P1] escribir como sumatorias conocidas

a) $-1+2-3+4-5+\ldots$ hasta el n-ésimo término

$$= \sum_{k=1}^{n} (-1)^k \cdot k$$

* en este caso k puede partir desde el 0 ó desde el 1

b) $n + n-1 + n-2 + \ldots + 3 + 2 + 1$

$$= \sum_{k=0}^{n-1} (n-k)$$

c) $$\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \ldots + \frac{1}{2n} = \sum_{k=1}^{n} \frac{1}{2k}$$

d) $$a_0 + a_2 + a_4 + \ldots + a_{2n} = \sum_{k=0}^{n} a_{2k}$$

e) $1 + (1+2) + (1+2+3) + \ldots + (1+2+3+\ldots+n)$

$$= \sum_{i=1}^{n} \sum_{k=1}^{i} k$$

P2] Calcular las siguiente sumatorias

a) $$\sum_{k=3}^{n-1} (k-2)(k+1) = \sum_{k=1}^{n-3} k(k+3) = \sum_{k=1}^{n-3} k^2 + 3k$$

$$= \sum_{k=1}^{n-3} k^2 + \sum_{k=1}^{n-3} 3k = \sum_{k=1}^{n-3} k^2 + 3\sum_{k=1}^{n-3} k$$

$$= \frac{(n-3)(n-2)(2n-5)}{6} + 3\,\frac{(n-3)(n-2)}{2}$$

$$= \frac{(n-3)(n-2)}{2}\left[\frac{2n-5}{3} + \frac{3\cdot 3}{3}\right]$$

$$= \frac{(n-3)(n-2)}{2} \cdot \frac{(2n-5+9)}{3} = \frac{(n-3)(n-2)(2n+4)}{2\cdot 3}$$

$$\ast \sum_{k=3}^{n-1} (k-2)(k+1) = \frac{(n-3)(n-2)(n+2)}{3}$$

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b) $$\sum_{k=1}^{n} 3(4^k+2k^2) = \sum_{k=1}^{n}(3\cdot 4^k + 3\cdot 2k^2)$$

$$= \sum_{k=1}^{n} 3\cdot 4^k + \sum_{k=1}^{n} 6k^2 = 3\sum_{k=1}^{n} 4^k + 6\sum_{k=1}^{n} k^2$$

$$* \sum_{k=1}^{n} 4^k = \sum_{k=0}^{n-1} 4^{k+1} = \sum_{k=0}^{n-1} 4\cdot 4^k = 4\sum_{k=0}^{n-1} 4^k$$

$$\to 3\cdot 4\sum_{k=0}^{n-1} 4^k + 6\sum_{k=1}^{n} k^2 = \frac{\cancel{12}(1-4^n)}{1-4=-\cancel{3}} + \cancel{6}\,\frac{n(n+1)(2n+1)}{\cancel{6}}$$

$$= -4(1-4^n) + n(n+1)(2n+1)$$

c) $$\sum_{k=1}^{n} b_k - b_{k+2} = \sum_{k=1}^{n} b_k - b_{k+2} + (b_{k+1} - b_{k+1})$$

$$= \sum_{k=1}^{n} b_k - b_{k+1} + b_{k+1} - b_{k+2}$$

telescopicos
$$\to = \underbrace{\sum_{k=1}^{n} b_k - b_{k+1}}_{b_1 - b_{n+1}} + \underbrace{\sum_{k=1}^{n} b_{k+1} - b_{k+2}}_{b_2 - b_{n+2}} = b_1 - b_{n+1} + b_2 - b_{n+2}$$

d) $$\sum_{k=1}^{n} \frac{1}{\sqrt{k}(k+1) + k\sqrt{k+1}} = \sum_{k=1}^{n} \frac{1}{\sqrt{k}(k+1)+k\sqrt{k+1}} \cdot \frac{\sqrt{k}(k+1) - k\sqrt{k+1}}{\sqrt{k}(k+1) - k\sqrt{k+1}}$$

$$= \sum_{k=1}^{n} \frac{\sqrt{k}(k+1) - k\sqrt{k+1}}{k(k+1)^2 - k^2(k+1)} = \sum_{k=1}^{n} \frac{\sqrt{k}(k+1) - k\sqrt{k+1}}{k(k+1)(k+1-k)}$$

$$= \sum_{k=1}^{n} \frac{\sqrt{k}(k+1) - k\sqrt{k+1}}{k(k+1)} = \sum_{k=1}^{n} \frac{\sqrt{k}\,\cancel{(k+1)}}{k\cancel{(k+1)}} - \frac{\cancel{k}\sqrt{k+1}}{\cancel{k}(k+1)}$$

$$* \frac{\sqrt{k}}{k} = \frac{\sqrt{k}}{\sqrt{k^2}} = \sqrt{\frac{k}{k^2}} = \sqrt{\frac{1}{k}} = \frac{1}{\sqrt{k}}$$

$$\sum_{k=1}^{n} \frac{\sqrt{k}}{k} - \frac{\sqrt{k+1}}{k+1} = \sum_{k=1}^{n} \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} = 1 - \frac{1}{n+1}$$

← telescopico

c) $\sum_{k=1}^{n} (k+1)^2 k! = \sum_{k=1}^{n} (k+1)(k+1)k!$ ③

$= \sum_{k=1}^{n} (k+1)(k+1)! = \sum_{k=1}^{n} (k+1+1-1)(k+1)!$

$= \sum_{k=1}^{n} ((k+2)-1)(k+1)! = \sum_{k=1}^{n} (k+2)(k+1)! - 1\cdot(k+1)!$

telescópica $\rightarrow$ $= \sum_{k=1}^{n} (k+2)! - (k+1)! = (n+2)! - 2!$

f) $\sum_{k=0}^{n} 2^k \cdot k$

* usaremos una suma telescópica auxiliar

$\rightarrow \sum_{k=0}^{n} 2^{k+1}(k+1) - 2^k k = 2^{n+1}(n+1) - \cancel{2^0 0} = 2^{n+1}(n+1)$

por otro lado, $\sum_{k=0}^{n} 2^{k+1}(k+1) - 2^k k = \sum_{k=0}^{n} 2^{k+1}k + 2^{k+1} - 2^k k$

$= \sum_{k=0}^{n} 2^k \cdot 2 \cdot k - 2^k k + 2^{k+1}$

$= \sum_{k=0}^{n} 2^k k (\underbrace{2-1}_{1}) + \sum_{k=0}^{n} 2^{k+1}$

$= \sum_{k=0}^{n} 2^k k + \sum_{k=0}^{n} 2 \cdot 2^k = \sum_{k=0}^{n} 2^k k + 2\sum_{k=0}^{n} 2^k$

suma buscada $\rightarrow$

* $\sum_{k=0}^{n} 2^k = \frac{1-2^{n+1}}{1-2}$

$$\Rightarrow \sum_{k=0}^{n} 2^k k + 2\frac{(1-2^{n+1})}{-1} = \sum_{k=0}^{n} 2^k k - 2(1-2^{n+1})$$

Pero, esta suma es igual a $2^{n+1}(n+1)$, entonces

$$\sum_{k=0}^{n} 2^k k - 2(1-2^{n+1}) = 2^{n+1}(n+1)$$

$$\Rightarrow \sum_{k=0}^{n} 2^k \cdot k = 2^{n+1}(n+1) + 2(1-2^{n+1}) = 2^{n+1}\cdot n + 2^{n+1} + 2 - 2\cdot 2^{n+1}$$

$$= 2^{n+1}\cdot n - 2^{n+1} + 2 = 2(2^n \cdot n - 2^n + 1)_{//}$$

g) $$\sum_{k=1}^{n} \ln\left(1+\frac{1}{k}\right) = \sum_{k=1}^{n} \ln\left(\frac{k+1}{k}\right) = \sum_{k=1}^{n} \ln(k+1) - \ln(k)$$

$$= \ln(n+1) - \cancel{\ln(1)}^{0}$$

$$= \underline{\ln(n+1)}$$

h) $$\sum_{k=1}^{n} \frac{1}{k(k+2)}$$

* veamos que pasa con $\frac{1}{k} - \frac{1}{k+2}$

$$= \frac{k+2-k}{k(k+2)} = \frac{2}{k(k+2)}$$, lo que se parece a nuestra sumatoria

$$\Rightarrow \sum_{k=1}^{n} \frac{1}{k(k+2)}\cdot\frac{2}{2} = \sum_{k=1}^{n} \frac{1}{2}\cdot\frac{2}{k(k+2)} = \frac{1}{2}\sum_{k=1}^{n} \frac{2}{k(k+2)}$$

$$= \frac{1}{2}\sum_{k=1}^{n} \frac{1}{k} - \frac{1}{k+2}$$, ahora tenemos algo parecido a una telescopica, y agregaremos $\frac{1}{k+1} - \frac{1}{k+1}$

$$\Rightarrow \frac{1}{2}\sum_{k=1}^{n} \frac{1}{k} - \frac{1}{k+2} + \frac{1}{k+1} - \frac{1}{k+1} = \frac{1}{2}\sum_{k=1}^{n} \frac{1}{k} - \frac{1}{k+1} + \frac{1}{k+1} - \frac{1}{k+2}$$

$$= \frac{1}{2}\Bigg( \underbrace{\sum_{k=1}^{n} \frac{1}{k} - \frac{1}{k+1}}_{1-\frac{1}{n+1}} + \underbrace{\sum_{k=1}^{n} \frac{1}{k+1} - \frac{1}{k+2}}_{\frac{1}{2}-\frac{1}{n+2}} \Bigg)$$ en donde formamos dos telescópicas

$$= \frac{1}{2}\left( 1 - \frac{1}{n+1} + \frac{1}{2} - \frac{1}{n+2} \right) = \frac{1}{2}\left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) = \frac{1}{2}\left(\frac{3}{2} - \frac{2n+3}{(n+1)(n+2)}\right)$$

P3] $\sum_{k=1}^{n} a_k = \frac{1}{3}(n^2+5n)$ averiguar $a_n$.

$$\frac{1}{3}(n^2+5n) = \sum_{k=1}^{n} a_k = \underbrace{a_1 + a_2 + a_3 + \dots + a_{n-2} + a_{n-1}}_{\sum_{k=1}^{n-1} a_k} + a_n$$

$$\frac{1}{3}(n^2+5n) = \sum_{k=1}^{n-1} a_k + a_n$$

* Para resolver $\sum_{k=1}^{n-1} a_k$ podemos usar lo que nos da el enunciado, es decir, $\sum_{k=1}^{n-1} a_k = \frac{1}{3}((n-1)^2 + 5(n-1))$

entonces $\frac{1}{3}(n^2+5n) = \frac{1}{3}((n-1)^2+5(n-1)) + a_n$, despejando $a_n$.

$$a_n = \frac{1}{3}(n^2+5n) - \frac{1}{3}((n-1)^2+5(n-1)) = \frac{1}{3}(n^2+5n-(n-1)^2-5(n-1))$$

$$= \frac{1}{3}(n^2 + \cancel{5n} - (n^2-2n+1) - \cancel{5n} + 5) = \frac{1}{3}(\cancel{n^2} - \cancel{n^2} + 2n - 1 + 5)$$

$$= \frac{1}{3}(2n+4) = \frac{2}{3}(n+2) = a_n$$

P4] Primer término: $\ln(2)$
diferencia $=\ln(4)$ $\Rightarrow$ Progresión $a_k = \ln(2) + k\ln(4)$

demostrar $S_{n-1} = n^2 \ln(2)$

· $S_n = \sum_{k=0}^{n} a + kd = \sum_{k=0}^{n} \ln(2) + \ln(4)k$, pero nos están pidiendo $S_{n-1}$

$$\Rightarrow S_{n-1} = \sum_{k=0}^{n-1} \ln(2) + \ln(4)k = \sum_{k=0}^{n-1} \ln(2) + \sum_{k=0}^{n-1} \ln(4)k$$

$$= \ln(2) \underbrace{\sum_{k=0}^{n-1} 1}_{\substack{n-1-0+1 \\ =n}} + \ln(4) \underbrace{\sum_{k=0}^{n-1} k}_{\frac{(n-1)(n-\cancel{1}+\cancel{1})}{2}}$$

$$= \ln(2)n + \ln(4)\frac{n(n-1)}{2}$$

* Por propiedad del ln se sabe que $\ln(4) = \ln(2^2) = 2\ln(2)$

$$= \ln(2)n + \frac{\cancel{2}\ln(2)n(n-1)}{\cancel{2}} = \ln(2)n + \ln(2)n(n-1)$$

$$= \ln(2)n(\cancel{1} + n - \cancel{1}) = \ln(2)n \cdot n$$

$$\Rightarrow S_{n-1} = \ln(2)n^2$$

P5] $$S = 1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \dots + \frac{1+2+3+\dots+n}{n}$$

$$= \sum_{i=1}^{n} \frac{\sum_{k=1}^{i} k}{i} = \sum_{i=1}^{n} \frac{1}{i} \underbrace{\sum_{k=1}^{i} k}_{\frac{i(i+1)}{2}}$$

$$= \sum_{i=1}^{n} \frac{1}{\cancel{i}} \cdot \frac{\cancel{i}(i+1)}{2} = \sum_{i=1}^{n} \frac{1}{2}(i+1) = \frac{1}{2}\sum_{i=1}^{n} i+1$$

$$= \frac{1}{2}\Big( \underbrace{\sum_{i=1}^{n} i}_{\frac{n(n+1)}{2}} + \underbrace{\sum_{i=1}^{n} 1}_{n-\cancel{1}+\cancel{1}} \Big) = \frac{1}{2}\left(\frac{n(n+1)}{2} + n\right) = S$$