$\boxed{P\ 1.6}$

$$\sum_{\nu=0}^{N-1} \cos(\alpha+\nu\beta) = \frac{\sin\frac{N\beta}{2}}{\sin\frac{\beta}{2}} \cos\left(\alpha+\frac{N-1}{2}\beta\right)$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

$$\Rightarrow \boxed{\sin(x+y) - \sin(x-y) = 2\cos x \sin y} \quad (*)$$

con $x = \alpha + \nu\beta \qquad y = \frac{\beta}{2}$

$$\Rightarrow \cos(\alpha+\nu\beta)\sin\frac{\beta}{2} = \frac{1}{2}\left[\sin\left(\alpha+\left(\nu+\tfrac{1}{2}\right)\beta\right) - \sin\left(\alpha+\left(\nu-\tfrac{1}{2}\right)\beta\right)\right]$$

$$\Rightarrow \cos(\alpha+\nu\beta) = \frac{1}{2\sin\frac{\beta}{2}}\left[\sin\left(\alpha+\left(\nu+\tfrac{1}{2}\right)\beta\right) - \sin\left(\alpha+\left(\nu-\tfrac{1}{2}\right)\beta\right)\right]$$

$$\Rightarrow \sum_{\nu=0}^{N-1}\cos(\alpha+\nu\beta) = \frac{1}{2\sin\frac{\beta}{2}}\sum_{\nu=0}^{N-1}\left[\sin\left(\alpha+\left(\nu+\tfrac{1}{2}\right)\beta\right) - \sin\left(\alpha+\left(\nu-\tfrac{1}{2}\right)\beta\right)\right]$$

$$= \frac{1}{2\sin\frac{\beta}{2}}\Big[\cancel{\sin\left(\alpha+\tfrac{\beta}{2}\right)} - \sin\left(\alpha-\tfrac{\beta}{2}\right)$$
$$+\cancel{\sin\left(\alpha+\tfrac{3}{2}\beta\right)} - \cancel{\sin\left(\alpha+\tfrac{\beta}{2}\right)}$$
$$+\dots$$
$$+\sin\left(\alpha+\left(N-\tfrac{1}{2}\right)\beta\right) - \cancel{\sin\left(\alpha+\left(N-\tfrac{3}{2}\right)\beta\right)}\Big]$$

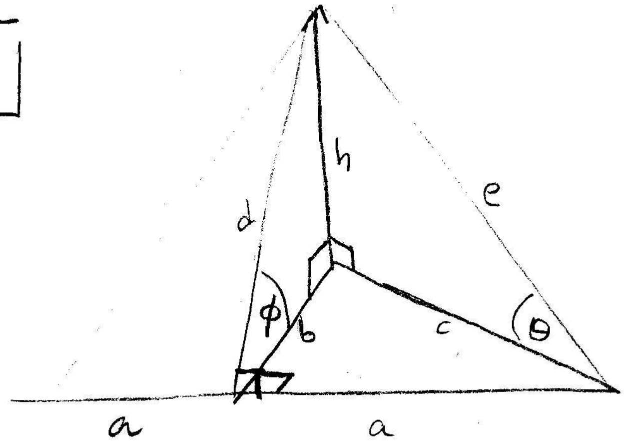
$$= \left[\sin\left(\alpha-\tfrac{\beta}{2}\right) - \sin\left(\alpha+\left(N-\tfrac{1}{2}\right)\beta\right)\right]\cdot\frac{1}{2\sin\beta/2}$$

$$\hat{=} \left(\sin(x+y) - \sin(x-y)\right)$$

$$\Rightarrow \left.\begin{aligned} x+y &= \alpha-\tfrac{\beta}{2} \\ x-y &= \alpha+\left(N-\tfrac{1}{2}\right)\beta \end{aligned}\right\} \Rightarrow \begin{aligned} x &= \alpha + \frac{(N-1)}{2}\beta \\ y &= \frac{N\beta}{2} \end{aligned}$$

$$\Rightarrow \sum_{\nu=0}^{N-1}\cos(\alpha+\nu\beta) = \frac{\sin\left(\frac{N\beta}{2}\right)}{\sin\beta/2}\cdot\cos\left(\alpha+\frac{(N-1)}{2}\beta\right)$$

P1



$$a^2 + b^2 = c^2$$
$$h^2 + c^2 = e^2$$
$$h^2 + b^2 = d^2$$
$$a^2 + d^2 = e^2$$

$$\sin\phi = \frac{h}{d}$$
$$\cos\phi = \frac{b}{d}$$
$$\sin\theta = \frac{h}{e}$$
$$\cos\phi = \frac{c}{e}$$

$$a \sin\theta \sin\phi \cdot \sqrt{\csc(\theta+\phi)\csc(\phi-\theta)}$$

$$= \frac{a\sin\theta\sin\phi}{\sqrt{\sin(\phi+\theta)\sin(\phi-\theta)}}$$

$$= \frac{a\sin\theta\sin\phi}{\sqrt{(\sin\phi\cos\theta + \cos\phi\sin\theta)(\sin\phi\cos\theta - \cos\phi\sin\theta)}}$$

$$= \frac{a\sin\theta\sin\phi}{\sqrt{(\sin^2\phi\cos^2\theta - \cos^2\phi\sin^2\theta)}}$$

$$= \frac{a\cdot(h/e)\cdot(h/d)}{\sqrt{\frac{h^2}{d^2}\cdot\frac{c^2}{e^2} - \frac{b^2}{d^2}\cdot\frac{h^2}{e^2}}}$$

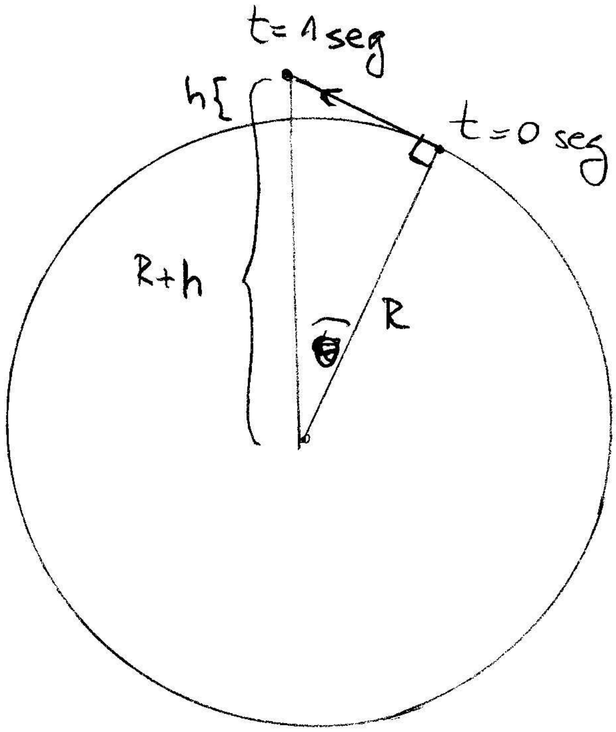
$$= \frac{ah^2/ed}{\sqrt{\frac{h^2}{e^2d^2}(c^2-b^2)}}$$

$$= \frac{ah^2/e\cdot d}{\frac{h}{e\cdot d}\sqrt{c^2-b^2}}$$

$$= \frac{ah}{\sqrt{c^2-b^2}} = \frac{ah}{\sqrt{a^2}} = h$$

$$\boxed{\therefore\ h = a\sin\theta\sin\phi\sqrt{\csc(\phi+\theta)\csc(\phi-\theta)}}$$

$\boxed{4.2}$ Primero hay que calcular el ángulo que avanza la Luna en 1 seg:



trayectoria de la Luna si siguiera una linea recta.

En 1 mes da 1 vuelta (2π) $\Rightarrow$

$$\frac{2\pi}{1\,\text{mes}} = \frac{\theta}{1\,\text{seg}}$$

$$\Rightarrow \theta = \frac{2\pi\ \text{seg.}}{30\times 24\times 60\times 60\ \text{seg}}$$

$$\Rightarrow \theta \approx 2{,}5\times 10^{-6}$$

Ahora para calcular la dist. h que cae, usando trigonometria

$$\cos\theta = \frac{R}{R+h} = \frac{R+h-h}{R+h} = 1 - \frac{h}{R+h}$$

Considerando ahora que h es pequeño $\Rightarrow$ θ es pequeño $\Rightarrow$

$$\left.\begin{array}{l} \cos\theta \approx 1 - \dfrac{\theta^2}{2} \\ 1 - \dfrac{h}{R+h} \approx 1 - \dfrac{h}{R} \end{array}\right\} \Rightarrow 1 - \frac{h}{R} = 1 - \frac{\theta^2}{2} \Rightarrow h \approx \frac{R}{2}\cdot\theta^2$$

$$\Rightarrow h \approx \frac{384\times 10^3\ \text{km}}{2}\cdot(2{,}5\times 10^{-6})^2 \approx \boxed{1{,}2\times 10^{-6}\ \text{km} = h}$$

En la sup. de la tierra un objeto cae

$x(t) = -g\frac{t^2}{2}$, en 1 seg $\Delta x = \frac{9{,}8}{2} \times 10^{-3}$ km/s $\sim 5 \times 10^{-3}$ km/s

luego $\frac{h}{\Delta x} \approx \frac{1 \times 10^{-6}}{5 \times 10^{-3}} \approx 2{,}4 \times 10^{-4}$

Ahora veamos que Radio Terrestre ≈ 6400 km

Radio orbita L. ≈ 384.000 km

$$\frac{(1/384.000)^2}{(1/6400)^2} \approx 2{,}7 \times 10^{-4} \approx \boxed{\frac{(1/R_L)^2}{(1/R_T)^2} \approx \frac{h}{\Delta x}}$$

lo que es logico pensando que la accel de gravedad depende como $\frac{1}{R^2}$ $\left(a_g = -\frac{GM}{R^2} \right)$

$\boxed{\text{Ej: } 5}$

dem. $\cos\theta < \frac{\theta^2}{2}$,

$$\cos\theta = \cos\left(\frac{\theta}{2}+\frac{\theta}{2}\right)$$
$$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$
$$= 1 - 2\sin^2\frac{\theta}{2}$$

consideremos ahora $\sin x < x$ (para $x > 0$)

$$\Rightarrow \sin^2 x < x^2$$
$$\Rightarrow -2\sin^2 x > -2x^2$$

$$\Rightarrow \cos\theta = 1 - 2\sin^2\frac{\theta}{2}$$
$$> 1 - 2\left(\frac{\theta}{2}\right)^2 = 1 - \frac{\theta^2}{2} \quad \Rightarrow \boxed{\cos\theta > 1 - \frac{\theta^2}{2}}$$

---

dem. $\sin\theta > \theta - \frac{\theta^3}{4}$

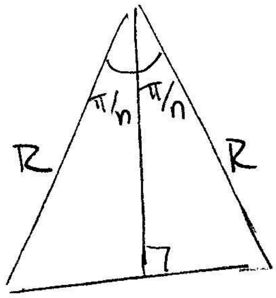
$$\sin\theta = \sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right) = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$
$$= 2\tan\frac{\theta}{2}\cdot\cos^2\frac{\theta}{2}$$

Usando $\tan x > x$ $\Rightarrow$

$$\sin\theta = 2\tan\frac{\theta}{2}\cos^2\frac{\theta}{2}$$
$$> 2\cdot\frac{\theta}{2}\cos^2\frac{\theta}{2}$$
$$= \theta\cdot\left(1 - \sin^2\frac{\theta}{2}\right)$$
$$> \theta\left(1 - \frac{\theta^2}{4}\right) \quad \Rightarrow \sin\theta > \theta - \frac{\theta^3}{4}$$

Ej: 6.2

Cuando hay n triangulos, primero debo calcular el area de cada triangulo,



cortando el triangulo por la mitad, formamos 2 triangulos rectangulos

$$\text{Area} = 2 \times \frac{R\sin(\pi/n) \cdot R\cos(\pi/n)}{2}$$

Entonces si hay n triangulos, el area total es:

$$A_n = n \cdot R^2 \sin(\pi/n)\cos(\pi/n)$$

$$= nR^2 \cdot \frac{1}{2}\sin\left(\frac{\pi}{n} + \frac{\pi}{n}\right)$$

$$A_n = \frac{nR^2}{2}\sin\left(\frac{2\pi}{n}\right)$$

Ahora $\lim\limits_{n\to\infty} A_n = \lim\limits_{n\to\infty} \frac{R^2}{2} n \sin\left(\frac{2\pi}{n}\right)$

Aca $\frac{2\pi}{n}$ es muy pequeño, entonces se ocupa la aprox $\sin\theta \sim \theta$

$$\Rightarrow \lim_{n\to\infty} A_n = \lim_{n\to\infty} \frac{R^2}{2} n \cdot \frac{2\pi}{n} = \pi R^2 = \text{Area del circulo}$$