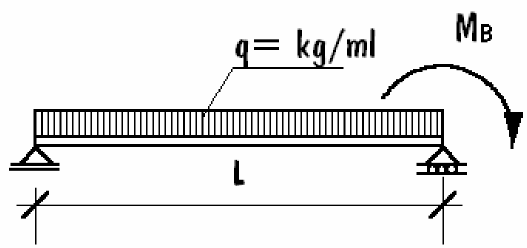
por ángulos opuestos por el vértice

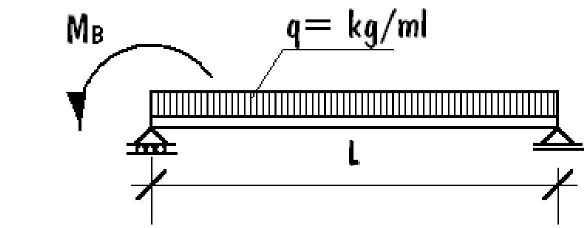
VIGA DE DOS TRAMOS CON CARGA UNIFORMEMENTE REPARTIDA

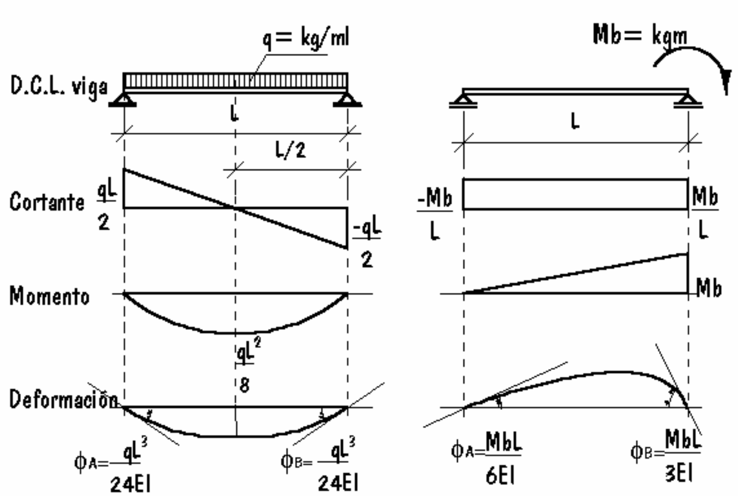


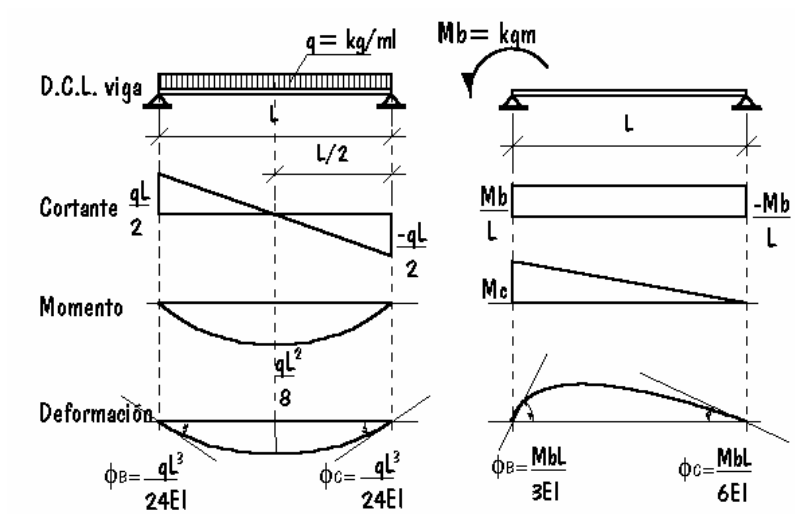
$$\phi B_{izquierdo} = - \phi B_{derecho}$$

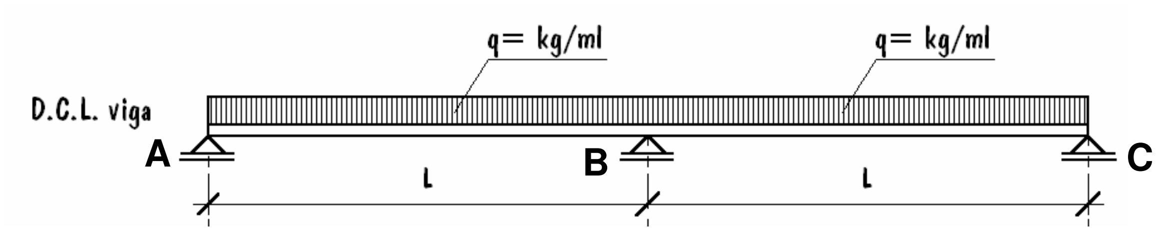

D.C.L. viga
q= kg/ml
q= kg/ml
L
L


q= kg/ml
MB
L


MB
q= kg/ml
L


D.C.L. viga
q= kg/ml
Mb= kgm
L
L/2
Cortante
qL/2
-qL/2
-Mb/L
Mb/L
Momento
Mb
qL²/8
Deformación
φA= qL³/24EI
φB= qL³/24EI
φA= MbL/6EI
φB= MbL/3EI


D.C.L. viga
q= kg/ml
Mb= kgm
L
L/2
Cortante
qL/2
-qL/2
Mb/L
-Mb/L
Momento
Mc
qL²/8
Deformación
φB= qL³/24EI
φC= qL³/24EI
φB= MbL/3EI
φC= MbL/6EI

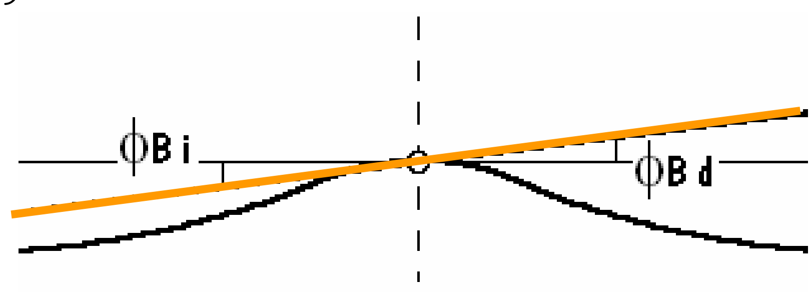


Se igualan los valores de ángulos a ambos lados del apoyo B para determinar el momento de continuidad entre ambos tramos.

$$\Sigma \phi_{B \text{ Izquierdo}} = -\Sigma \phi_{B \text{ derecho}}$$

$$-\frac{qL^3}{24EI} + \frac{M_B L}{3EI} = -\left(-\frac{qL^3}{24EI} + \frac{M_B L}{3EI}\right)$$

$$\frac{2M_B L}{3EI} = \frac{qL^3}{12EI} \qquad \ldots * EI/L$$



$$\boxed{M_B = \frac{qL^2}{8EI}}$$

$$R_B = \frac{qL}{2} + \frac{M_B}{L} = \frac{5qL}{8}$$

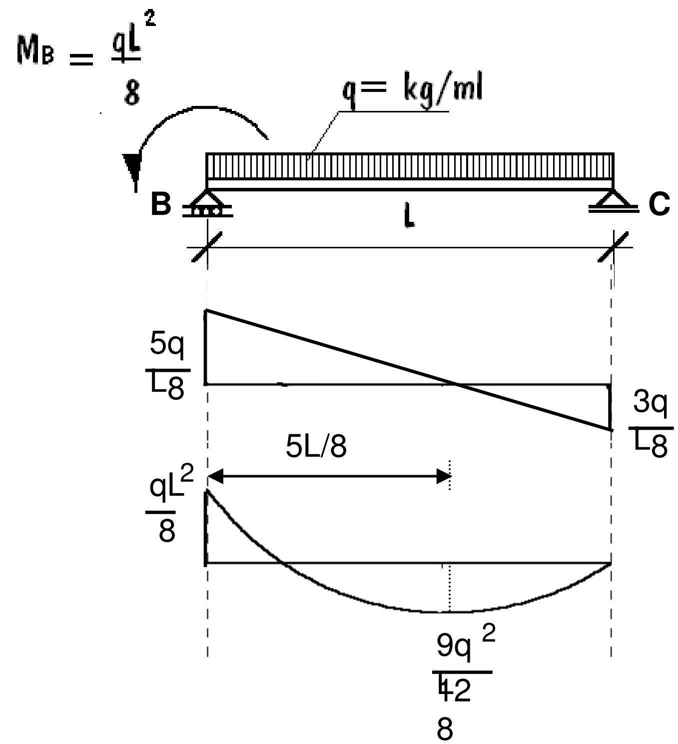
$$R_C = \frac{qL}{2} - \frac{M_B}{L} = \frac{3qL}{8}$$

$$Mx = \frac{5qLx}{8} - \frac{qx^2}{2} - \frac{qL^2}{8}$$

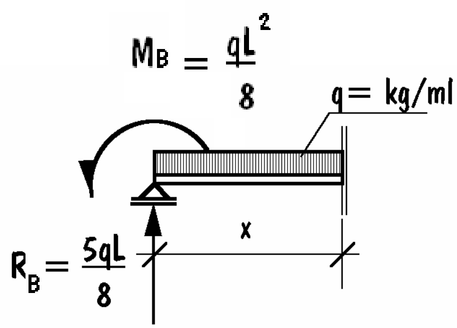
$$V_x = 0$$

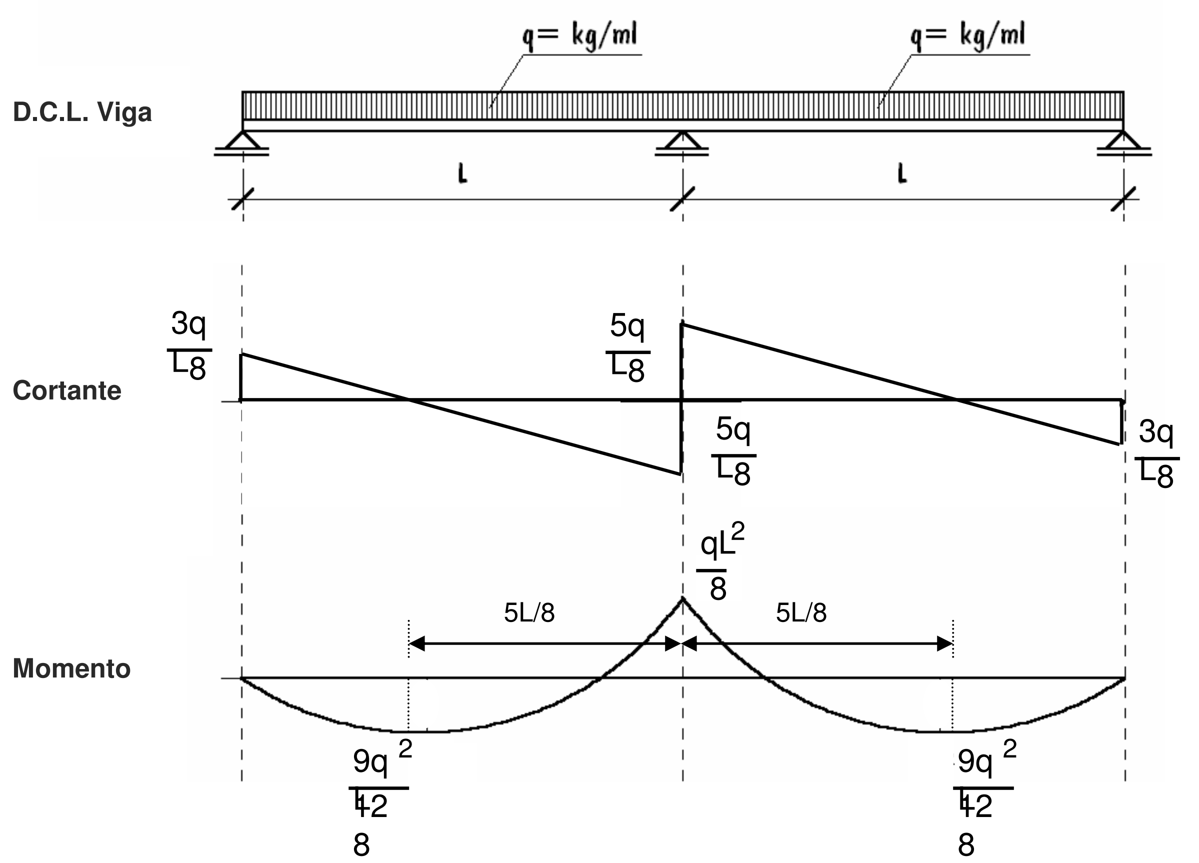
$$\frac{5qL}{8} - q\,x = 0 \qquad X = \frac{5L}{8}$$

$$M_{MAX} = \frac{25qL^2}{64} - \frac{25qL^2}{128} - \frac{qL^2}{8}$$



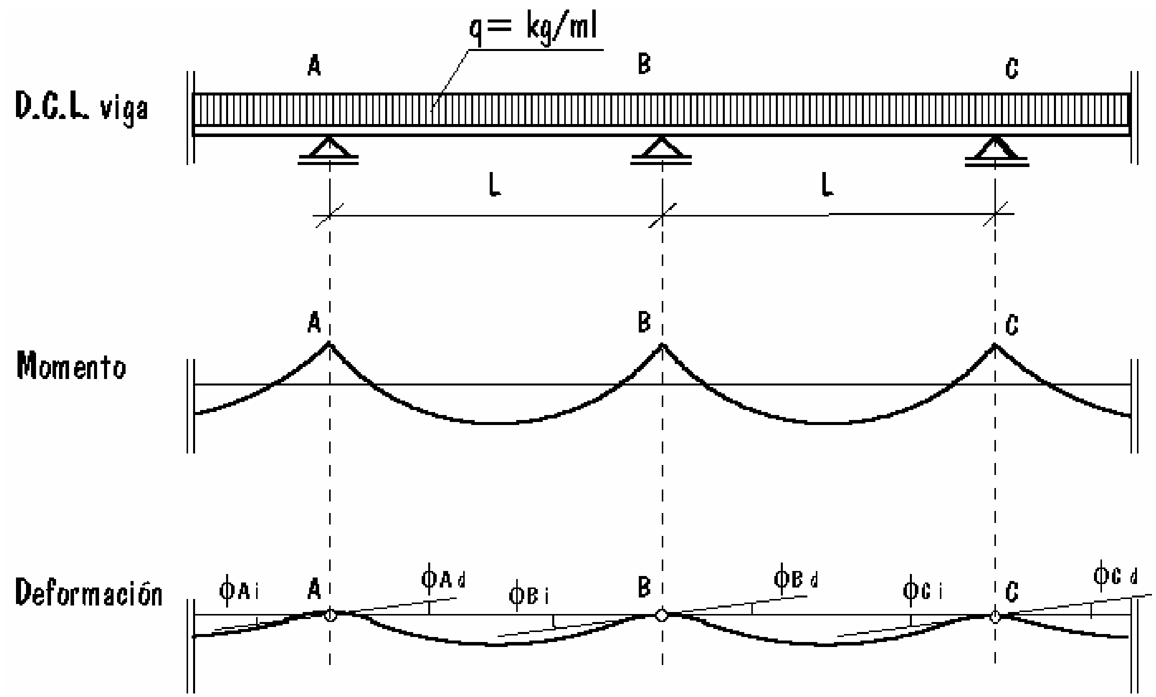
$$\boxed{M_{MAX} = \frac{9qL^2}{128}}$$

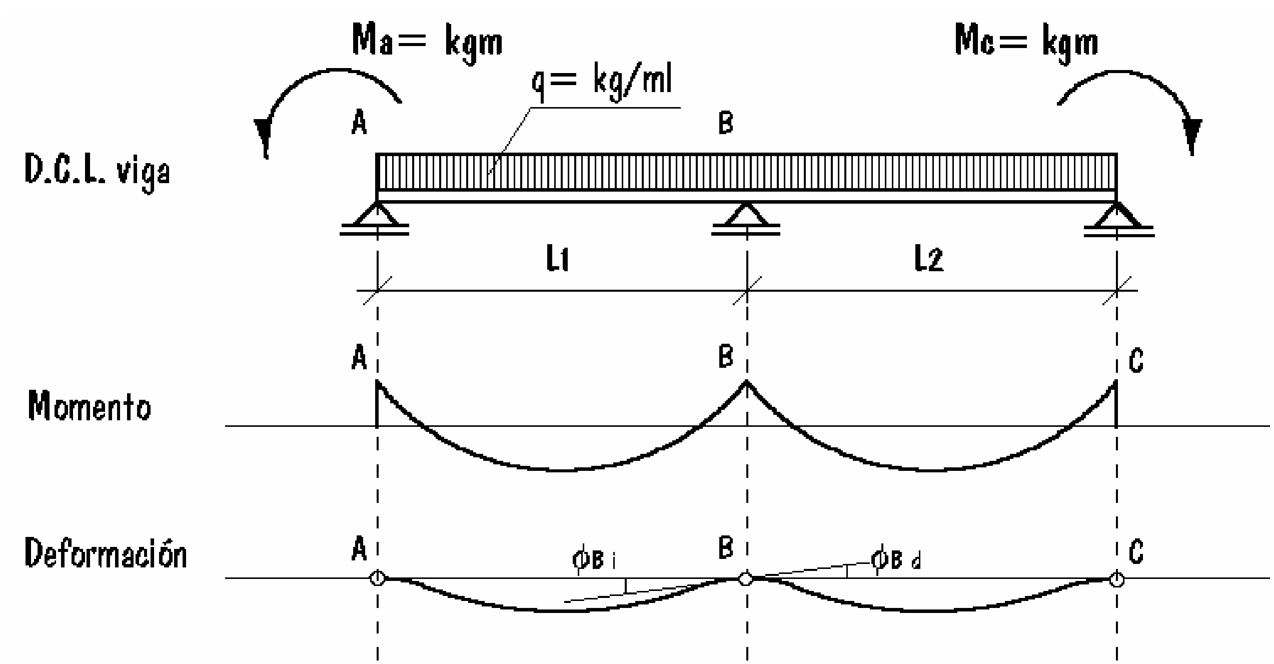




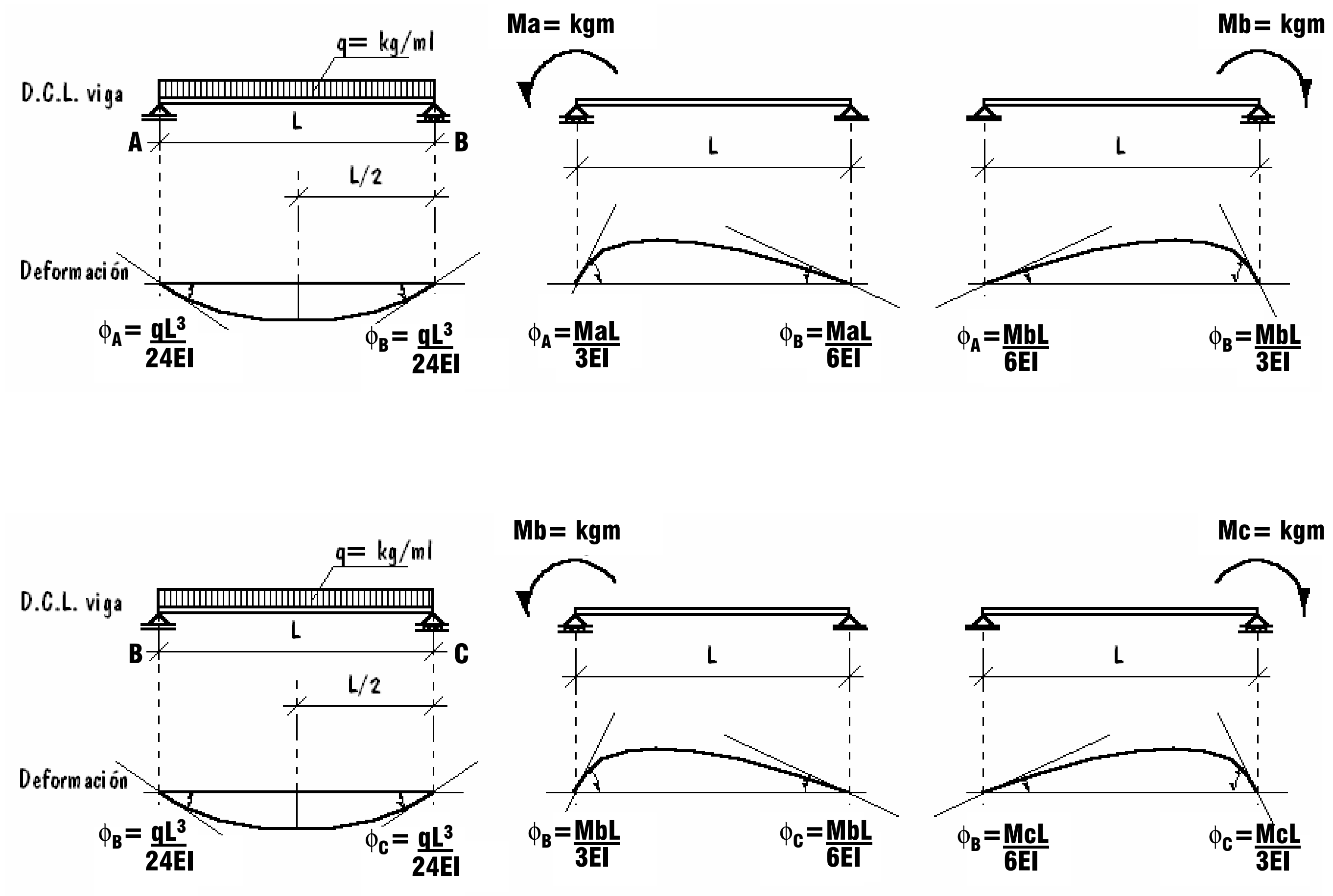
D.C.L. Viga
q= kg/ml
q= kg/ml
L
L
Cortante
3q
L8
5q
L8
5q
L8
3q
L8
Momento
qL2
8
5L/8
5L/8
9q 2
L2
8
9q 2
L2
8

TEOREMA DE LOS TRES MOMENTOS O CLAPEYRON





$\Sigma\phi B$ **izquierdo=** $-\Sigma\phi B$ **derecho**


q= kg/ml
D.C.L. viga
L
A
B
L/2
Deformación
φA= gL3 / 24EI
φB= gL3 / 24EI
Ma= kgm
L
φA=MaL / 3EI
φB=MaL / 6EI
Mb= kgm
L
φA=MbL / 6EI
φB=MbL / 3EI
q= kg/ml
D.C.L. viga
L
B
C
L/2
Deformación
φB= gL3 / 24EI
φC= gL3 / 24EI
Mb= kgm
L
φB=MbL / 3EI
φC=MbL / 6EI
Mc= kgm
L
φB=McL / 6EI
φC=McL / 3EI

$$\Sigma\phi B \text{ izquierdo} = -\Sigma\phi B \text{ derecho}$$

$$-\frac{qL_1^3}{24EI} + \frac{MaL_1}{6EI} + \frac{MbL_1}{3EI} = -\left[-\frac{qL_2^3}{24EI} + \frac{MbL_2}{3EI} + \frac{McL_2}{6EI}\right]$$

$$\frac{MaL_1}{6EI} + \frac{MbL_1}{3EI} + \frac{MbL_2}{3EI} + \frac{McL_2}{6EI} = \frac{qL_1^3}{24EI} + \frac{qL_2^3}{24EI}$$

Reemplazando L/EI por λ (módulo de flexibilidad)

$$\frac{Ma\lambda_1}{6} + \frac{Mb\lambda_1}{3} + \frac{Mb\lambda_2}{3} + \frac{Mc\lambda_2}{6} = \frac{qL_1^2\lambda_1}{24} + \frac{qL_2^2\lambda_2}{24} \quad /{*}6$$

Al amplificar la expresión 6 veces se obtiene

$$Ma\lambda_1 + 2Mb\lambda_1 + 2Mb\lambda_2 + Mc\lambda_2 = 6\left[\frac{qL_1^2\lambda_1}{24} + \frac{qL_2^2\lambda_2}{24}\right]$$

$$Ma\lambda_1 + 2Mb(\lambda_1+\lambda_2) + Mc\lambda_2 = 6\left[\frac{qL_1^2\lambda_1}{24} + \frac{qL_2^2\lambda_2}{24}\right]$$

Si EI = constante y λ = L/EI λ = L

Reemplazando λ = L en la ecuación se tiene:

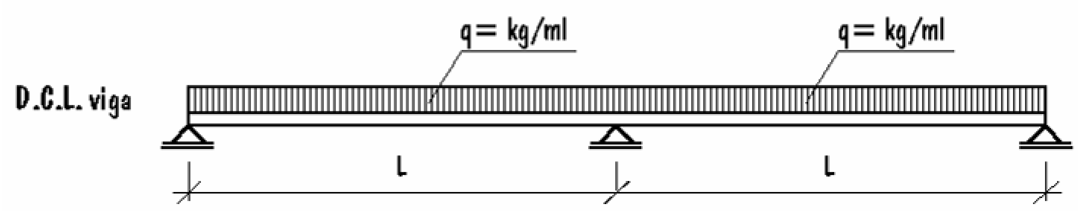
$$MaL_1 + 2Mb\,(L_1 + L_2) + McL_2 = \frac{6}{24}\left[\frac{qL_1^3}{24} + \frac{qL_2^3}{24}\right]$$

Reemplazando $\frac{qL_1^3}{24}$ **por** Tc_1 **y** $\frac{qL_2^3}{24}$ **por** Tc_2

$$\boxed{MaL_1 + 2Mb\,(L_1+L_2) + McL_2 = 6(Tc_1 + Tc_2)}$$

APLICACIÓN DEL TEOREMA DE CLAPEYRON

VIGA DE DOS TRAMOS CON CARGA UNIFORMEMENTE REPARTIDA



$$M_aL_1 + 2M_b\,(L_1+L_2) + M_cL_2 = 6\,(Tc_1 + Tc_2)$$

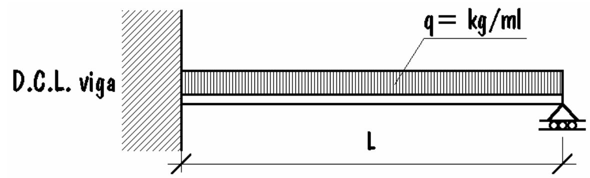
$$0L_1 + 2M_b\,(L_1+L_2) + 0L_2 = 6\left[\frac{qL_1^3}{24} + \frac{qL_2^3}{24}\right]$$

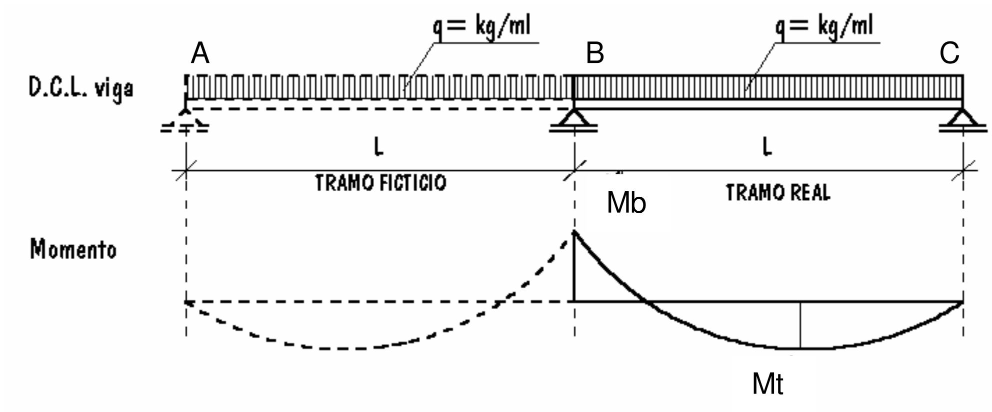
$$2M_b\,(L_1+L_2) = \frac{qL_1^3}{4} + \frac{qL_2^3}{4} \qquad \textbf{Si} \quad \mathbf{L_1=L_2}$$

$$2M_b\,2L = \frac{qL^3}{2}$$

$$\boxed{M_b = \frac{qL^2}{8}}$$

VIGA EMPOTRADA EN UN EXTREMO Y APOYADA EN EL OTRO CON CARGA UNIFORMEMENTE REPARTIDA





$$M_aL_1 + 2M_b(L_1+L_2) + M_cL_2 = 6(T_{c1} + T_{c2})$$

$$00 + 2M_b(0+L) + 0L = 6\left[0 + \frac{qL^3}{24}\right]$$

$$2M_b L = \frac{qL^3}{4}$$

$$\boxed{M_b = \frac{qL^2}{8}}$$