

# Population and the Environment: A Parable of Firewood and Other Tales

Marc Nerlove

Few issues, it is safe to say, excite as much interest or concern in contemporary economics as those related to environment and to unpriced resources of all kinds. At bottom, many long-term environmental problems, whether they derive proximately from use of modern agricultural technology to augment food production or too rapid exploitation of exhaustible energy and other natural resources, stem ultimately from the pressure of human population and human desires for subsistence, if not greater, levels of creature comforts. Economists and others are acutely aware of the role which population pressure plays in causing environmental degradation in developing countries and, since Hardin's justly famous "The Tragedy of the Commons," of how the unpriced or underpriced character of environmental and other natural resources leads to overexploitation and ultimate degradation. The other side of this intellectual coin is the role which environmental degradation and natural resources depletion may play in producing the very same population pressure which lies behind such degradation and depletion especially in developing countries. This role is less widely appreciated and understood.

In this essay I shall focus on this latter aspect of the relation between population and environment. I shall do so in the context of a rather abstract dynamic planar system of two nonlinear difference equations, one of which reflects the way in which population pressure affects the state of the environment of future generations and its evolution over time and the other of which characterizes human fertility behavior in terms of optimization subject to environmental con-

straints.<sup>1</sup> While there is much to say about how population pressure may affect environment in different environmental and institutional contexts, I shall adopt the not entirely uncontroversial position that it does so adversely, in order to emphasize the way in which human fertility and population growth may react to environmental degradation.

The principal conclusion of this analysis is that the possibilities for a stable equilibrium between human population and its environment are quite limited. Even given a relatively favorable relationship between population pressure and the evolution of environmental degradation over time, a stable equilibrium can be achieved only if fertility responds negatively to environmental degradation and then only if the response is sufficiently large in absolute magnitude in relation to the dynamic response of environment to population pressure. Under exceptionally adverse environmental circumstances, rising death rates can ultimately bring a halt to further environmental deterioration and/or lead to human extinction.

Unfortunately, there are ample reasons to suppose that in much of the Third World fertility is likely to react positively to increasing environmental degradation because parents perceive the benefits of having more children to be higher under environmentally more adverse cir-

<sup>1</sup> Analysis of the dynamics of planar systems is discussed in an appendix to this lecture available from the author on request. Many important economic problems can be cast in the form of a planar system, the mathematics of which is now becoming known among mathematical economists (Grandmont). Basic references are Guckenheimer and Holmes and Looss and Joseph. A more complete exposition of the dynamics of planar systems with applications to population, environmental, and natural resource economics is currently in preparation.

One important characteristic of problems in the economics of natural resources, shared by the analysis of the present lecture, is that the economic decision making is one sided: exploitation of resources, population growth, or similar variables are the result of human behavioral processes, whereas the regeneration or degradation of the resource is subject to natural law. Clearly, it is possible to imagine situations in which this need not be the case, for example, if investment can be made to ameliorate the adverse effects on environment caused by population growth.

Frederick V. Waugh Memorial Lecture

Marc Nerlove is a professor in the Department of Economics, University of Pennsylvania.

This paper is based on earlier work joint with Anke Meyer and with the able research assistance of Viktória Dalkó. It has been supported by the International Food Policy Research Institute, Washington, D.C.

The author is also indebted to Christain Gourieroux, Heinz Koenig, and Efraim Sadka for helpful discussions in connection with a number of previous drafts.

cumstances than under more favorable ones. For example, if the probability of a child surviving to adulthood and thus to care for its parents in their old age is lower in poorer quality environments, parents may be induced to bear greater numbers, which, until infant and child mortality levels rise sufficiently to offset the greater number of births, will generally result in higher, not lower, rates of population growth under environmentally adverse circumstances. Along similar lines, it may be argued that if children participate actively in agricultural production in the Third World, their comparative advantage is enhanced relative to adults in poor environments because animal husbandry supplants crop production, where children arguably have less advantage in the heavy work of tilling and harvesting than they have in herding livestock.

This theme is developed at some length in a series of metaphors based on a two-period overlapping generations (OLG) model, in which parents value children solely for the contribution they may make to their own selfishly perceived welfare. Notwithstanding, it is possible to argue that, as environment deteriorates, the relationship between the state of environmental degradation and the rate of population growth must eventually turn from positive to negative; that is, further deterioration in the state of the environment will be accompanied by a lesser rate of growth of population, thus opening the way to a stable, albeit possibly unpleasant, equilibrium between population and environment.

While one may question the desirability of such an equilibrium from some grand and humane ethical view, altruism or love is perhaps a luxury in which only the relatively well-off can afford to indulge. In any case, I do not believe in the efficacy of addressing such issues from any point of view other than that of the present generation. The reason we care about environmental deterioration as it affects future generations is precisely because we care about the welfare of our own children and possibly those of others. Without introducing altruism or love in some manner, we cannot hope to make precise the welfare losses which result from excessive population growth leading to environmental deterioration. And, in a two-period OLG model without altruism, there is no scope for a discussion of welfare losses to the present generation resulting from the unpriced character of environmental and natural resources because those members of the present generation are no longer around to experience the effects of en-

vironmental deterioration and do not care about their children's welfare.<sup>2</sup>

Next, in the context of a metaphor in which the perceived benefits of having children relate to their role in agricultural production, which I call "a parable of firewood," I show that parental altruism toward their children can only make matters worse, if socially unchecked, by leading to an increase of the birthrate in every environmental state in comparison with that which would occur in its absence.<sup>3</sup> In other words, love is not enough to lead to stability and may even result in a worse outcome for members of the current present generation over time than in its absence. Nonetheless, love in the sense of parental concern for the future welfare of their offspring, opens the door to social intervention. This is because environmental quality being an unpriced resource does not enter into individual parents' calculation of the costs and benefits of having additional children.

The failure of *laissez-faire* to attain a Pareto-optimal situation for members of the present generation represents an externality, the usual theoretical remedy for which is a system of Pigouvian taxes and subsidies which force parents to internalize the true social costs and benefits of having additional children. In other words, the unpriced nature of environmental resources drives a wedge between the marginal social cost and the marginal private cost of a child for the parents which a per capita tax on children accompanied by a lump sum payment to parents of the social total of the proceeds might correct.

The next step in the analysis is to see how a tax on children repaid lump sum to parents will change the equilibrium birthrate for a given level

<sup>2</sup> There is also no scope for bequests in the form of physical capital which, in this context, could take the form of investments in "clean-up" facilities or other projects designed to improve future environmental quality. It should also be noted that the metaphors explored here assume no adverse environmental effects of population pressure on the present generation, i.e., their environment is what it is and they can do nothing to affect it.

<sup>3</sup> This is in part a consequence of the neglect of possibilities for investment in the human capital of children to improve their future welfare, or in bequests in the form of physical capital. If such possibilities were introduced along with parental altruism, improvements in the rates of return associated with bequests would act to cause parents to substitute away from child numbers towards child "quality." In an early paper (Nerlove) I offered this as one explanation of the so-called "demographic transition," since, arguably, falling infant, child, and adult mortality enhances the rate of return to investment in human capital (because such capital dies with its owner). Increasing bequests in the form of human capital act, à la Becker-Lewis, to augment the "price" of additional child numbers leading parents, *ceteris paribus*, to substitute "quality" for "quantity."

of environmental quality with or without altruism. In general, it is possible to design such a tax/subsidy scheme to achieve any desired birth rate including one which might stabilize the equilibrium between population and the environment. This leads naturally to the question of what a Pareto-optimal birthrate from the standpoint of the present generation might be. Unfortunately, determination of such a rate depends crucially on parents' beliefs about the future states of the environment as well as the degree to which they weight their children's welfare in their private calculation of costs and benefits. Except in a state of stationary equilibrium, it is unreasonable to suppose that such beliefs reflect reality with any degree of accuracy. This observation, then, leads naturally to a welfare comparison between two or more steady states occurring at different levels of environmental degradation. It is easy to argue that each member of the present generation will be better off at an equilibrium with better environmental quality than at one of lesser quality. What is not so clear is that members of the present generation would be better off at such an equilibrium than in a nonequilibrium and possibly unstable situation. On this pessimistic note I close.<sup>4</sup>

The plan of the remainder of the paper is as follows: In the second section the two-equation OLG model of population and the environment is established and the questions of the existence and local stability of steady-state solutions are addressed, with some remarks on the global evolution of the system. Mathematical details are relegated to an appendix on the dynamics of planar systems available from the author on request. In the third section, the focus shifts to the relation between environment and population when fertility is endogenously determined to maximize parents' utility from surviving children. I argue that an absolute limit to the number of children a woman can have in her lifetime will ultimately reduce the rate of growth of population if survival probabilities decline sufficiently, even if the number of births continues to rise. The manner in which children generate utility for their parents is taken up in the fourth section, in which a similar conclusion follows even when survival probability is assumed to be unaffected by environmental quality. Taxes and

subsidies which can achieve and maintain a given birthrate then are introduced. Parental altruism is introduced and shown to lead to a higher birthrate than would occur in its absence for any given state of environmental quality. Although altruism generally leads to a failure to achieve a Pareto optimum from the standpoint of the present generation, it is difficult to specify such an optimum. The paper concludes with some remarks about the possibility of investment in human and physical capital to improve the welfare of future generations and ameliorate the effects of population pressure on environment.

### The Dynamics of Population and Environment When Fertility Is Endogenous

Consider a two-period overlapping-generations (OLG) model in which people live as children in the first period. At the end of the first period, they reproduce in order to maximize their utility of life in the next period as adults, a utility which may or may not reflect concern for the future welfare of their offspring. Their decisions are assumed to reflect their perception of the state of the environment. To avoid the complications of sexual reproduction and marriage, reproduction is assumed to occur by parthenogenesis.

Let  $Z_t$  represent the state of the environmental degradation at time  $t$ . Thus, the larger  $Z_t$ , the lower the level of environmental quality. Let  $N_t$  be the number of children alive at the end of period  $t$  who instantly become parents at that moment. The current state of the environment is assumed to depend only on its state in the previous period, as that interacts with that part of the population who are then children:

$$(2.1) \quad Z_t = g(Z_{t-1}, N_{t-1}),$$

where  $\partial g / \partial Z_{t-1} > 0$  and  $\partial g / \partial N_{t-1} > 0$ .<sup>5</sup> The assumption that  $\partial g / \partial N > 0$  is not uncontroversial. It has been suggested (McNicholl; Jodha;

<sup>5</sup> In an earlier formulation (Nerlove and Meyer 1990), we assumed more specifically that  $g$  was of such a form that

$$(2.1^*) \quad Z_t / Z_{t-1} = g^*(N_{t-1}).$$

Taking logs, it is easy to see that this implies

$$\xi_Z = \frac{\partial g}{\partial Z_{t-1}} \frac{Z_{t-1}}{Z_t} = 1.$$

Our more specialized assumption was motivated largely by a desire to facilitate a graphical determination of global trajectories. Unfortunately,  $\xi_Z = 1$  virtually guarantees local instability of any stationary point.

<sup>4</sup> Less pessimistic conclusions would likely result from the introduction of the possibilities of investment in human and physical capital together with parental altruism (see the previous footnote). Models with such possibilities will be explored in subsequent research.

Simon, pp. 82–107) that in certain types of environments, given the proper institutional and social organization, population growth can affect environmental quality favorably. Moreover, such a favorable relationship may be characteristic at low levels of population and relatively high levels of environmental quality, but not when population pressure is great. This is not, however, the stuff of current concerns nor the focus here. A stationary environmental state, if one exists for some level of population size  $N$ , is characterized by  $Z_t = Z_{t-1} = \bar{Z}$  such that

$$(2.2) \quad \bar{Z} = g(\bar{Z}, N).$$

At such a point, if one exists, the elasticities are defined as

$$(2.3) \quad \xi_Z = \frac{\partial g}{\partial Z} \frac{\bar{Z}}{\bar{Z}} \quad \text{and} \quad \xi_N = \frac{\partial g}{\partial N} \frac{\bar{N}}{\bar{N}}.$$

In the following sections of this paper, the focus is on how family decisions with respect to births, or endogenous fertility, as shaped by environmental circumstances or constraints, relate to the rate of growth of population,  $N_t/N_{t-1}$ . It is natural, therefore, to treat this matter on a per family basis so that individual decisions are translated into social outcomes proportionately to the numbers of decision makers. At this point, let me write generally

$$(2.4) \quad N_t/N_{t-1} = h(Z_{t-1}),$$

where  $h'$  is to be determined.<sup>6</sup> A stationary population, if one exists for a level of environmental quality  $\bar{Z}$ , is characterized by  $N_t = N_{t-1} = \bar{N}$  such that

$$(2.5) \quad 1 = h(\bar{Z}).$$

Because  $h$  may not be monotonic or there may exist no value of  $Z$  for which  $h(Z) = 1$ , equation (2.5) may have multiple solutions, one solution, or no solutions at all. I argue below that plausible models of family decision making and/or the effects of rising death rates with increasing environmental deterioration make it likely that if, for relatively good environmental quality,  $h' > 0$ , eventually as environment deteriorates  $h' < 0$ . Thus, if  $h'$  changes sign just once, there are either two solutions to (2.5) or none. If a

solution exists the elasticities are defined as

$$(2.6) \quad \eta_Z = h' \bar{Z} / \bar{N} \quad \text{and} \quad \eta_N = h \bar{N} / \bar{N} = 1;$$

that is, because of the form of the relation between the current level of population, its past value, and environmental quality,  $\eta_N$  is always 1 at a stationary population level. This fact plays a crucial role in further analysis.

In order for a stationary solution, in which both population and environmental quality are unchanging, to exist, points  $(\bar{Z}, \bar{N})$  must exist for which (2.2) and (2.5) simultaneously hold:

$$(2.7) \quad \bar{Z} = g(\bar{Z}, \bar{N}) \quad \text{and} \quad 1 = h(\bar{Z}).$$

Clearly, even if solutions to (2.5) exist, none may be characterized by  $\bar{N}$  such that (2.2) holds. The opposite is not true, however; any solution to (2.2) must be characterized by unchanging  $N$ , except possibly when the function  $g$  is pathological. The appendix shows how to find stationary points graphically, if they exist, and how to determine points along the trajectory beginning from any other point. Here, the only comment is that if  $\xi_Z = 1$ , a property which will guarantee local instability of any stationary points which may exist, then it is easy to impose plausible conditions on  $g$  which guarantee a simultaneous solution  $(\bar{Z}, \bar{N})$  to (2.7).

Equations (2.1) and (2.4) constitute a system of two nonlinear difference equations which determine the trajectories of the two variables  $Z_t$  and  $N_t$  starting from some initial value  $(Z_0, N_0)$  and subject to some boundary conditions, a so-called planar system. Systems of nonlinear difference equations may be analyzed qualitatively in the vicinity of a stationary point  $(\bar{Z}, \bar{N})$  by standard methods. See, for example, looss and Joseph (chap. 4, pp. 42–58). Analysis of global behavior is considerably more difficult and can be exceedingly complex (Guckenheimer and Holmes). The appendix contains a complete analysis of the local dynamics of planar systems with application to the system (2.1) and (2.4) and some remarks on global properties, including the diagrammatic analysis referred to above. The discussion of global dynamics also serves to illustrate how the existence of stationary points may be determined.

The general analysis of local stability is supplemented by an analysis incorporating the special assumptions concerning the functions  $g$  and  $h$  in (2.1) and (2.4). These are that at a stationary point  $(\bar{Z}, \bar{N})$ , if one exists,

$$\begin{aligned} \xi_Z &> 0 \\ \xi_N &> 0 \end{aligned}$$

<sup>6</sup> In Nerlove and Meyer (1991), we made family decisions at the end of period  $t - 1$  depend on environmental quality in period  $t$ :

$$(2.4^*) \quad \frac{N_t}{N_{t-1}} = h^*(Z_t).$$

This only complicates the algebra without adding anything essential to the analysis.



$\eta_Z$  may be either positive or negative  
 $\eta_N = 1$ .

The analysis shows that, if  $\xi_Z \geq 1$ , no stationary point can ever be locally stable. Consequently, in studying the possibility of a locally stable solution, we can rule this case out and assume  $\xi_Z < 1$ . The analysis then yields the following conclusions:

**PROPOSITION 1.** *If, at a stationary point  $S = (\bar{Z}, \bar{N})$ ,  $\eta_Z > 0$ , i.e., the rate of change of population responds positively to environmental deterioration, the point  $S$  is unstable. It is an unstable saddle when*

$$\xi_N \eta_Z < 2(1 + \xi_Z)$$

*and an unstable source when*

$$\xi_N \eta_Z > 2(1 + \xi_Z).$$

**Discussion.** Because we have already restricted consideration to the range  $0 < \xi_Z < 1$ , the bound on  $\xi_N \eta_Z$  is a number between 2 and 4. Thus, which type of instability we encounter when  $\xi_N > 0$  and  $\eta_Z > 0$  depends on how responsive environmental quality is to population pressure. If it is relatively unresponsive, the rate of growth of population can be more responsive to environmental deterioration and still yield a locally unstable saddle rather than an unstable source.

**PROPOSITION 2.** *If, at a stationary point  $S$ ,  $\eta_Z < 0$ , it is possible that  $S$  is stable.*

**Discussion.** I will argue in the next section that if  $\eta_Z > 0$  at low levels of  $Z$ , that is relatively good environments,  $\eta_Z$  is likely to fall and eventually become negative, thus opening up the possibility that there exists another stationary point with a larger population and a poorer environment which is a stable equilibrium of the system. The argument is based on the fact that there is some fixed upper bound to the number of children a woman can have. In the following section, a similar argument is given based on the presumption that environmental quality becomes more like an ordinary factor of production, in the sense that deterioration reduces rather than enhances the perceived benefits of having additional children, when environmental quality becomes sufficiently bad.

**PROPOSITION 3.** *Even when  $\eta_Z < 0$  at a stationary point  $S$ , if*

$$\xi_Z - \xi_N \eta_Z > 1$$

*or, put another way, since  $0 < \xi_Z < 1$  and  $\xi_N > 0$ ,  $\eta_Z < 0$ , when*

$$\xi_N(-\eta_Z) > 1 - \xi_Z > 0,$$

*$S$  is an unstable spiral. Otherwise  $S$  is a stable spiral node or a stable simple node depending how small  $\xi_N(-\eta_Z)$  is in relation to  $\xi_Z$ .*

**Discussion.** Because  $\xi_Z$  is bounded between 0 and 1, so is  $1 - \xi_Z$ . Thus, the possibility that  $S$  is stable in either way is reduced as  $\xi_Z$  approaches 1. Even under the most favorable of circumstances, stability can be achieved only when

$$\xi_N(-\eta_Z) < 1.$$

Thus if  $\xi_N$  is small,  $\eta_Z$  can be larger in absolute value and stability may be attained. If  $\xi_N$  is large, however, then the rate of population growth must be rather insensitive to environmental deterioration. The balance is delicate: For any fixed  $\xi_Z$  between zero and one, either too large  $\xi_N$  or  $(-\eta_Z)$  produce instability. In the last section of this paper I show how social intervention in the form of a head tax on children and a lump-sum subsidy to parents can be used to force the system to equilibrium by altering the function  $h$ , and thus  $\eta_Z$ , and to maintain it there. But, such intervention may not represent a gain in the welfare of parents even when they are altruistic with respect to their children.

## Endogenous Fertility 1: Ensuring Survivors for Old Age Security

In this section, a relation is derived between the rate of growth of population and environmental quality based on a model in which parents determine their fertility in order to maximize the expectation of their utility which is a concave function of the number of surviving children. The optimal number of births can be shown, under rather general circumstances, to be negatively related to the probability of survival which, in turn, is assumed to be related to environmental quality, through the effects of environmental deterioration, pollution, and overcrowding on infant and child mortality. The introduction of *ex ante* costs of births which are positively correlated with the rate of infant and child mortality as well as with the number of actual births may reduce, or even reverse, the sign of the relation between environmental deterioration and the endogenously determined birthrate. But, in any case, the rate of growth of population de-

depends not merely on the optimal birthrate for a particular survival rate which, in turn, depends on the level of environmental quality. It also depends on the elasticity of the birthrate with respect to survival probability. While generally negative, this elasticity must approach zero as environment deteriorates because of an absolute limitation to the number of births a woman can have. This means that if  $\eta_Z$  of the preceding section begins positive, it must eventually become negative. Adult death rates (where death occurs prior to or limits reproduction), which rise as environment deteriorates, have a similar effect.

The assumption of a two-period OLG model with parthenogenesis made in the previous section is maintained. In the first period of life, children support their parents, in the second, each child becomes herself a parent and lives off the fruit of her children's labors. The analysis is an extension of the so-called "old-age security hypothesis" (Neher, T. W. Schultz, Willis). Nerlove, Razin, and Sadka (1987a, b) show that parental altruism may alter the usual conclusion about the way in which means other than children for transferring resources from youth to old age may affect fertility.

It has been argued that lower birthrates are empirically associated with lower infant and child mortality rates, which in turn are associated with generally falling death rates (Freedman; Preston 1976, 1978; T. P. Schultz). Such a relationship serves as a basis for many theories of the so-called demographic transition. Nonetheless, demonstration of the relationship from a simple economic model of fertility choice has proved elusive. Recent theoretical work of Sah, and to a lesser extent the pioneering numerical work of Wolpin, have provided a significant breakthrough in the development of a satisfactory analytical characterization of the observed empirical regularity in terms of the structure of the parent's utility function and the existence of *ex ante* costs. For purposes here, it is assumed that the utility-maximizing parent determines the optimal number of births she has as a function of the probability a birth survives and the *ex ante* marginal costs of births, which may or may not be correlated negatively with child survival probabilities.<sup>7</sup>

I will first assume that *ex ante* costs of births are absent and assume that, *ex post*, the parent's utility is a function of the number of surviving children,  $n_t$ , which if we assume that children who survive birth also survive to reproduce, is the rate of growth of the number of parents:

$$(3.1) \quad n_{t-1} = N_t/N_{t-1}.$$

After the analysis of this case, I will examine what difference adult deaths prior to reproduction may make and the effects of *ex ante* costs of births.

In this model, however, all births do not survive. If the proportion of survivors is  $s$ , then, on average,

$$(3.2) \quad n_t = sb_t.$$

Sah shows that, when utility is a concave function of surviving births and when there are no *ex ante* costs of births or absolute limitations to the number of births a woman can have, the optimal utility-maximizing value of  $b_t$  as a function of  $s$  is nonincreasing. Sah's proof makes essential use of the discrete nature of the variable  $b$  at the individual level. For our purpose, assume that the function which relates the optimal number of births to the survival probability is

$$(3.3) \quad b = \hat{b}(s) = \hat{b}[s(Z)] = H(Z),$$

where  $\hat{b}' < 0$  and  $s' < 0$ , so that  $H' > 0$ . Here,  $s$  is a decreasing function of environmental deterioration. Thus,  $b$  is an increasing function of environmental deterioration. Time subscripts have been dropped for simplicity.

From (3.2) it follows that

$$(3.4) \quad h'(Z) = s'(Z)H(Z)[1 + \sigma(Z)],$$

where

$$n = s\hat{b}(s) = s(Z)H(Z) \quad \text{and} \\ \sigma(Z) = b'[s(Z)]s(Z)]/b[s(Z)],$$

is the elasticity of births with respect to the survival rate. Because  $s' < 0$ ,  $H > 0$ , and  $\sigma < 0$ ,

$$(3.5) \quad h' \leq 0 \quad \text{according as} \quad \sigma \leq -1.$$

Thus,  $\eta_Z$ , the elasticity of the rate of growth of population with respect to environmental deterioration, is positive or negative depending on whether the elasticity of births with respect to the survival rate, which is negative, is greater than or less than one in absolute value. This clearly depends on the shape of the parent's utility function, a matter to be investigated in a particular context in the next section.

What is the effect of adult deaths prior to re-

<sup>7</sup> In currently ongoing research, Viktória Dalkó and I are exploring 3-period OLG models in which investment in the human capital of children at the expense of parental consumption in the first period of the child's life can enhance the probability that a child will survive to the second period of life to contribute to the support of her parents. This is a way of introducing human capital investments without parental altruism. It also shows the *ex ante* costs are controllable by the parents independently of the number of births.

production on the sign of the derivative of  $n$  with respect to environmental deterioration? Suppose that the adult death rate  $\delta$  is an increasing function of  $Z$ . Then

$$(3.6) \quad dn/dZ = h'(1 - \delta) - h\delta'.$$

Because  $0 < \delta < 1$  and if  $\delta' > 0$ , the rate of growth of population will be reduced by environmental deterioration below that which it would have been in the absence of such "Malthusian" effects; in particular, if  $\eta_Z$  starts out positive, it may eventually become negative as adult deaths increase.

Another reason to suppose that if  $\eta_Z$  begins positive it must eventually turn negative involves a factor which has not been explicitly introduced into the model. This is an absolute limitation on the number of births an individual woman can have in her lifetime. Such an absolute limitation implies that as environment deteriorates and the number of derived births increases, the elasticity of births with respect to the survival rate must be approaching 0 from below. Thus, if  $\sigma$  starts out less than  $-1$ , it will eventually become greater than  $-1$ ; that is, if the rate of rise of optimal births is sufficiently rapid initially to make  $\sigma < -1$ , this rate must slow as the maximum number of births per woman is approached so that eventually  $\sigma$  must become less than one in absolute value.

What are the effects of *ex ante* child-bearing costs? We must distinguish between movements along the curve relating such costs to the number of births and shifts in the entire curve resulting from factors such as environmental deterioration which also affect child survival probability. Holding the *ex ante* cost curve fixed, we can easily see that the effects of decreasing survival in increasing the optimal number of births are partially offset if the marginal cost of additional births is positive. Thus, for any given utility function, the optimal number of births will be lower and will increase at a lesser rate as survival probabilities fall. We cannot say whether  $\sigma$  will increase or decrease in absolute value. If the marginal *ex ante* costs of an additional birth are constant, only the optimal number of births will be reduced; the effect of a fall in survival probability should be unchanged. Thus,  $\sigma$  may be higher in absolute value (more negative) for any given level of survival probability.

Suppose now that the entire *ex ante* cost curve shifts upward, marginal costs unchanged, as survival probabilities fall. Whatever the optimal response of birthrates in the absence of such shifts, it will be less when they occur; that is, the elasticity of births with respect to survival

probability will be reduced in absolute value. Thus, *ex ante* costs of births, the level of which increases with environmental deterioration, makes it more likely rather than less that  $\eta_Z < 0$  rather than  $\eta_Z > 0$ . But, of course, environmental deterioration may also change the slope of the *ex ante* cost curve and thus alter the elasticity  $\eta_Z$  in other ways.

To reach more definitive conclusions, it is necessary to specify more exactly the utility function of parents and the nature of the costs incurred in having children. I assume that all children born survive and *ex ante* costs are absent, and turn to the parable of firewood.

### Endogenous Fertility 2: A Parable of Firewood

Suppose that all children survive birth and in the first period of life support their parents by gathering firewood which is shared out equally among family members. Any other sharing rule would do equally well. (There are no other *ex post* or *ex ante* costs of having an additional child other than the requirement that family production be shared.) Because all children survive,  $b_i = n_i$ .

Environmental deterioration is, as previously assumed, related to the size of the parents' generation, i.e., to the number of children working in the previous period. The larger this cohort, the greater the rate of environmental deterioration is assumed to be. Below a certain population, environment is assumed to be improving. The existing state of the environment is assumed to affect parents' perceptions of the benefit of having an additional child by affecting their expectations of the marginal productivity of an additional family member in gathering firewood and their perceptions of the level of the family production function. These expectations are formed at the time each parent decides how many children to have. While such decisions are assumed to reflect a correct perception of the relation between the size of her cohort and the future state of the environment, I do not assume that they take into account the possible behavior of other mothers or the effects of the number of children born on future states of the environment. Indeed, because parents are assumed not to care about their children's welfare, this behavior is consistent because the effects of the size of the children's cohort will be realized only after the parents are dead. Each mother decides how many children to have on the basis of her perception of the relationship between family income, the number of children she has,

and the state of the environment. This relationship embodies her beliefs about the productivity of her children and a correct assessment of the state of the environment during the period  $t - 1$ .

Note that the "production function" in this model is purely a family affair, but it could be, and likely is, related to some economy-wide production function having objective reality. Such a relationship is required if we are to interpret the function empirically. A "Nash-equilibrium" argument should suffice to establish the connection between such an economy-wide production function and the perceptions of individual parents.

In this context, it is assumed that the perceived marginal product of an additional child is positive but diminishing. Increased environmental deterioration reduces the family's harvest, *ceteris paribus*, but its effect on the perceived marginal product of an additional child may be either positive or negative.

Let  $x_t$  be the family's expected harvest, then

$$(4.1) \quad x_t = f(b_{t-1}, Z_{t-1}), \quad f(0, Z_{t-1}) = 0,$$

and where  $f \geq 0$ ,  $f_1 > 0$ ,  $f_{11} < 0$ , and  $f_2 < 0$ , for all  $b \geq 0$  and  $Z > 0$ .

If the state of environmental degradation,  $Z$ , could be assumed to behave like the inverse of an ordinary factor of production, it would be plausible to argue that

$$f_{12} < 0,$$

so that increases in environmental quality would be expected to increase the marginal productivity of the other factor, namely, children. Similarly, the marginal effect of further environmental deterioration might be expected to diminish at greater levels of poorer quality environment, i.e., the better environmental quality, the greater the marginal effect of deterioration:

$$f_{22} < 0.$$

On the other hand, an argument can be made for opposite signs: For example, as forests recede up the mountain sides, parents may perceive a greater benefit of having an additional child to gather firewood. More realistically, in a poor agricultural setting, lower quality environments may be associated with a greater livestock component in total production, whereas higher quality environments may be associated with a greater crop component. Arguably, children have a comparative advantage over adults

in tending livestock in contrast to the heavier labor of planting, tilling, and harvesting crops. Thus, environmental deterioration may well enhance the marginal productivity of children, at least relative to total family productivity. Similarly, environmental deterioration may accelerate the perceived adverse effects on family income: In very poor quality environments the effects of a given change in the quality of the environment may be larger than when the environment is in good shape.

Parents are assumed to choose the number of their offspring so as to maximize their own selfishly determined utility in the retirement period. Utility is assumed to be a monotone-increasing function of the parent's own consumption of firewood and nothing else. Thus, the parent maximizes

$$\frac{x_t}{b_{t-1} + 1} = \frac{f(b_{t-1}, Z_{t-1})}{b_{t-1} + 1}$$

with respect to  $b_{t-1}$ , taking  $Z_{t-1}$  as given. We drop temporal subscripts in what follows.

The first-order condition for a maximum of the per capita income in the family is

$$(4.2) \quad \frac{(b+1)f_1(b, Z) - f(b, Z)}{(b+1)^2} = 0, \text{ or}$$

$$b = \frac{f(b, Z)}{f_1(b, Z)} - 1, \text{ or} \\ f_1 = f/(b+1).$$

Because  $b = 0$  implies  $f(0, Z) = 0$ ,  $b = 0$  cannot be a solution to (4.2) as long as  $f_1$  is strictly positive, even at  $b = 0$ . For  $b > 0$ ,

$$\frac{\partial f/f_1}{\partial b} = \frac{f_1^2 - ff_{11}}{f_1^2} = 1 - \frac{ff_{11}}{f_1^2} > 1,$$

as a consequence of the diminishing marginal product of children and  $f(b, Z) > 0$  for  $b > 0$ . Provided also  $f/f_1$  is convex, the curve  $f/f_1$  must cross the line  $b + 1$  at some value  $\bar{b}$  and thus (4.2) determines a unique value of the birthrate which is always positive, but which may be greater than, equal to, or less than 1 according to whether  $f/f_1 \geq 2$  at  $b = 1$  (see fig. 1). And, thus, (4.2) defines a function  $h$  which relates each state of the environment  $Z_{t-1}$  to a unique positive value of the birthrate at the end of period  $t - 1$ :

$$(4.3) \quad b_{t-1} = h(Z_{t-1}) > 0.$$

It can happen, however, even in this case that



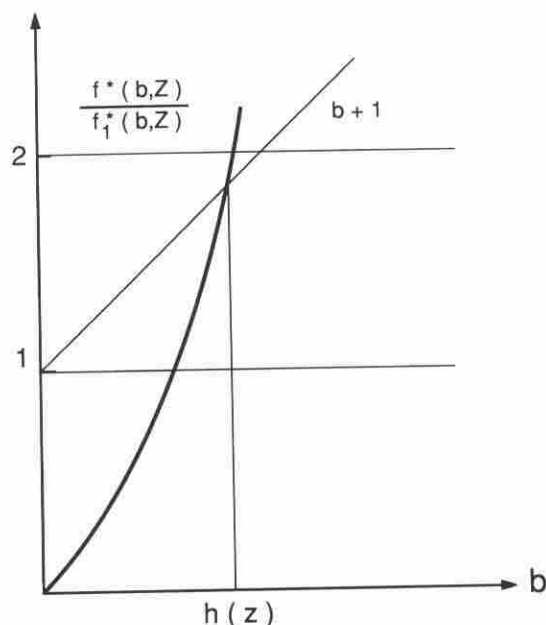


Figure 1. Determination of optimal birth-rate

no solution to the equation  $h(Z) = 1$  exists. Clearly, if

$$h(Z) > 1, \text{ or } h(Z) < 1,$$

for all  $Z$ , no solution exists;  $h(Z) > 1$  for all  $Z$  can occur if

$$(4.4) \quad f(1, Z) < 2f_1(1, Z)$$

for all  $Z$ . Similarly,  $h(Z) < 1$  for all  $Z$  if for all  $Z$ ,

$$(4.5) \quad f(1, Z) > 2f_1(1, Z).$$

To guarantee a solution to  $h(Z) = 1$  if  $h(Z)$  is continuous, it is sufficient that (4.4) holds for some values of  $Z$  and (4.5) for other values.

Before we examine the further properties of  $h$ , note that the second-order condition for a maximum is always satisfied because  $f_{11} < 0$ :

$$\frac{\partial^2 [f(b, Z)/(b+1)]}{\partial b^2} = \frac{(b+1)^2 \frac{\partial}{\partial b} [(b+1)f_1 - f] - 2[(b+1)f_1 - f](b+1)}{(b+1)^4} = \frac{(b+1)f_{11}}{(b+1)^2} = \frac{f_{11}}{b+1} < 0, \text{ for } b > 0,$$

making use of the first-order condition.

How are the properties of the function  $h(\cdot)$  in

(4.3) related to those of  $f(\cdot)$ , which is the basis for parents' decision making with respect to fertility? First, let us determine  $h'$ . From (4.2),

$$h' = \frac{db}{dZ} = \frac{d}{dZ} \left[ \frac{f}{f_1} \right] = \frac{\partial}{\partial b} \left[ \frac{f}{f_1} \right] \frac{db}{dZ} + \frac{\partial}{\partial Z} \left[ \frac{f}{f_1} \right],$$

so that

$$h' = \frac{\frac{\partial}{\partial Z} \left[ \frac{f}{f_1} \right]}{1 - \frac{\partial}{\partial b} \left[ \frac{f}{f_1} \right]}.$$

Now,

$$\frac{\partial [f/f_1]}{\partial Z} = \frac{f_1 f_2 - f f_{12}}{f_1^2} \quad \text{and} \quad \frac{\partial [f/f_1]}{\partial b} = \frac{f_1^2 - f f_{11}}{f_1^2} = 1 - \frac{f f_{11}}{f_1^2}.$$

Thus,

$$(4.6) \quad h' = \frac{f_1 f_2 - f f_{12}}{f f_{11}}.$$

Because  $f_{11} < 0$  and  $f > 0$ ,  $b > 0$ , the sign of  $h'$  is determined by

$$(4.7) \quad \text{sign } h' = - \text{sign}(f_1 f_2 - f f_{12}),$$

and because  $f_1 > 0$ , and  $f_2 < 0$ , and  $f > 0$ , for  $b > 0$ , we can say unambiguously

$$(4.8) \quad h' > 0, \text{ if } f_{12} > 0,$$

that is, if the perceived marginal product of an additional child increases as environment deteriorates, birthrates increase as the environment worsens. In this case, good environmental quality is not like an ordinary factor of production. However, a more general interaction of environment and the marginal product of a child is possible because environmental quality may, at some levels, be like an ordinary factor of pro-

duction, but in any case is not under the control of the decision-making parent.

From (4.6), we find after some manipulation that

$$(4.9) \quad h' = \frac{f_{12}f_2}{ff_{11}} [1 - \varphi/\psi],$$

where

$$\varphi = f_{12} \frac{Z}{f_1}$$

is the elasticity of the marginal product of a child with respect to environmental deterioration and

$$\psi = f_2 \frac{Z}{f}$$

is the elasticity of family output with respect to environmental deterioration. We assume  $f_1 > 0$ ,  $f_2 < 0$ ,  $f > 0$  and  $f_{11} < 0$  for all  $b > 0$  and, hence, for all  $Z$ . Thus,

$$(4.10) \quad h' \geq 0 \text{ according as } 1 \geq \frac{\varphi}{\psi}.$$

Because  $f_2 < 0$ , then  $\psi < 0$ . As we saw,  $f_{12} > 0$  implies  $h' > 0$  and also  $\varphi > 0$  but  $f_{12} < 0$  implies  $\varphi < 0$ . So, only when  $f_{12} < 0$ , that is, when environmental deterioration reduces the perceived marginal benefit of having an additional child, is the possibility open that such deterioration reduces the birthrate. But, even in this case, the effect is not unambiguous because, by (4.10),

$$(4.11) \quad h' \geq 0 \text{ according as } |\psi| \geq |\varphi|;$$

that is, if the elasticity of family income is greater in absolute value than the elasticity of the marginal product of a child,  $h' > 0$ , whereas, if the opposite is true,  $h' < 0$ . Moreover, these elasticities may change as environment deteriorates, which opens up the possibility that  $h'$  changes sign.

Suppose that  $Z$  is not like an ordinary factor of production when environmental quality is relatively good, that is  $f_{12} > 0$ , then environmental deterioration enhances the marginal productivity of a child so that at this level of  $Z$ ,  $h' > 0$ . Now, however, suppose that as environment deteriorates it becomes more and more like an ordinary factor of production and further deterioration reduces the marginal productivity of a child as well as total family product. A possible explanation for the change in the nature of environment as a factor of production lies outside the narrow confines of the model; casual observation suggests that, under severe environmental circum-

stances, children actually become a drag on their parents' chances for survival. In any case, even if  $h'$  remains positive, rising adult death rates would ultimately lead to a fall in the rate of population growth.

### Taxes, Subsidies, and Social Intervention

In the preceding two sections I have argued that multiple, at least two, stationary solutions to the dynamic system relating population and environmental quality are likely. The first of these, if there are two, is likely to be characterized by a positive response of fertility and the rate of growth of population to environmental deterioration and the second by a negative response. Only when there is such a negative response is there any possibility of obtaining a stable stationary solution under other plausible assumptions about the parameters of the system. This section considers how social intervention through a system of per capita taxes on children and lump-sum subsidies to parents can achieve any socially desired birthrate. Provided environment can eventually recover from the effects of excessive population growth, such a system of taxes and subsidies could be used to achieve and maintain any specified birthrate. In particular, social intervention could induce parents to determine their fertility in order, first, to reach a stationary solution—for example, the one with the lower level of population and better environment—and then to maintain that equilibrium despite its local instability. Clearly, those alive in all generations at the environmentally better equilibrium are better off than all who live at the environmentally worse equilibrium. But that does not mean that individual parents are better off; indeed, if, in the absence of taxes and subsidies, they would choose different levels of fertility, they are clearly worse off than if left to exercise their unfettered preferences. The welfare economics of environmental/population interactions is left to the next section, however, where parental altruism toward their children, or love, is discussed.

Reformulate the parent's optimization problem of the preceding section as follows: If the parent has  $b$  children, the family can gather  $x = f(b, Z)$  units of firewood. However, firewood must be shared among family members, so each child "costs" the parent  $p_b = x/(b + 1)$ . The parent gets what is left over,  $y$ . So the parent's problem is

$$(5.1) \quad \max_b y \quad \text{subject to} \quad p_b b + y = x.$$

Now it is easy to see what the effect is of a tax,  $\tau$ , per birth coupled with a lump-sum subsidy,  $\sigma$ , per parent, which need not be shared with her children. The budget constraint in (5.1) is now

$$(5.2) \quad (p_b + \tau)b + y = x + \sigma,$$

where  $p_b$  is  $x/(b + 1)$ , as before. The parent takes  $\tau$  and  $\sigma$  as given. Differentiating the expression

$$y = x - (p_b + \tau)b + \sigma$$

with respect to  $b$  yields the appropriate first-order condition:

$$(5.3) \quad \frac{\partial x}{\partial b} - (p_b + \tau) - b \frac{\partial p_b}{\partial b} \\ = f_1 - f/(b + 1) - \tau \\ - b\{f(b + 1)f_1 - f\}/(b + 1)^2 \\ = f_1/(b + 1) - \tau - f/(b + 1)^2 \\ = 0.$$

All taxes are returned to parents as a subsidy  $\sigma$ , so the subsidy per parent must equal  $b\tau$ . It follows that  $\tau$  and  $\sigma$  are determined for any given  $b = b^*$  as

$$(5.4) \quad \tau^* = \frac{f_1(b^*, Z)(b^* + 1) - f(b^*, Z)}{(b^* + 1)^2} \\ \sigma^* = \tau^* b^*.$$

$\tau^*$  and  $\sigma^*$  thus induce the parents to choose to have  $b^*$  children per family.

It is instructive to compare this solution with the result of unfettered parental preferences derived in the preceding section. There, the result was

$$b = f/f_1 - 1.$$

After some manipulation, we find

$$(5.5) \quad b^* = f/(f_1 - [\tau^* + \sigma^*]) - 1;$$

so the tax/subsidy pair  $(\tau^*, \sigma^*)$  works by reducing (augmenting if we allow  $\tau^*$  and  $\sigma^*$  to be negative, i.e., to be a subsidy per child and a lump-sum tax per parent) the perceived benefit,  $f_1$ , of having an additional child.

Given a sequence of environmental qualities  $Z_{-1}, Z_0, Z_1, \dots$ , the social planner can choose a sequence  $(\tau_0^*, \sigma_0^*), \dots$  to solve (5.5) for the sequence  $b_0^*, b_1^*, \dots$ . In general, the planner can force the system to a particular stationary point of the system from any initial point away from that stationary point under general conditions, for example, provided that the sequence  $\{b_i^*\}$

necessary to achieve this result were bounded away from zero, and population itself never became zero.

### Parental Altruism: Love Is Not Enough

I now take up the difficult welfare issues which arise when there is parental altruism in the sense that parents love and value their children and their children's welfare over and above what their children may do for them. As remarked above, if parents do not care about their children's welfare and if all die at the end of the second period in our OLG model, they cannot be concerned about the consequence of their actions on future environmental quality because they are no longer alive to experience such effects when they occur. If, however, parents also love their children, matters are different. In this section, I argue that such love, in the absence of any goods other than firewood or children from which to obtain utility, can only increase parents' fertility over and above what they would choose in the absence of such altruism. Now, however, it becomes clear that unfettered parental choice may not lead to a Pareto optimum in which no parent can be made better off without making some others worse off, because the environmental resource which contributes to children's welfare is unpriced. Preferences are assumed stationary across generations; that is, parents and children have identical utility functions which are known to parents who can make interpersonal comparisons of utility—or, at least, believe they can. A consistent system of what are called "normally benevolent" utility functions (see Bergstrom) are the additively separable preferences

$$(6.1) \quad U_t = u(c_t) + aU_{t+1}, \quad 0 < a < 1,$$

where  $U_t$  is the utility of the  $i$ th generation (see also Barro, and Razin and Ben-Zion). It is well known that this system of interdependent utility functions induces independent utility functions for each generation  $t$  which take the form

$$(6.2) \quad U_t = u(c_t) + \sum_{j>t} a^{j-t} u(c_j).$$

In the present context, it is plausible that a parent derives utility from each of her children in proportion to their maximized utility. Thus, (6.1) may be rewritten as

$$(6.3) \quad V_t = \max_{b_{t-1}} \{u(c_t) + ab_{t-1}V_{t+1}\}.$$

Some conditions, such as boundedness of the sequence  $\{(ab_{j-2})^{j-t}, j = t+1, \dots\}$ , must be imposed to ensure the existence of  $V_t$  defined in this way. The consumption of a parent of generation  $t$  is given in section 5:  $c_t = f(b_{t-1}, Z_{t-1})/(b_{t-1} + 1)$ , and the utility-maximizing birthrate in the absence of altruism is illustrated in figure 2. Suppressing time subscripts, setting  $u$  equal to its argument and replacing  $c_t$  in (6.3) by its value, we obtain the specific parent's problem in our context:

$$V_t = \max_b \{f(b, Z)/(b+1) + abV_{t+1}\}.$$

Now  $V_{t+1}$  is clearly a function of what parents of this generation believe the future course of environment will be. In general, these beliefs will be erroneous. Each birth a parent chooses to have will diminish environmental quality in the future and thus the welfare of future generations. The parent cannot be expected to take this into account in making her individual decision, because her action alone has a negligible effect on future environmental quality. Using the system (2.1) and (2.4) to calculate the trajectories of  $N$  and  $Z$ , and the utility function of a

parent (6.4), which a social planner would know only if he could calculate  $V_{t+1}$  for each generation, he then could, in principle, calculate an optimal sequence of birthrates so as to maximize the utility of a parent in the present generation. Such a calculation would take into account the true marginal cost of an additional birth in terms of its effects, through environmental deterioration, on the welfare of the next, and through them, subsequent generations. The difficulty for the planner is in knowing the beliefs of parents in the present generation. In effect, to solve the problem, the planner needs to know both what  $V_{t+1}$  is and what it ought to be, given a correct forecast of the future trajectory of environmental quality.

The problem of parents' beliefs could be resolved by introducing a model of expectations formulation, for example, static expectations in which parents assume the present state of environment will continue forever. This, however, hardly seems plausible except when the system is at a stationary point. At stationary points, we can indeed compute the difference in welfare, and the planner can, as suggested in the preceding section, choose a sequence of tax/sub-

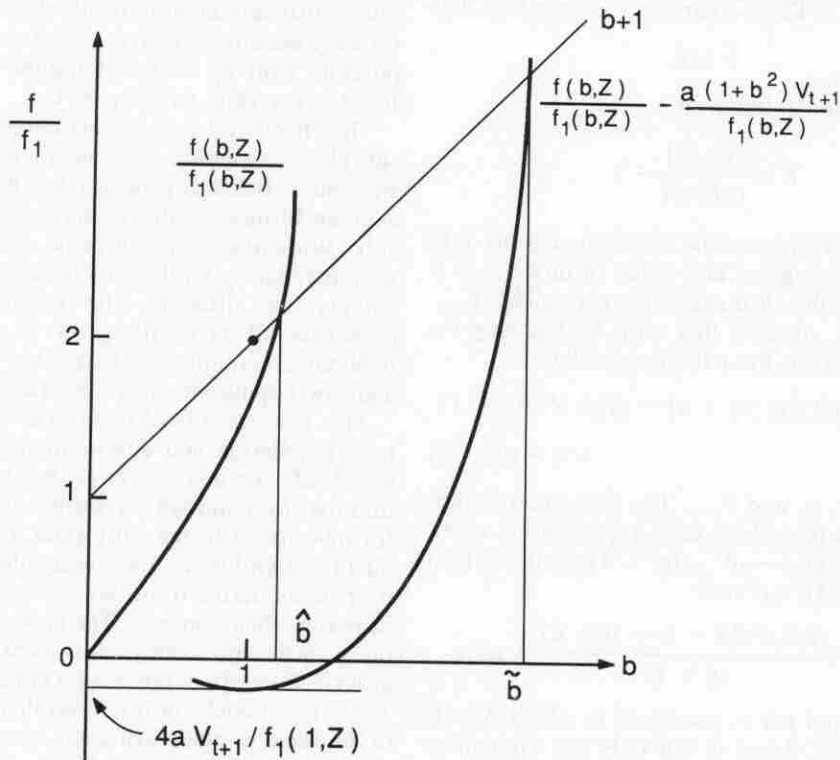


Figure 2. Effects of parental altruism on optimal birthrate



sidies to achieve and maintain the stationary state with the highest per capita utility for members of each generation. The only difference between these tax/subsidies and those of the preceding analysis is that birthrates will always be higher, for any given level of environmental quality, with parental altruism than without. Thus, per child taxes must be higher at a stationary point to maintain the system there with love than without.

To show the optimal level of births must be higher with altruism than without, compare the first-order conditions (4.2) with those obtained in the maximization problem (6.4), holding  $V_{t+1}$  fixed for the current generation:

$$(6.5) \quad \frac{(\bar{b} + 1)f_1(\bar{b}, Z) - f(\bar{b}, Z)}{(\bar{b} + 1)^2} + aV_{t+1} = 0,$$

where  $\bar{b}$  is the maximizing value for given  $Z$  and fixed  $V_{t+1}$ . Thus,

$$(6.6) \quad \bar{b} = \frac{f(\bar{b}, Z)}{f_1(\bar{b}, Z)} - \left\{ 1 + \frac{a(\bar{b} + 1)^2 V_{t+1}}{f_1(\bar{b}, Z)} \right\}.$$

Thus, the effect is to lower the curve  $f(b, Z)/f_1(b, Z)$  in figure 2 everywhere by an amount which depends on  $a$  and  $V_{t+1}$  and a factor increasing in  $b$ . From figure 2, it is apparent that

$$(6.7) \quad \bar{b} \geq \hat{b},$$

where  $\hat{b}$  is the solution to

$$\hat{b} = \frac{f(\hat{b}, Z)}{f_1(\hat{b}, Z)} - 1.$$

When there is parental altruism, the tax rate necessary to achieve any given birthrate  $b = \beta$  is higher by the altruistic effect per child,  $aV_{t+1}$ . To see this, observe that with tax/subsidy ( $\tau$ ,  $\sigma$ ) and altruism, the parent's problem is

$$(6.8) \quad \max_b \{ [f(b, Z) + \sigma] - [f(b, Z)/(b + 1) + \tau - aV_{t+1}]b \}$$

for fixed  $\sigma$ ,  $\tau$ , and  $V_{t+1}$ . The first-order conditions follow from those derived previously, (5.3), by substituting  $\tau - aV_{t+1}$  for  $\tau$ . Thus, in contrast to  $\tau^*$  in (5.4), we have

$$(6.9) \quad \bar{\tau} = \frac{f_1(\beta, Z)(\beta + 1) - f(\beta, Z)}{(\beta + 1)^2} + aV_{t+1}.$$

An additional tax is necessary to offset the effects of love. Love is not only not enough to stabilize the relation between population and environment, it makes matters worse!

## Conclusions

The dynamics of population/environmental interaction were developed in the context of a two-equation model in which the size of population is related to the past value of population and the value of environmental quality, and the level of environmental quality is related to its past value and to the past value of population. For a model of endogenous fertility, in which parents choose the number of births they have to maximize their utility and in which environmental degradation,  $Z$ , would converge for a fixed population and responds unfavorably to rising population, the relevant range of elasticities is

$$(7.1) \quad \begin{aligned} 0 < \xi_Z < 1: & \text{Elasticity of environmental quality} \\ & \text{with respect to its past value.} \\ 0 < \xi_N: & \text{Elasticity of environmental quality} \\ & \text{with respect to population.} \\ \eta_N = 1: & \text{Elasticity of population size with} \\ & \text{with respect to its past value.} \end{aligned}$$

The response of population size to environmental quality may be positive or negative,  $\eta_Z$  indeterminate. In a series of metaphors, I argued that  $\eta_Z$  might well start out positive but would, given sufficient environmental deterioration, ultimately become negative. Local stability is impossible with  $\eta_Z > 0$ , but limited possibilities for stability exist when  $\eta_Z < 0$ .

The effects of taxes on children and lump-sum subsidies to parents were next investigated. It was shown that it is generally possible to achieve a given birthrate with an appropriate tax/subsidy. Without parental altruism, such a policy is welfare reducing for the present generation. When parents are altruistic, the welfare-enhancing properties of taxes designed to achieve and maintain an equilibrium with good environment and low population are problematic.

This leads to a final point: The model relates only population and environment. There is no possibility for investment in physical capital to improve or maintain environmental quality or for investment in human capital to improve the quality of children's lives and enhance the utility parents derive from having children without increasing their numbers. The bulk of recent work on growth and endogenous fertility revolves around these two types of capital formation. There is no doubt in my mind that introduction of physical capital formation to offset the environmentally adverse effects of population pressure and of human capital formation to en-

hance the quality of individual children would result in far more optimistic conclusions. The mathematics of introducing such possibilities, however, is more complicated than the analysis of dynamic planar system on which the analysis of this paper is based. Much remains to be done, but the present work offers a beginning to the study of the difficult issues of intergenerational welfare economics which must be confronted in any attempt to understand and cope with problems of population/environmental interaction.

## References

- Barro, R. J. "Are Government Bonds Net Wealth?" *J. Polit. Econ.* 82(1974):1095-1117.
- Bergstrom, T. "Systems of Benevolent Utility Interdependence." Unpublished, 1990.
- Freedman, R. *The Sociology of Human Fertility*. New York: Irvington, 1975.
- Grantmont, J. M. "On Endogenous Competitive Business Cycles." *Econometrica* 53(1985):995-1045.
- Guckenheimer, J., and P. Holmes. *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. New York: Springer-Verlag, 1986.
- Hardin, G. "The Tragedy of the Commons." *Science* 162(1968):1243-48.
- Iooss, J., and D. D. Joseph. *Elementary Stability and Bifurcation Theory*. New York: Springer-Verlag, 1990.
- Jodha, N. S. "Rural Common Property Resources: Contributions and Crisis." Foundation Day Lecture, May 1990, Society for the Promotion of Wastelands Development.
- McNicholl, G. "Social Organization and Ecological Stability Under Demographic Stress." The Population Council work. pap. no. 11, 1990.
- Neher, P. A. "Peasants, Procreation, and Pensions." *Amer. Econ. Rev.* 61(1971):380-89.
- Nerlove, M. "Household and Economy: Toward a New Theory of Population and Economic Growth." *J. Polit. Econ.* 82(1974):S200-218.
- Nerlove, M., and A. Meyer. "Population and the Environment: Some Dynamics." Paper presented at a Conference on Nonlinear Dynamics and Econometrics, University of California, Los Angeles, April 1991.
- Nerlove, M., and A. Meyer (with V. Dalkó). "Endogenous Fertility and the Environment: A Parable of Firewood." Paper presented at a UN University/WIDER Conference on Economic Development and the Environment, Helsinki, Finland, Sep. 1990.
- Nerlove, M., A. Razin, and E. Sadka. *Household and Economy: Welfare Economics of Endogenous Fertility*. New York: Academic Press, 1987a.
- . *Population Policy and Individual Choice: A Theoretical Investigation*. Washington DC: International Food Policy Research Institute Res. Rep. No. 60, 1987b.
- Preston, S. H. *Mortality Patterns in National Populations*. New York: Academic Press, 1976.
- Preston, S. H., ed. *The Effects of Infant and Child Mortality on Fertility*. New York: Academic Press, 1978.
- Razin, A., and U. Ben-Zion. "An Intergenerational Model of Population Growth." *Amer. Econ. Rev.* 65(1975):923-33.
- Sah, R. K. "The Effects of Child Mortality Changes on Fertility Choice and Parental Welfare." *J. Polit. Econ.* 99(1991):582-606.
- Schultz, T. P. *Economics of Population*. Reading MA: Addison-Wesley Publishing Co., 1981.
- Schultz, T. W. *Economics of the Family: Marriage, Children and Human Capital*. Chicago: University of Chicago Press for the National Bureau of Economic Research, 1974.
- Simon, J. L. *The Economics of Population Growth*. Princeton NJ: Princeton University Press, 1977.
- Willis, R. J. "The Old-Age Security Hypothesis and Population Growth." *Demographic Behavior: Interdisciplinary Perspectives on Decision Making*, ed. T. K. Burch. Boulder CO: Westview Press, 1980.
- Wolpin, K. I. "An Estimable Dynamic Stochastic Model of Fertility and Child Mortality." *J. Polit. Econ.* 92(1984):852-74.