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Solution Manual
Game Theory: An Introduction

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strategy, their payoff from 0 is zero, and hence their expected payoff from any choice in equilibrium must be zero. ■

iv. Show that if two such strategies are a mixed strategy Nash equilibrium then it must be that $\overline{x}_1 = \overline{x}_2 = 1$.

Answer: Assume not so that $\overline{x}_1 = \overline{x}_2 = \overline{x} < 1$. From (iii) above the expected payoff from any bid in $[0, \overline{x}]$ is equal to zero. If one of the players deviates from this strategy and choose to bid $\overline{x}+\varepsilon < 1$ then he will win with probability 1 and receive a payoff of $1-(\overline{x}+\varepsilon) > 0$ contradicting that $\overline{x}_1 = \overline{x}_2 = \overline{x} < 1$ is an equilibrium. ■

v. Show that $F_i(x)$ being uniform over $[0, 1]$ is a symmetric Nash equilibrium of this game.

Answer: Imagine that player 2 is playing according to the proposed strategy $F_2(x)$ uniform over $[0, 1]$. If player 1 bids some value $s_1 \in [0, 1]$ then his expected payoff is

$$\Pr\{s_1 > s_2\}(1-s_1)+\Pr\{s_1 < s_2\}(-s_1) = s_1(1-s_1)+(1-s_1)(-s_1) = 0$$

implying that player 1 is willing to bid any value in the $[0, 1]$ interval, and in particular, choosing a bid according to $F_1(x)$ uniform over $[0, 1]$. Hence, this is a symmetric Nash equilibrium. ■

11.

12. **The Tax Man:** A citizen (player 1) must choose whether or not to file taxes honestly or whether to cheat. The tax man (player 2) decides how much effort to invest in auditing and can choose $a \in [0, 1]$, and the cost to the tax man of investing at a level a is $c(a) = 100a^2$. If the citizen is honest then he receives the benchmark payoff of 0, and the tax man pays the auditing costs without any benefit from the audit, yielding him a payoff of $(-100a^2)$. If the citizen cheats then his payoff depends on whether he is caught. If he is caught then his payoff is (-100) and the tax man's payoff is $100 - 100a^2$. If he is not caught then his payoff is 50 while the tax man's payoff is $(-100a^2)$. If the

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citizen cheats and the tax man chooses to audit at level a then the citizen is caught with probability a and is not caught with probability $(1-a)$.

(a) If the tax man believes that the citizen is cheating for sure, what is his best response level of a?

Answer: The tax man maximizes $a(100-100a^2)+(1-a)(0-100a^2) = 100a - 100a^2$. The first-order optimality condition is $100 - 200a = 0$ yielding $a = \frac{1}{2}$. ■

(b) If the tax man believes that the citizen is honest for sure, what is his best response level of a?

Answer: The tax man maximizes $-100a^2$ which is maximized at $a = 0$ ■

(c) If the tax man believes that the citizen is honest with probability p what is his best response level of a as a function of p?

Answer: The tax man maximizes $p(-100a^2)+(1-p)(100a-100a^2) = 100(1-p)a-100a^2$. The first-order optimality condition is $100(1-p)-200a = 0$, yielding the best response function $a^*(p) = \frac{1-p}{2}$. ■

(d) Is there a pure strategy Nash equilibrium of this game? Why or why not?

Answer: There is no pure strategy Nash equilibrium. To see this, consider the best response of player 1 who believes that player 2 chooses some level $a \in [0,1]$. His payoff from being honest is 0 while his payoff from cheating is $a(-100)+(1-a)50 = 50-150a$. Hence, he prefers to be honest if and only if $0 \geq 50 - 150a$, or $a \geq \frac{1}{3}$. Letting $p^*(a)$ denote the best response correspondence of player 1 as the probability that he is honest, we have that

$$p^*(a) = \begin{cases} 1 & \text{if } a > \frac{1}{3} \\ [0,1] & \text{if } a = \frac{1}{3} \\ 0 & \text{if } a < \frac{1}{3} \end{cases}$$

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and it is easy to see that there are no values of a and p for which both players are playing mutual best responses. ■

(e) Is there a mixed strategy Nash equilibrium of this game? Why or why not?

Answer: From (d) above we know that player 1 is willing to mix if and only if $a = \frac{1}{3}$, which must therefore hold true in a mixed strategy Nash equilibrium. For player 2 to be willing to play $a = \frac{1}{3}$ we use his best response from part (c), $\frac{1}{3} = \frac{1-p}{2}$, which yields, $p = \frac{1}{3}$. Hence, the unique mixed strategy Nash equilibrium has player 1 being honest with probability $\frac{1}{3}$ and player 2 choosing $a = \frac{1}{3}$. ■