

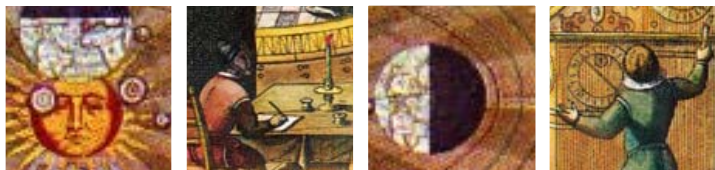
TRIGONOMETRÍA

UNIDADES :

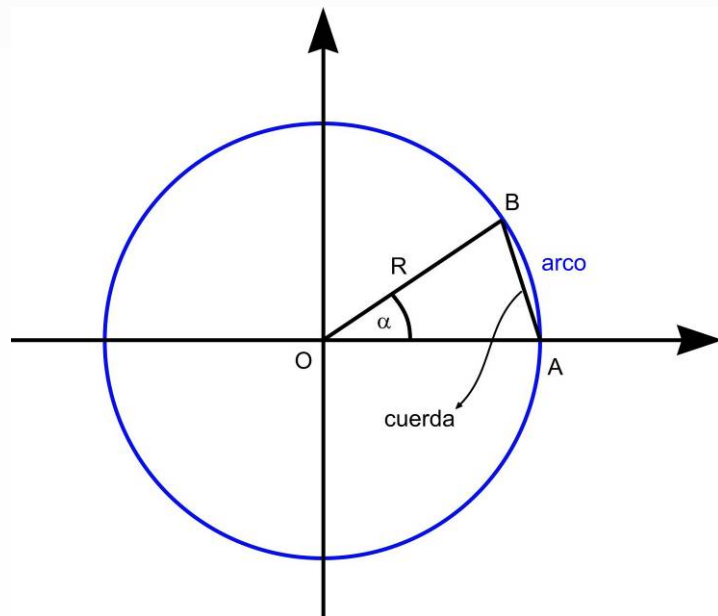
- GRADOS ($^{\circ}$), MINUTOS ($'$)
- RADIANTES

EQUIVALENCIA :

GRADOS	RADIANTES	
360°	2π	GIRO COMPLETO
180°	π	$\frac{1}{2}$ GIRO
90°	$\frac{\pi}{2}$	$\frac{1}{4}$ GIRO
1°	$\frac{\pi}{180}$	$\frac{1}{360}$ GIRO

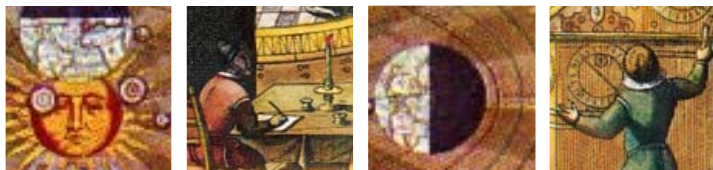


RADIANTES



$$\frac{\text{longitud}}{\text{radio}} = \frac{2\pi R}{R} = 2\pi$$

↑
independiente del
radio de la circunf.



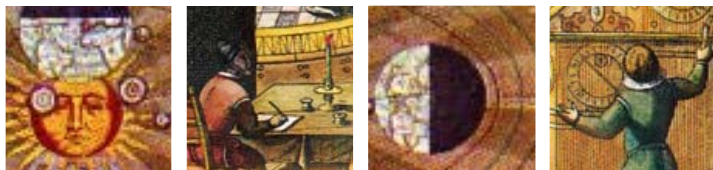
DEFINICIÓN

La magnitud de un ángulo en RADIANTES es igual a la razón entre la longitud del arco de circunferencia que subtiende y el valor del radio de dicha circunferencia

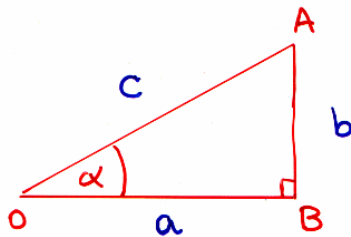
Ej. $\theta = 360^\circ \Rightarrow \theta = \frac{2\pi R}{R} = 2\pi \text{ rad}$

$$1 \text{ rad} = 57,3^\circ$$

$$\boxed{\begin{array}{l} \text{LONGITUD} \\ \text{ARCO} \end{array} = \alpha R}$$



Funciones seno, coseno y tangente



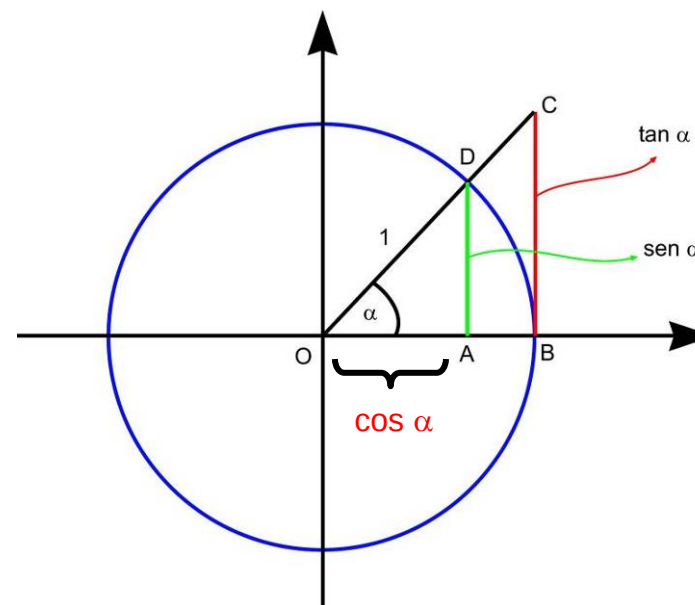
$$\text{sen } \alpha \equiv \frac{|AB|}{|OA|} = \frac{b}{c}$$

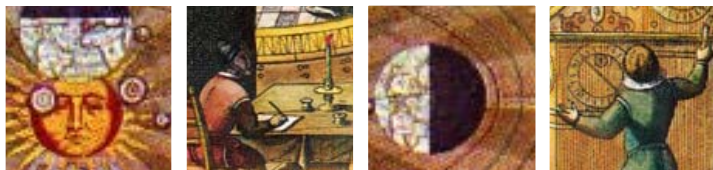
$$\text{cos } \alpha \equiv \frac{|OB|}{|OA|} = \frac{a}{c} \quad \tan \alpha \equiv \frac{\text{sen } \alpha}{\text{cos } \alpha} = \frac{b}{a}$$

PROPIEDADES

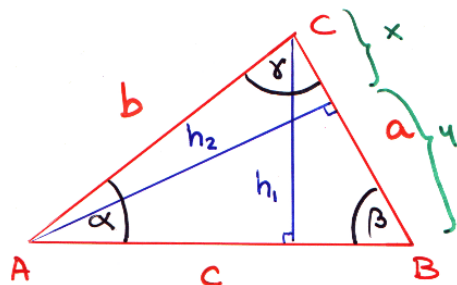
- $\text{sen}^2 \alpha + \text{cos}^2 \alpha = 1$
- $\text{sen}(\frac{\pi}{2} - \alpha) = \text{cos } \alpha$
- $\text{cos}(\frac{\pi}{2} - \alpha) = \text{sen } \alpha$

Otra manera...





Teoremas del seno y del coseno



$$h_1 = b \sin \alpha$$

$$h_1 = a \sin \beta$$

$$\Rightarrow b \sin \alpha = a \sin \beta$$

$$\frac{b}{\sin \beta} = \frac{a}{\sin \alpha}$$

POR OTRO LADO

$$h_2 = c \sin \beta$$

$$h_2 = b \sin \gamma$$

$$\Rightarrow \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$



Teorema del seno

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

TEOREMA DEL COSENO

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

USANDO TEOREMA DE PITÁGORAS

$$h_2^2 + x^2 = b^2 \Rightarrow h_2^2 = b^2 - x^2$$

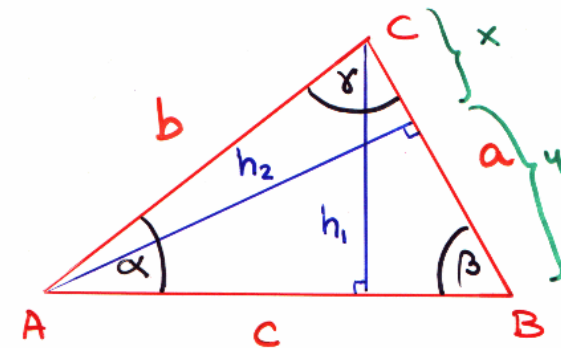
$$h_2^2 + y^2 = c^2 \Rightarrow h_2^2 = c^2 - y^2$$

PERO $x + y = a \Rightarrow y = a - x$

ENTONCES

$$b^2 - x^2 = c^2 - (a - x)^2$$

$$b^2 - \cancel{x^2} = c^2 - a^2 + 2ax - \cancel{x^2}$$





$$b^2 = c^2 - a^2 + 2ax$$

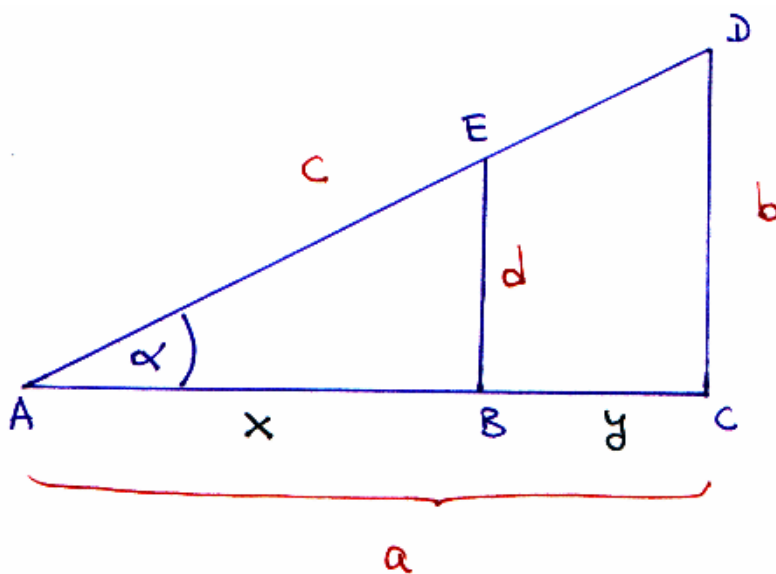
PERO $x = b \cos \gamma$

$$\therefore c^2 = a^2 + b^2 - 2ab \cos \gamma$$

ANALOGAMENTE

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$



Teorema de Tales de Mileto

$$\frac{\overline{BE}}{\overline{CD}} = \frac{d}{b} = \frac{x}{a} = \frac{\overline{AB}}{\overline{AC}}$$

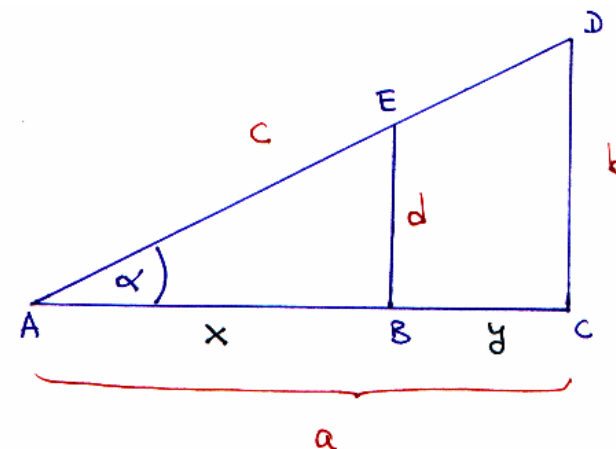


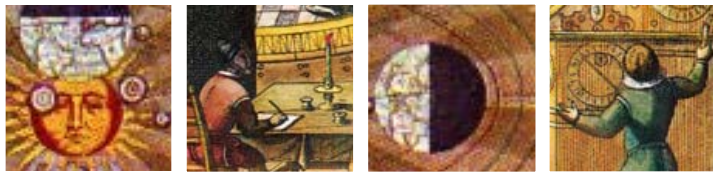
Dem:

$$\tan \alpha \equiv \frac{\overline{BE}}{\overline{AB}} = \frac{d}{x}$$

$$\tan \alpha = \frac{\overline{CD}}{\overline{AC}} = \frac{b}{a}$$

$$\Rightarrow \frac{d}{x} = \frac{b}{a} \Rightarrow \boxed{\frac{d}{b} = \frac{x}{a}}$$





SERIES

$$(1+x)^{\alpha} \equiv 1 + \frac{\alpha}{1!} x + \frac{\alpha(\alpha-1)}{2!} x^2 +$$

$$\frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots$$

Factorial :

$$0! = 1$$

$$1! = 1$$

$$2! = 1 \cdot 2 = 2$$

$$3! = 1 \cdot 2 \cdot 3 = 6$$

$$4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$$

⋮

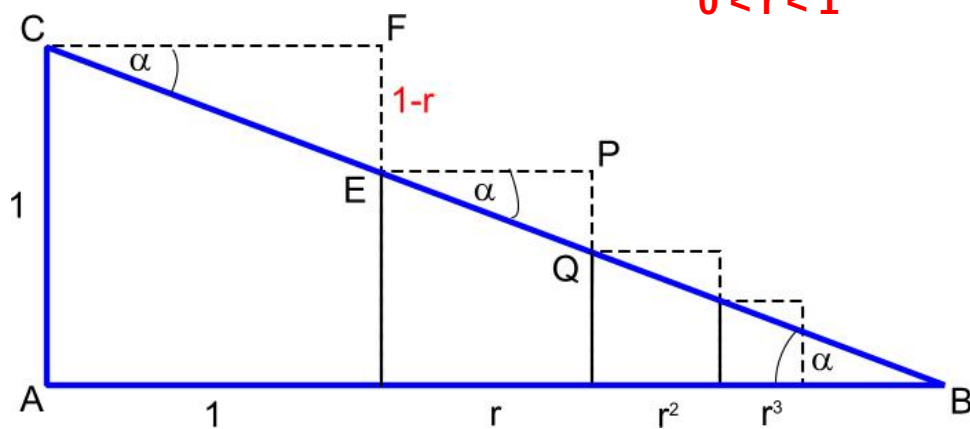


Demuestre que

$$\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \dots = \frac{1}{1-r}$$

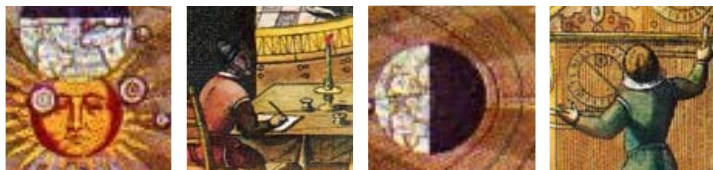
Dem:

$$0 < r < 1$$



$$\overline{EP} = r \quad \overline{CF} = 1 \quad \overline{EF} = 1 - r$$

$$\overline{PQ} = ?$$



En el $\triangle CEF$:

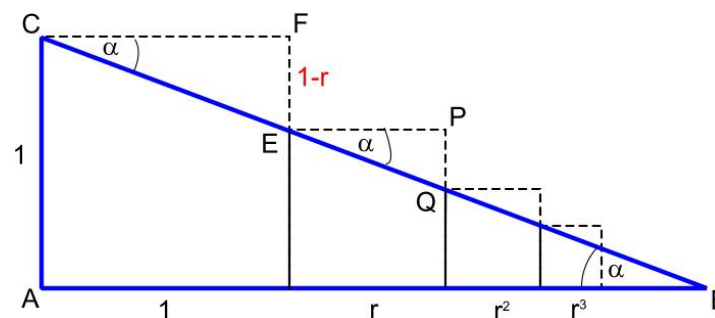
$$\tan \alpha = \frac{\overline{EF}}{\overline{CF}} = \frac{1-r}{1} = 1-r$$

En el $\triangle EPQ$:

$$\tan \alpha = \frac{\overline{PQ}}{\overline{EP}} = \frac{\overline{PQ}}{r}$$

$$\Rightarrow \quad 1-r = \frac{\overline{PQ}}{r}$$

$$\overline{PQ} = r(1-r)$$





En el $\triangle ABC$:

$$\tan \alpha = \frac{\overline{AC}}{\overline{AB}} = \frac{1}{\overline{AB}}$$

pero

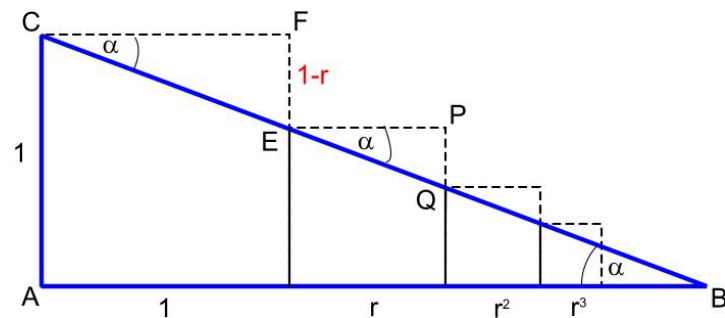
$$\tan \alpha = \frac{\overline{PQ}}{r} = 1 - r$$

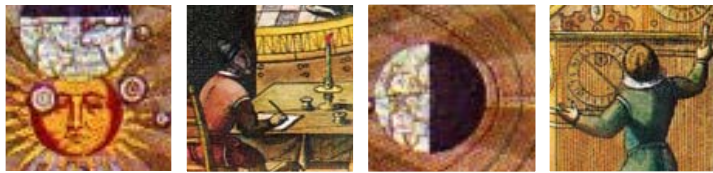
$$\Rightarrow 1 - r = \frac{1}{\overline{AB}}$$

$$\overline{AB} = \frac{1}{1 - r}$$

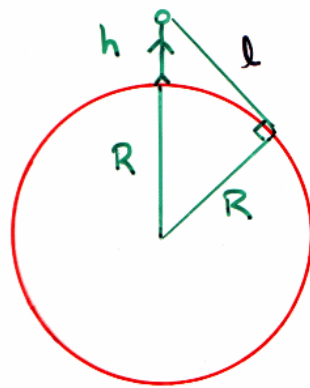
$$1 + r + r^2 + r^3 + \dots = \frac{1}{1 - r}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1 - r}$$





ALCANCE VISUAL SOBRE EL HORIZONTE



$$R = 6400 \text{ km}$$

$$l^2 + R^2 = (R + h)^2$$

$$l^2 + \cancel{R^2} = \cancel{R^2} + 2Rh + h^2$$

$$l = \sqrt{2Rh + h^2}$$

$$l = \sqrt{2Rh} \left(1 + \frac{h}{2R}\right)^{1/2}$$



APROXIMACIÓN

$$\text{Si } x \ll 1 \quad (1+x)^\alpha \approx 1 + \alpha x$$

EN NUESTRO CASO $h \approx 2 \text{ m}$

$$\Rightarrow \frac{h}{2R} = \frac{1}{6.4 \times 10^6} \approx 10^{-7} \ll 1$$

POR LO TANTO

$$l \approx \sqrt{2Rh} \left(1 + \frac{h}{4R} \right)$$

EVALUANDO

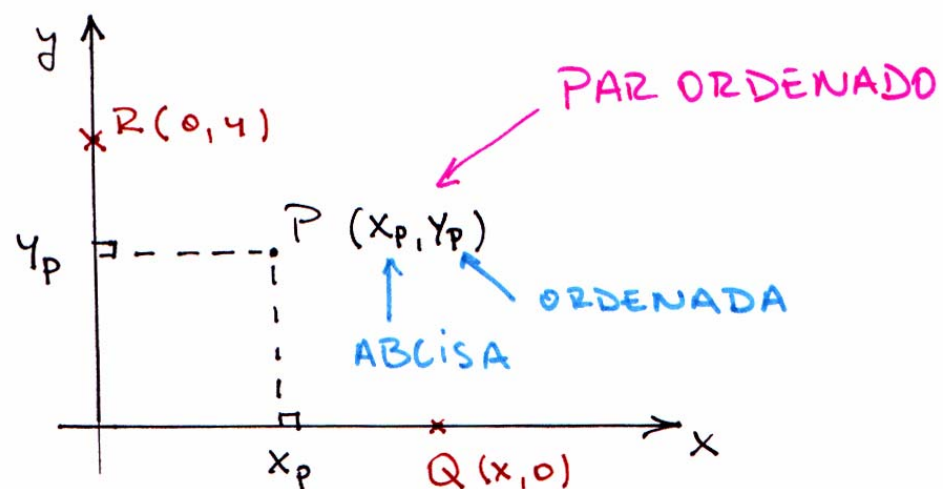
$$l \approx \sqrt{2 \times 6.4 \times 10^6 \times 2} \times 1 \text{ m}$$

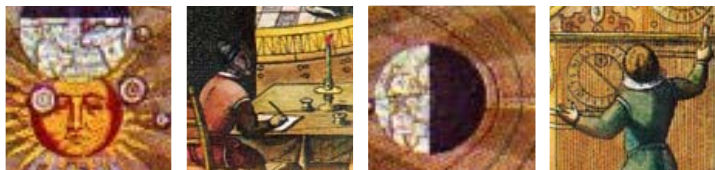
$$l \approx 2 \times 10^3 \sqrt{6.4} \text{ m}$$

$$l \approx 5 \text{ km}$$

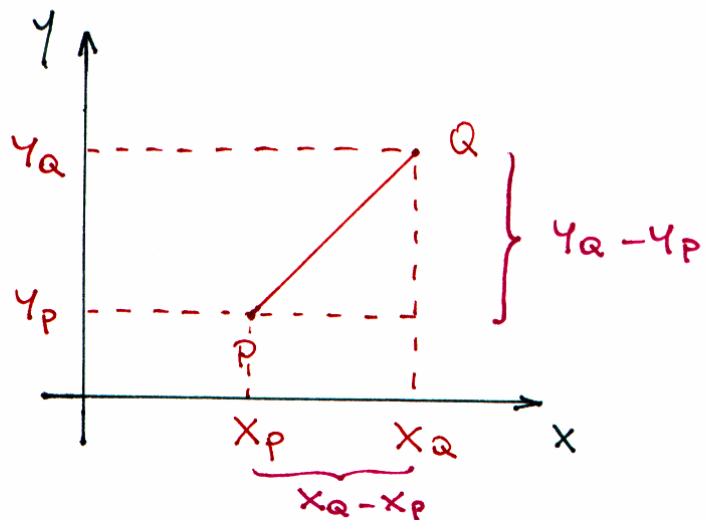


COORDENADAS

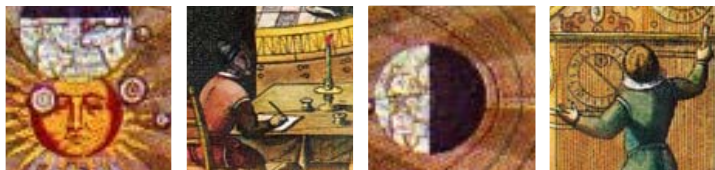




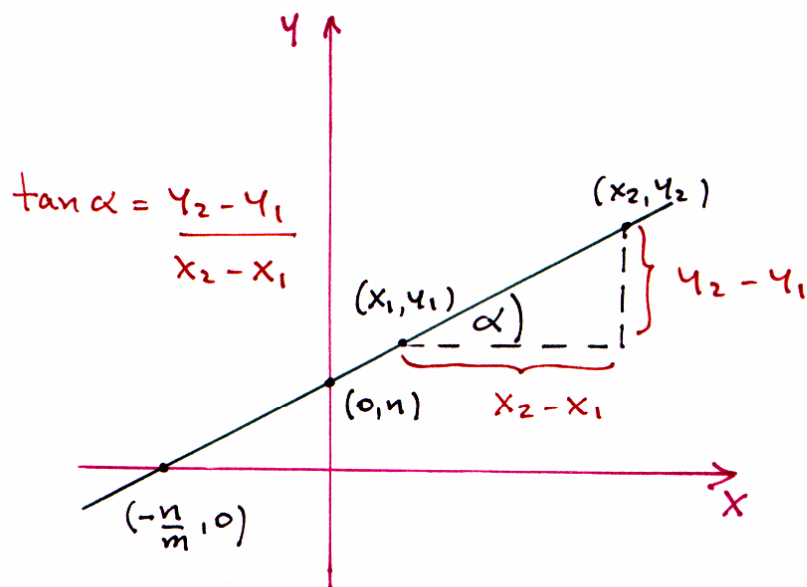
DISTANCIA ENTRE 2 PUNTOS



$$|PQ| = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2}$$



ECUACIÓN DE LA RECTA



$$y = mx + n$$

m : PENDIENTE ($m = \tan \alpha$)

n : VALOR DE LA COORDENADA
DONDE LA RECTA CORTA EL EJE y