P1 c) $-1+2-3+4-\dots$ (n veces)

2 casos: n par o n impar

1) n par:

$$\left.\begin{aligned} -1+2 &= 1 \\ -3+4 &= 1 \\ -5+6 &= 1 \\ &\vdots \\ -(n-1)+n &= 1 \end{aligned}\right\} \frac{n}{2} \text{ parejas que suman } 1$$

$\therefore$ para n par, la suma resulta $\frac{n}{2}$ ■

2) n impar: quitamos n y procedemos con $n-1$ (que sabemos es par)

$$\left.\begin{aligned} -1+2 &= 1 \\ -3+4 &= 1 \\ &\vdots \\ -(n-2)+(n-1) &= 1 \end{aligned}\right\} \frac{n-1}{2} \text{ parejas de } 1\text{'s.}$$

y le restamos n.

$\therefore$ para n impar, la suma resulta $\frac{(n-1)}{2} - n$ ■

Aux #1

P1) a) $\sum_{k=1}^{n} k(k+1) = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = \frac{n(n+1)(n+2)}{3}$

b) $\sum_{k=1}^{n} 2k+1 = 2\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 = n(n+1) + n = (n+2)n$ //

c) $\sum_{n=1}^{n} (-1)^k k =$

d) $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$ //

P3) Tenemos $\sum_{k=1}^{n} a_k a_{k+1} = b_n$

i) $\sum_{k=1}^{n} a_k a_{k-1} = \sum_{k=0}^{n-1} a_{k+1} a_k = b_n + a_0 a_1 - a_{n+1} a_n$

ii) $\sum_{k=n}^{2n+1} a_k a_{k+1} = \sum_{k=1}^{2n+1} a_k a_{k+1} - \sum_{k=1}^{n-1} a_k a_{k+1} = b_{2n+1} - b_{n-1}$ //

P4) $S_n = 1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \dots + \frac{1+2+3+\dots+n}{n}$

$\Rightarrow S_n = \sum_{k=1}^{n} \frac{\sum_{l=1}^{k} l}{k} = \sum_{k=1}^{n} \frac{\frac{k(k+1)}{2}}{k} = \sum_{k=1}^{n} \frac{k+1}{2} = \frac{1}{2}\left(\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1\right)$

$\therefore S_n = \frac{1}{2}\left(\frac{n(n+1)}{2} + n\right) = \frac{n}{2}\left(\frac{n+1}{2} + 1\right) = \frac{n}{2}\frac{(n+3)}{2} = \frac{n(n+3)}{4}$ //

P5) Sabemos $\sum_{k=1}^{n} a_k = \frac{1}{3}(n^2 + 5n)$, buscar a_n

* Evaluaremos en distintos n para buscar la relación

n=1) $\sum_{k=1}^{1} a_k = \frac{1}{3}(1+5) \iff a_1 = 2$

n=2) $\sum_{k=1}^{2} a_k = \frac{1}{3}(4+10) \iff a_1 + a_2 = \frac{14}{3} \iff a_2 = \frac{8}{3}$

n=3) $\sum_{k=1}^{3} a_k = \frac{1}{3}(9+15) \iff a_1 + a_2 + a_3 = \frac{24}{3} \iff a_3 = \frac{10}{3}$

n=4) $\sum_{k=1}^{4} a_k = \frac{1}{3}(16+20) \iff a_1 + a_2 + a_3 + a_4 = \frac{36}{3} \iff a_4 = \frac{12}{3}$

Ya podemos ver la recurrencia:

Definimos $a_{n+1} = a_n + \frac{2}{3}$; $a_1 = \frac{6}{3} = 2$

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P2) a) $\sum_{k=3}^{n-1}(k-2)(k+1) = \sum_{k=1}^{n-3}(k+3)k = \sum_{k=1}^{n-3} k^2+3k = \sum_{k=1}^{n-3}k^2 + 3\sum_{k=1}^{n-3}k$

$= \frac{(n-3)(n-2)(2n-5)}{6} + 3\frac{(n-3)(n-2)}{2}$ //

b) $\sum_{k=1}^{n} 3(4^k+2k^2)-4k^3 = 3\sum_{k=1}^{n} 4^k + 6\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k^3$

$=3\left(\frac{4^{n+1}-1}{3}\right)+6\frac{(n+1)n(2n+1)}{6}-4\left(\frac{n(n+1)}{2}\right)^2$

$=4^{n+1}-1+n(n+1)(2n+1)-n^2(n+1)^2$

c) $\sum_{k=1}^{n} \frac{1}{\sqrt{k}(k+1)+k\sqrt{k+1}} = \cancel{\frac{1}{k(k+1)\left(\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}\right)}} = \cancel{\frac{1}{k(k+1)}+\frac{1}{\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}}}$

$= \cancel{\frac{1}{k(k+1)\frac{\sqrt{k+1}+\sqrt{k}}{\sqrt{k}\sqrt{k+1}}}} = \cancel{\frac{1}{\sqrt{k}\sqrt{k+1}(\sqrt{k+1}+\sqrt{k})}} = \cancel{\frac{1}{\sqrt{k}\sqrt{k+1}}+\frac{1}{\sqrt{k+1}+\sqrt{k}}}$

$\Rightarrow \sum_{k=1}^{n} \frac{\sqrt{k}(k+1)-k\sqrt{k+1}}{k(k+1)^2-(k+1)k^2} = \sum_{k=1}^{n} \frac{\sqrt{k}(k+1)-k\sqrt{k+1}}{k(k+1)((k+1)-k)}$

$=\sum_{k=1}^{n} \frac{\sqrt{k}(k+1)}{k(k+1)(} - \frac{k\sqrt{k+1}}{k(k+1)} = \sum_{k=1}^{n} \frac{1}{\sqrt{k}}-\frac{1}{\sqrt{k+1}} = 1-\frac{1}{\sqrt{n+1}}$

d) $\sum_{k=1}^{n}(k+1)^2 k! = \sum_{k=1}^{n}(k+1)(k+1)! = \sum_{k=1}^{n}(k+2-1)(k+1)!$

$= \sum_{k=1}^{n}(k+2)(k+1)! - (k+1)! = \sum_{k=1}^{n}(k+2)! - (k+1)! = (n+2)! - 2!$

e) $\sum_{k=0}^{n} 2^k \cdot k = \sum_{k=0}^{n}(2^{k+1}-2^k)k = \sum_{k=0}^{n} k2^{k+1} - k2^k$

$= \sum_{k=0}^{n}(k+1)2^{k+1} - k2^k - 2^{k+1} = \sum_{k=0}^{n}\left[(k+1)2^{k+1} - k2^k\right] - \sum_{k=0}^{n}2^{k+1}$

$= (n+1)2^{n+1} - \sum_{k=1}^{n+1}2^k = (n+1)2^{n+1} - \left(\sum_{k=0}^{n+1}2^k - 1\right)$

$= (n+1)2^{n+1} + 1 - \frac{2^{n+2}-1}{2-1} = (n+1)2^{n+1} + 2 - 2^{n+2}$

f) $\sum_{k=1}^{n}\ln\left(1+\frac{1}{k}\right) = \sum_{k=1}^{n}\ln\left(\frac{k+1}{k}\right) = \sum_{k=1}^{n}\ln(k+1) - \ln(k) = \ln(n+1)$ //

g) $\sum_{k=1}^{n}\frac{1}{k(k+2)} = \sum_{k=1}^{n}\frac{A}{k}+\frac{B}{k+2} = \sum_{k=1}^{n}\frac{A(k+2)+Bk}{k(k+2)} = \sum_{k=1}^{n}\frac{(A+B)k+2A}{k(k+2)}$

$\Rightarrow A+B=0 \rightarrow B=-1/2$

$2A=1 \rightarrow A=1/2$

$\Rightarrow \sum_{k=1}^{n}\frac{1/2}{k} + \frac{-1/2}{k+2} = \frac{1}{2}\sum_{k=1}^{n}\frac{1}{k} - \frac{1}{k+2}$

$= \frac{1}{2}\sum_{k=1}^{n}\frac{1}{k} - \frac{1}{k+1} + \frac{1}{k+1} - \frac{1}{k+2} = \frac{1}{2}\left(\sum_{k=1}^{n}\frac{1}{k} - \frac{1}{k+1} + \sum_{k=1}^{n}\frac{1}{k+1} - \frac{1}{k+2}\right)$

$= \frac{1}{2}\left(1 - \frac{1}{n+1} + \frac{1}{2} - \frac{1}{n+2}\right)$

h) $\sum_{k=m}^{n} b_k - b_{k+2} = \sum_{k=m}^{n} b_k - b_{k+1} + b_{k+1} - b_{k+2}$

$= \sum_{k=m}^{n} b_k - b_{k+1} + \sum_{k=m}^{n} b_{k+1} - b_{k+2} = b_m - b_{n+1} + b_{m+1} - b_{n+2}$