Mill Feed Optimization for Multiple Processing Facilities Using Integer Linear Programming

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ABSTRACT: A considerable amount of work has been done on measuring and modeling the mineral processing characteristics such as work index, grindability, hardness and metallurgical recovery as a function of the geological setting and the blasting process. Nevertheless it is still unclear the way how different non linear relationships can be fed into very well established mine planning methods and procedures. Thus the traditional approach is to plan a mine according to the standard procedure and on the final production schedule estimate energy consumption, liberation size, availability of the plant facility and ultimate metallurgical recovery. Nevertheless these characteristics do not play any role on the construction of the production schedule, in particular, regarding the decision of allocating different blocks of ore to multiple processing facilities during a particular period of time of the operation of the mine.

This paper discusses the development of an algorithm to allocate ore produced from an Open Pit mine to two mills with different settings. The allocation of the ore is done through an Integer Linear algorithm that takes into account grindability of the rock, lithology, grades and economic parameters. A second algorithm consists of introducing a linear relationship in the objective function to account for potential production scenarios using variable throughput. Both algorithms have been tested at a full scale at Grasberg, Freeport Indonesia, reporting a considerable benefit respect to the current heuristic approach used at the mine site. The paper ends with a discussion of the methods and the assumptions made along the development of these tools.

1 INTRODUCTION

The mine planning process of an open pit mine defines the ore body depletion strategy over time. This process has been extensively discussed by Dagdelen (1994), Fytas et al (1987) and Kim et al (1994), more sophisticated optimization methods used on the production scheduling part of the process have been presented by Halatchev (2002). One important component of the production scheduling process on defining the optimal production strategy is the definition of cut off grades over time, which defines the limit between economic ore and waste. Variable cut off grade over time technique was first introduced by Lane (1988) and later there have been several industrial applications presented by Whittle and Wharton (1995) and Asad (2005). Even though there has been a considerable amount of research conducted on the area of open pit production planning very little has been done in order to introduce on the optimization techniques models that could represent the behaviour of the ore body through the mining system. This requires incorporating in the ore body model some mining and metallurgical variables that can represent the behaviour of the ore body in the processing facilities. Also the optimization model requires having the characteristics of different plant facilities such as different mills configurations that would have different production characteristics according to the ore body treated at the time. The introduction of these elements in the optimization method would change the way how the mine is sequenced and the resulting production schedule. Nevertheless the current commercial optimization tools do not include the facilities to introduce in an efficient way the allocation of ore blocks to different facilities when the underlying strategic objective is to maximize NPV. Thus the detailed task of allocating ore blocks to different process facilities or stockpiles with different properties has been left to the short term planning. Usually the short term planning consists of defining the operational resource requirements to achieve the medium term plan. Then this planning horizon needs to be as near as possible to the operational performance of the mine. At this planning horizon is when the blending and the multi process allocation happen as shown by Cai (2005).

The traditional approach to allocate ore blocks with different metallurgical characteristics to a multi-process facility has been done heuristically using differential cut off grades for the different processing units. Even in some cases the detailed allocation of mineral reserves to different processing facilities is ignored letting the operation of the mine to deal with this decision. The investigation presented in this paper shows that a binary linear and non linear model can be used to facilitate the engineering decision of ore allocation.

2 THE MATHEMATICAL MODEL

The objective of using mathematical programming to support mine planning decisions is to maximize maximize ore body utilization and recovery by introducing in the processing decision ore body characteristics that are often forgotten or left aside of the mine planning process. The objective function of the mathematical model presented in this section of the paper intends to maximize the short term mine revenue by sending different packets of rock to different concentrating facilities that hold different settings. Then the problem is to find the correct allocation of ore blocks that contained different metallurgical and throughput characteristics to a set of available processing facilities.

The proposed model aims to answer the following questions:

- Which material goes to which circuit?
- If the throughput of a circuit is increased by the addition of lower grade material, will there be a net benefit, or will the reduction in recovery, due to the additional material, be more than the additional metal from the lower grade components.

Figure 1 shows a schematic representation of the problem to be solved. This figure shows that the destination of the blocks will be defined as result of the optimization problem. The model will produce an automated way of defining which block goes to which mill.

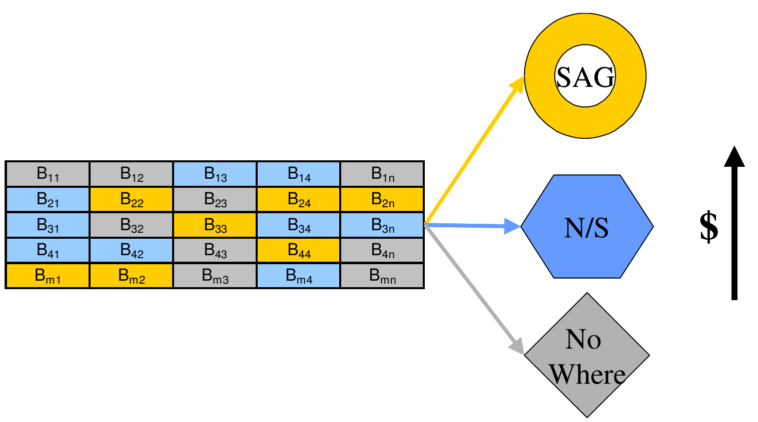

Figure 1 Conceptual diagram of the problem to optimize

The model developed in this research was set up to deal with the decision of allocating ore blocks to a conventional or a SAG mill. Then for every block of ore the revenue of processing the block through either a conventional mill or a SAG mill are computed. These revenues are a function of the metallurgical recovery which is also a function of the mill size product and the mineralogy of the ore block. Kelebec (2000) and Caceres (2006) show a clear evidence of correlations between the mill product size and metallurgical recovery as shown in Figure 2.

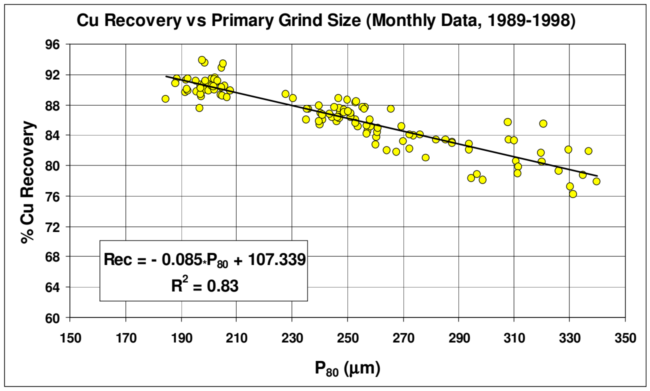

Figure 2. Copper recovery as a function of the mill product size from Kelebec (2000)

Based on the above results one can postulate a relationship to compute the metallurgical recovery of a block of ore as a function of the rock hardness and the mineralogy expressed in the rock type of the block. Thus, the following relationship can be used to compute the metallurgical recovery of block *i* processed through mill *k*, which is embedded in the calculation of the block revenue:

$$R_i^k = f^k\left(MOTC_i, RT_i\right) \tag{1}$$

Where R_i^k is the metallurgical recovery of block *i* processed through mill *k*, *i* is an index to identify the block of ore, *k* is an index to identify the milling process, $MOTC_i$ is the material operating throughput of block *i* which is directly a function of the rock hardness measured as the bond work index (Bond, 1961), RT_i is the rock type of block *i.* The main parameters used to formulate the mathematical programming model are: the revenues earned by processing a block of ore through either of the processing facilities and the time taken to process a block of ore through either of the processing facilities. The definitions needed to formulate the mathematical programming model are presented as follows:

N	is the total number of blocks to optimize
R_SAG_i	is the revenue earned if the block *i* is sent to the SAG mill
R_NS_i	is the revenue earned if the block *i* is sent to the N/S mill
v_i	is a binary variable that is equal to 1 if the block *i* is processed through the SAG mill or 0 otherwise
w_i	is a binary variable that is equal to 1 if block *i* is processed through the N/S mill or 0 otherwise
T_SAG_i	is the time that would take to process block *i* through the SAG mill
T_NS_i	is the time that would take to process block *i* through the N/S mill
$TSAG$	is the total available time at the SAG mill at a given time period. This is based on the mill availability and the length of the period to evaluate
TNS	is the total available time at the N/S mill at a given time period. This is based on the mill availability and the length of the period to evaluate

The objective function to be optimized in a given period of time is presented as follows:

$$\underset{v_i, w_i}{Max}\left\{\sum_{i=1}^{N}\left(R\_SAG_i \cdot v_i + R\_NS_i \cdot w_i\right)\right\} \tag{2}$$

The set of constraints that defines the feasible space is presented as follows:

$$\sum_{i=1}^{N} \left(T\_SAG_i \cdot v_i \right) \le TSAG$$

$$\sum_{i=1}^{N} \left(T\_NS_i \cdot v_i \right) \le TNS$$

$$v_i + w_i \le 1 \;\; \forall \;\; i = 1..N$$

$$v_i, w_i \;\; \text{binary variables (optional)}$$

The model outlined above intends to find a blend of blocks that would maximize the revenue earned by processing the run of mine production in a plant setting within the time available at different milling facilities in a given period of time.

The total number of variables is twice the number of blocks. These variables could be binary or not depending on the accuracy of the solution desired. Usually by solving the relaxation of the problem (taking variables as real) will be much faster than solving it with integer variables. In the above formulation the relaxation of the problem will be very often close to the integer solution. The number of constraints of the above formulation will be one per block plus the global constraints that control time availability at the mills.

The objective function is linear and could be solved using a linear optimizer. In this case the simplex algorithm will be used to optimize the problem using real variables. However if it is desired to use binary variables the algorithm to solve the problem will be branch and bound.

3 MODEL IMPLEMENTATION

The implementation of the model presented in the previous section of the paper was conducted at Grasberg mine of PT Freeport Indonesia located in West Papua, Indonesia. The tonnage to be allocated per month is about 8 Mt of ore concentrated into 1300 blocks to schedule per month. Every block would have copper and gold grades, tonnage, an index of hardness (MOTC) and metallurgical recoveries. Before implementing the model presented in this paper the short term planning department of the mine was using a heuristic approach, in which at a given cut off grade of copper equivalent the SAG mill will be filled up at its maximum throughput capacity. Once the SAG mill is filled at its maximum capacity, the conventional mill (N/S) is filled with the blocks left until the N/S mill capacity is reached.

The problem found with the above approach is that it does not take into account the processing time per block per mill. Perhaps one could find a block of ore that shows a high revenue potential and also shows a long processing time, in this case the block should not be attractive for processing given the mills time availability characteristics.

The model was constructed in the Excel solver add in which is commercialized by Frontline Systems. The model used to implement the mathematical model is the "Premium Solver Platform" that can deal with linear objective functions, several variables and constraints. The main model input are the ore blocks with their processing properties as shown Table 2.

Table 2. Ore blocks to be scheduled in a month period

BLOCKS

x	y	z	MOTC	TYPE	TONNES	CU	AU	CuEq
1	1	1	3	3	7725	0.14%	0.06	0.19%
1	1	1	3	3	7665	0.42%	0.06	0.47%
1	1	1	3	3	9217	0.43%	0.1	0.50%
1	1	1	3	3	9160	0.35%	0.077	0.41%
1	1	1	3	3	8883	0.26%	0.117	0.34%
1	1	1	3	3	8984	0.24%	0.092	0.30%
1	1	1	3	3	10179	0.26%	0.08	0.32%
1	1	1	3	3	10493	0.28%	0.077	0.33%
1	1	1	3	3	10162	0.27%	0.073	0.32%
1	1	1	3	3	10962	0.29%	0.075	0.34%

The MOTC throughput relationship for both the SAG and the N/S mills were constructed performing a three years production back analysis, these relationships are shown in Table 3.

Table 3. SAG and N/S productivities for different MOTC

MOTC	SAG tph	Size
1	8931	23.18
2	7671	16.38
3	6241	8.66

MOTC	N/S tph	Size
1	3510	30.25
2	3510	30.25
3	3510	30.25

Also the metallurgical recovery as a function of the MOTC and the Rock Type was obtained from a back analysis process and are presented in Table 4.

Table 4. SAG and N/S processing lines metallurgical recoveries

Metallurgical Recoveries for SAG Processing line

MOTC/TYPE	1	2	3	4
1	96.5%	86.4%	76.2%	86.4%
2	97.5%	88.9%	78.8%	88.9%
3	98.7%	91.8%	81.7%	91.8%

Metallurgical Recoveries for N/S Processing line

MOTC/TYPE	1	2	3	4
1	95.5%	83.8%	73.5%	83.8%
2	95.5%	83.8%	73.5%	83.8%
3	95.5%	83.8%	73.5%	83.8%

The processing time available per mill in hours over a year is shown below:

Table 5. Processing time available per mill per period

(Hrs)	*TSAG*	*TNS*
Jan	724	540
Feb	722	556
Mar	716	568
Aprl	704	581
May	762	586
Jun	734	597
Jul	753	590
Aug	750	587
Sep	746	570
Oct	761	592
Nov	760	589
Dec	749	563

The model was computed over a year time and the main results are shown ion Table 6.

Table 6. Comparative results of Binary optimization against heuristic methods

KUS$	Manual	Manual w Time Constraint	Optimized w time constraint
Jan	163,610	163,072	164,876
Feb	189,212	162,554	185,543
Mar	228,963	229,418	229,797
Aprl	186,339	180,214	185,961
May	213,429	201,095	212,305
Jun	213,937	195,528	212,438
Jul	197,362	182,402	195,819
Aug	203,317	194,737	203,021
Sep	186,245	174,996	185,289
Oct	179,043	167,145	177,500
Nov	176,324	160,169	173,985
Dec	170,664	164,252	170,137
Total	2,308,446	2,175,581	2,296,672

Table 6 shows in its second column the revenues earned based on a heuristic block allocation that disregards the time available per mill. The second column shows the revenues earned based on a heuristic allocation but in this case integrating the time constraint per mill. The fourth column shows the revenues earned using the mathematical programming model presented in this paper. The results shown in table 6 are encouraging since there is an improvement of 120 M$ between the heuristic and the mathematical programming model.

A way to implement at the mine the results found above is by using a variable cut off grade by material type as shown in Table 7.

Table 7. Copper Cut off grade by material type

	SAG Cut-Off grade by Material Type			
	SAG t1	SAG t2	SAG t3	NS
Jan	0.42%	0.91%	1.17%	0.35%
Feb	0.36%	0.79%	2.05%	0.38%
Mar	0.30%	0.40%	0.56%	0.39%
Aprl	0.41%	0.51%	0.59%	0.29%
May	0.29%	0.47%	0.49%	0.37%
Jun	0.27%	0.34%	0.70%	0.33%
Jul	0.36%	0.42%	0.71%	0.45%
Aug	0.31%	0.53%	0.73%	0.31%
Sep	0.32%	0.36%	0.73%	0.40%
Oct	0.35%	0.39%	0.79%	0.43%
Nov	0.34%	0.39%	0.89%	0.39%
Dec	0.35%	0.66%	0.86%	0.32%

Another interesting result found in the research summarized in this paper is that the variables affecting the decision of where to send a block of ore are the copper equivalent grade and the time taken to process a given block of ore. A graphical representation of the dual variable cut off grade is shown in Figure 3.

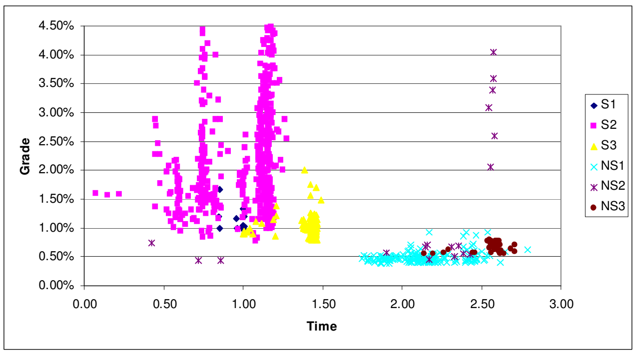

Figure 3. Grade/ Time relationship to define processing method

4 CONCLUSIONS

Mine planning process shall try continuously to represent the actual behaviour of the ore body through the mining system. A fundamental component of the mining system is the processing facility which often defines in great part the utilization an recovery of the ore body. At the moment at the optimization and sequencing part of the mine planning process there are no multi element multi facility optimization methodologies that can be used to approach the problem framed in this paper. Thus, short term planning models must contain block allocation algorithms that go beyond the scope of the traditional short term planning tasks.

Mathematical programming models can help to easily find a feasible solution that can be further analyzed introducing the mining sequence and other operational constraints. These models should be interpreted as a tool to facilitate the analyses and the construction of a production schedule. It should not be taken as it is a full automated production scheduler, since there are many planning and operational constraints that are very difficult to program in a mathematical way.

The research presented in this paper is a prototype of a model that can be used to guide the selection of blocks to feed a multi ore processing facility. Nevertheless the future work should concentrate on using the model embedded into the sequencing routines or the life of mine production schedules to provide a more comprehensive set of guidelines to the operation of the mine.

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