FM1003-2 Matematica III: Límites y Derivadas
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Para estudiantes de Educación Básica y Media.

Solución P4 d)

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P4.- d)

$$sin(\alpha)+sin(\beta)=2sin(\frac{\alpha+\beta}{2})cos(\frac{\alpha-\beta}{2})$$

Comenzaremos en el lado derecho y llegaremos al lado izquierdo!
Lo primero será utilizar las indicaciones del auxiliar 8 para separar las sumas de seno y coseno del lado derecho de la siguiente forma:

$$=2\{\overbrace{sin(\frac{\alpha}{2})cos(\frac{\beta}{2})+sin(\frac{\beta}{2})cos(\frac{\alpha}{2})]}^{\text{Esto es } sin(\frac{\alpha+\beta}{2})}\overbrace{[cos(\frac{\alpha}{2})cos(\frac{\beta}{2})+sin(\frac{\alpha}{2})sin(\frac{\beta}{2})}^{\text{Esto es } cos(\frac{\alpha-\beta}{)}2}\}$$

Aquí distribuiremos y reordenaremos, no hay nada extraño, por favor no se asusten (solo tienen que ser ordenados :))

$$=2\{\underbrace{sin^2(\frac{\alpha}{2})sin(\frac{\beta}{2})cos(\frac{\beta}{2})+cos^2(\frac{\alpha}{2})sin(\frac{\beta}{2})cos(\frac{\beta}{2})}_{\text{factorizaremos esto por } sin(\frac{\beta}{2})cos(\frac{\beta}{2})}$$

$$+\underbrace{sin^2(\frac{\beta}{2})sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})+sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})cos^2(\frac{\beta}{2})}_{\text{y factorizaremos esto por } sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})}\}$$

$$=2\{sin(\frac{\beta}{2})cos(\frac{\beta}{2})\quad\underbrace{[sin^2(\frac{\alpha}{2})+cos^2(\frac{\alpha}{2})]}_{\text{Esto da 1 por identidad fundamental!}}\quad+sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})\quad\underbrace{[sin^2(\frac{\beta}{2})+cos^2(\frac{\beta}{2})]}_{\text{De la misma manera, esto también es 1}}\quad\}$$

$$=2\{sin(\frac{\beta}{2})cos(\frac{\beta}{2})+sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})\}$$

$$=2sin(\frac{\beta}{2})cos(\frac{\beta}{2})+2sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})$$

$$=\underbrace{sin(\frac{\beta}{2})cos(\frac{\beta}{2})+sin(\frac{\beta}{2})cos(\frac{\beta}{2})}_{\text{Esto es justamente } sin(\frac{\beta}{2}+\frac{\beta}{2})}+\underbrace{sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})+sin(\frac{\alpha}{2})cos(\frac{\alpha}{2})}_{\text{Y esto es } sin(\frac{\alpha}{2}+\frac{\alpha}{2})}$$

$$=sin(\frac{\beta}{2}+\frac{\beta}{2})+sin(\frac{\alpha}{2}+\frac{\alpha}{2})$$

$$=sin(\alpha)+sin(\beta)$$

Llegando a lo que queríamos :)