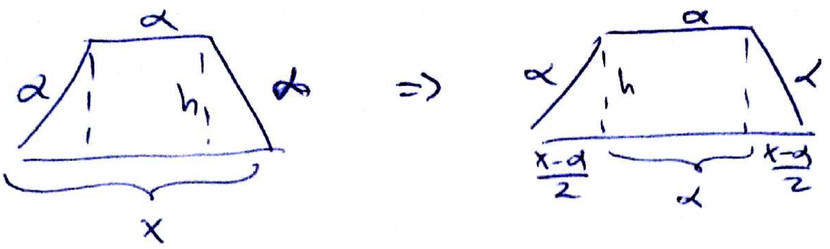
P6)



$$A(x) = \left(\frac{x+\alpha}{2}\right)h$$

$$A'(x) = \frac{1}{2}h + \left(\frac{x+\alpha}{2}\right)h'$$

$$h^2 = \alpha^2 - \left(\frac{x-\alpha}{2}\right)^2$$

$$h^2 = \alpha^2 - \frac{(x^2 - 2x\alpha + \alpha^2)}{4}$$

$$h^2 = \frac{4\alpha^2 - x^2 + 2x\alpha - \alpha^2}{4}$$

$$h^2 = \frac{3\alpha^2 + 2x\alpha - x^2}{4}$$

$$h = \frac{\sqrt{3\alpha^2 + 2x\alpha - x^2}}{2}$$

$$\hookrightarrow h' = \frac{1}{2}\left(\frac{1}{2}(3\alpha^2 + 2x\alpha - x^2)^{-1/2} \cdot (2\alpha - 2x)\right)$$

$$\Rightarrow A'(x) = \frac{1}{4}\sqrt{3\alpha^2 + 2x\alpha - x^2} + \frac{1}{8}(x+\alpha)(3\alpha^2 + 2x\alpha - x^2)^{-1/2}(2\alpha - 2x)$$

$$= \frac{1}{4}\sqrt{3\alpha^2 + 2x\alpha - x^2} + \frac{1}{4}(x+\alpha)(\alpha - x)(3\alpha^2 + 2x\alpha - x^2)^{-1/2}$$

$$= \frac{1}{4}\sqrt{3\alpha^2 + 2x\alpha - x^2} + \frac{1}{4}(\alpha^2 - x^2)(3\alpha^2 + 2x\alpha - x^2)^{-1/2} = 0$$

$$\Rightarrow \frac{1}{4}(3\alpha^2 + 2x\alpha - x^2) = -\frac{1}{4}(\alpha^2 - x^2)$$

$$\Rightarrow 3\alpha^2 + 2x\alpha - x^2 = (\alpha^2 - x^2) -$$

$$\Rightarrow 3\alpha^2 + 2x\alpha - x^2 = x^2 - \alpha^2$$

$$\Rightarrow 2x^2 - 2\alpha x - 4\alpha^2 = 0$$

$$\Rightarrow x^2 - \alpha x - 2\alpha^2 = 0$$

$$\underbrace{(x+\alpha)}_{\text{No sirve}}(x - 2\alpha) = 0$$

No sirve $\Rightarrow \boxed{x = 2\alpha}$

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P7) $f(x) = \left(a - \frac{1}{a} - x\right)(4 - 3x^2)$

$$\Rightarrow f'(x) = -(4-3x^2) + \left(a - \frac{1}{a} - x\right)(-6x)$$

$$= 3x^2 - 4 + 6x^2 - 6ax + \frac{6}{a}x$$

$$= 9x^2 + \left(\frac{6}{a} - 6a\right)x - 4$$

Igualo $f'(x) = 0 \Rightarrow 9x^2 + \left(\frac{6}{a} - 6a\right)x - 4 = 0$

$$9ax^2 + (6 - 6a^2)x - 4a = 0$$

$$ax^2 + \frac{6}{9}(1 - a^2)x - \frac{4}{9}a = 0$$

$$ax^2 + \frac{2}{3}(1 - a^2)x - \frac{4}{9}a = 0$$

$$\therefore \quad x = \frac{\frac{2}{3}(a^2-1) \pm \sqrt{\left(\frac{2}{3}(1-a^2)\right)^2 + \frac{16}{9}a^2}}{2a}$$

$$x = \frac{\frac{2}{3}(a^2-1) \pm \sqrt{\frac{4}{9}(1-a^2)^2 + \frac{16}{9}a^2}}{2a}$$

$$x = \frac{\frac{2}{3}(a^2-1) \pm \frac{\sqrt{4(1-a^2)^2 + 16a^2}}{3}}{2a}$$

$$x = \frac{2(a^2-1) \pm \sqrt{4(1-a^2)^2 + 16a^2}}{6a}$$

$$x = \frac{2(a^2-1) \pm 2\sqrt{(1-a^2)^2 + 4a^2}}{6a}$$

$$x = \frac{(a^2-1) \pm \sqrt{(1-a^2)^2 + 4a^2}}{3a}$$

$$x = \frac{(a^2-1) \pm \sqrt{a^4 - 2a^2 + 1 + 4a^2}}{3a}$$

$$x = \frac{(a^2-1) \pm \sqrt{a^4 + 2a^2 + 1}}{3a}$$

$$x = \frac{(a^2-1) \pm \sqrt{(a^2+1)^2}}{3a}$$

o $x = \frac{(a^2-1) \pm (a^2+1)}{3a}$

$$\to x_1 = \frac{a^2 - 1 + a^2 + 1}{3a} = \frac{2}{3}a$$

$$\to x_2 = \frac{a^2 - 1 - a^2 - 1}{3a} = \frac{-2}{3a}$$

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$$\therefore\ f(x_1) = \left(a - \frac{1}{a} - \frac{2}{3}a\right)\left(4 - 3\left(\frac{2}{3}a\right)^2\right)$$

$$= \left(\frac{1}{3}a - \frac{1}{a}\right)\left(4 - \frac{4}{3}a^2\right) = \frac{4}{3}a - \frac{4}{a} - \frac{4}{9}a^3 + \frac{4}{3}a$$

$$\Rightarrow f(x_1) = -\frac{4}{9}a^3 + \frac{8}{3}a - \frac{4}{a}$$

$$f(x_2) = \left(a - \frac{1}{a} + \frac{2}{3a}\right)\left(4 - 3\left(\frac{-2}{3a}\right)^2\right)$$

$$= \left(a - \frac{1}{3a}\right)\left(4 - \frac{4}{3a^2}\right) = 4a - \frac{4}{3a} - \frac{4}{3a} + \frac{4}{9a^3}$$

$$\Rightarrow f(x_2) = \frac{4}{9a^3} - \frac{8}{3a} + 4a$$

$$\Rightarrow f(x_2) - f(x_1) = \frac{4}{9a^3} - \frac{8}{3a} + 4a + \frac{4}{9}a^3 - \frac{8}{3}a + \frac{4}{a}$$

$$= \frac{4}{9}\left(\frac{1}{a^3} - \frac{6}{a} + \frac{9}{a} + a^3 - 6a + 9a\right)$$

$$= \frac{4}{9}\left(\frac{1}{a^3} + \frac{3}{a} + a^3 + 3a\right) = \frac{4}{9}\left(a + \frac{1}{a}\right)^3$$

* Sea $g(a) = \frac{4}{9}\left(a + 1/a\right)^3$

$$\Rightarrow g'(a) = \frac{4}{3}\left(a + 1/a\right)^2 \cdot \left(1 + 1/a^2\right) = \frac{4}{3}\left(a^2 + 2 + 1/a^2\right)\left(1 - 1/a^2\right)$$

$$= \frac{4}{3}\left(a^2 - 1 + 2 - 2/a^2 + 1/a^2 - 1/a^4\right) = \frac{4}{3}\left(a^2 + 1 - 1/a^2 - 1/a^4\right)$$

Igualamos $g'(a) = 0 \Rightarrow \frac{4}{3}\left(a^2 + 1 - 1/a^2 - 1/a^4\right) = 0$

$$\Rightarrow a^2 + 1 - 1/a^2 - 1/a^4 = 0 \Rightarrow a^6 + a^4 - a^2 - 1 = 0$$

$$\Rightarrow$$

$$\underbrace{(a^4 - 1)}\overbrace{(a^2 + 1)}^{\text{No sirve}} = 0$$

$$a^2 = \pm 1 \Rightarrow \boxed{\begin{array}{l} a = 1 \\ a = -1 \end{array}}$$

Es claro que el mín será $a = -1$, ya que dará el valor negativo.

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