Voting

King [to his princes]. "I'll be your father and your brother too; Let me but bear your love, I'll bear your cares."

—*Shakespeare's* Henry IV[1]

If men were angels, no government would be necessary. If angels were to govern men, neither external nor internal controls on government would be necessary. In framing a government which is to be administered by men over men, the great difficulty lies in this: you must first enable the government to control the governed; and in the next place oblige it to control itself.

—*James Madison*, The Federalist Papers[2]

SHAKESPEARE often depicts the nation as the king's family and the state as the king's household. All is well in the nation so long as the king's relatives and friends actually feel the love and affection that they proclaim toward each other, but let them fall out and strife overtakes the state. In this warm and intimate account of government, politics resembles the family. Love and affection, however, proved an unreliable foundation for politics. Most citizens these days do not regard themselves as the government's children, and they want political power restrained by something stronger than morality.

Eighteenth-century political theorists, including the founders of the United States, treated government as more like a machine than a household. They rejected the belief that politicians would act spontaneously in the public interest. Instead of family government, they wanted to design something like a market in which politicians would compete for votes, and this competition would direct politicians to do good as by an invisible hand.[3] Just as efficiency requires economic competition, so responsive politics requires political competition.

The vision of democracy as a market for votes proved useful and enduring, but the techniques for analyzing a market for votes changed little until recently, when economic theory was applied to politics. The basic techniques

[1] Henry IV, part 2, act 5, scene 2, ll. 57–58. Thanks to Robert Pearlman for this quote.

[2] Madison 1981b, p. 160. Thanks to David Lieberman for this quote.

[3] The relationship between public-choice theory and the political thought of Madison is discussed in Eskridge and Frickey 1988, pp. 37–38, 40–56.

for analyzing voting, which this chapter develops, offer fresh insights into questions such as these:

> *Example 1*: Some voters want government to be rich as fits the emblem of a great people, others want it starved into lethargy so it cannot hurt anyone, and most voters favor a position in between these extremes. Most politicians, however, just want to win elections. What political platform on government expenditure is most likely to command a majority of votes by citizens?

> *Example 2*: Minorities sometimes feel excluded from political power, and majorities sometimes feel that pivotal minorities wield excessive political power. What determines the degree of responsiveness of democratic politics to minorities?

> *Example 3*: When campaigning, some politicians are notoriously vague about their positions on particular policies. When does obfuscation help to win elections?

To begin to answer such questions, this chapter develops the economic theory of elections and applies it to the legislature and executive. The details of democratic institutions display as much variety as birdsong. To illustrate, elections are conducted by majority rule (winner receives at least half of votes), plurality rule (winner receives most votes), plurality run-off rule (two candidates receiving most votes in the primary stand against each other in the final election), super-majority rule (winner receives two-thirds of votes, as with constitutional amendments), sub-majority rule (party receiving, say, 10 percent of votes or more enjoys financial aid from state), pure proportional representation (parties receive seats in legislature in proportion to popular vote), and minimum proportional representation (parties receiving at least, say, 10 percent of the votes receive seats in legislature in proportion to popular vote). Citizens may elect the executive directly, as with presidents, or the legislature may elect the executive, as with a prime minister. Elections may occur at predetermined intervals or the executive may call elections at his discretion. Legislatures may have one house (unicameral) or two (bicameral). The legislature may amend bills proposed to it ("open rule"), or amendments may be forbidden ("closed rule"). The constitution may be explicitly written, with wide latitude for court interpretation, as in the United States, or the constitution may be unwritten, with little scope for court interpretation, as in Britain.

In spite of these differences, all elections share certain general features. This chapter abstracts from the differences and analyzes the general features of elections, proceeding along lines successfully applied to markets. Competition among firms seeking to satisfy consumers determines prices in a market. Similarly, competition among candidates seeking to satisfy voters determines public policies in a democracy. To develop this approach, I first explain how economics models the choice of voters among candidates (demand), and then I explain how candidates choose strategies to win elections (supply).

INDIVIDUAL VOTING

Citizens face several decisions in connection with voting. First I will assume that a rational person decides to participate in a vote and I will analyze how that person will vote. Second I will analyze how a rational citizen will decide whether to bother participating in a vote, and whether to abstain in voting on a particular issue. Third I will explain how to represent a voter's preferences by a utility function.

How to Vote: Self-Interest or Public Interest?

I like ice cream better than cabbage because of the taste, he likes San Diego better than Seattle because of the weather, and she likes the Republicans better than the Democrats because she is conservative. Among the many reasons that people have for their preferences, I will contrast two broad types. On the one hand, a citizen can vote based on material self-interest. A narrowly self-interested voter asks, "Which candidate will do more to increase my own wealth and power?"On the other hand, a public-interested voter asks, "Which candidate will benefit the country more according to my political philosophy?"

To supply efficient quantities of public goods, officials need information about the policy preferences of citizens. By supplying this information, self-interested voting sometimes promotes efficiency in the supply of public goods. All too often, however, citizens use politics to obtain advantages for themselves at the expense of others. The aim is redistribution, not efficiency. Thus banks want loan guarantees, farmers want price supports, unions want tariffs, artists want subsidies, taxis want fewer licenses for cabs, the elderly want property tax exemptions, and so forth. This kind of self-seeking wastes resources and oppresses the powerless. While people seldom criticize a consumer in the grocery store for following his self-interest when filling his shopping cart, people often criticize citizens for voting their self-interest.

Do most citizens vote their self-interest or the public interest? The determinants of voting behavior have been studied for many years. Survey research reveals that voters know little about issues or candidates, so they typically rely on guidance from political parties, ideology, and informed friends or associates. In spite of their ignorance, however, citizens tend to vote for candidates who promote the interests of the groups to which they belong. For example, farmers tend to vote for candidates who subsidize agriculture, ethnic groups tend to vote for candidates who benefit minorities, and investment bankers tend to vote for candidates who liberalize finance (Campbell et al. 1960).

Supporting candidates who advance a group's interests can benefit a person by showing solidarity with its members.[4] To illustrate, dairy farmers in a rural community may be more willing to cooperate with other dairy farmers who endorses milk subsidies. Conversely, an ethnic group may censor members who

---

[4] *Posner* forthcoming emphasizes this mechanism for creating social norms.

oppose preferential treatment for minorities.[5] In general, groups develop ideologies that advance a self-serving conception of the public interest, like the automobile worker who believes that "what's good for General Motors is good for America."

Assume that a political pollster asks me to rank three alternative political platforms by assigning the letter A to the platform that I like best, B to the middle platform, and C to the platform that I like least. Pollsters seldom ask whether self-interest or the public interest motivates my ranking. A utility function can represent a person's ranking of alternatives. The ability of a social scientist to represent preferences by a utility function has nothing to do with whether self-interest or a conception of the public interest generates the preferences. Later I will sketch a way to represent preferences by a utility function that applies to many types of voters.

> *Question:* Machiavelli's book *The Prince* scandalized sixteenth-century Europeans by describing immoral methods by which princes can enhance their power. Similarly, collective-choice theory scandalizes some people today by explaining the logic of self-interest in democratic politics. Why do you think the assumption of self-interest in politics troubles people so much? Discuss some of the advantages and disadvantages of using this assumption about motivation to explain political behavior.

Why Vote?

Journalists often deplore the fact that only about half of the eligible citizens vote in major U.S. elections, and participation has fallen since the nineteenth century.[6] Voter participation rates are similar in other countries, except where democracy is new, the law compels citizens to vote as in Australia and Argentina, or the names of nonvoting citizens are posted in public as in some Italian towns. Unlike journalists, however, economists find voter participation rates mysteriously *high*. Models of self-interest predict much lower voter participation rates than actually occur, and here is why. A self-interested citizen will decide whether or not to vote by comparing the cost of voting and his expected benefit. Given current rates of voter participation, the probability is negligible that a single vote in a large election will effect the outcome. So the effort required to vote exceeds the expected benefit for voters in large elections.

Some notation clarifies this point. The value of the time required to vote usually measures its opportunity cost, which I denote C_i for citizen i. For simplicity, assume that the citizen cares about who wins the election, not the margin of victory. Let p_i denote the probability that citizen i's vote decides the election's outcome. Let B_i denote the increase in citizen i's wealth or power obtained by

---

[5] For the dynamics of "ethnification," see Kuran 1997.

[6] Bumper sticker on pickup truck in Berkeley: "If God had intended us to vote, He would have given us candidates."

getting his preferred outcome in the election.[7] Thus the expected benefit from voting equals p_iB_i. According to the *self-interested theory of voter participation*, a citizen votes when $p_iB_i \geq C_i$, and a citizen does not vote when $p_iB_i < C_i$. The self-interested theory of voting predicts that voter participation rates will fall until p_iB_i approximately equals C_i. The *paradox of voting* refers to the fact that current levels of voter participation far exceed the rate at which p_iB_i equals C_i. If the self-interested theory of voting accurately described the behavior of most citizens, voter participation rates would fall far below current levels.

To illustrate, assume that having your preferred candidate win the election is worth \$1,000 to you. Assume that voting requires one hour of your time, which you value at \$10. Self-interest prompts you to vote if $p_i\$1,000 \geq \$10$, which implies $p_i \geq 1/100$. In large elections, the probability of any one vote being decisive is *much* smaller than 1/100. Computing the subjective probability of being decisive p, which is called the *power of a vote*, depends on what the voter thinks other voters will do (Palfrey and Rosenthal 1985). According to one calculation, the power of a vote in a typical U.S. general election approximately equals 10^{-8}.[8] Under any reasonable assumptions, the power of a vote is so small in a large election that purely self-interested citizens would not bother to vote at current rates.

If narrow self-interest does not explain why people vote at current rates, what does? An important tradition in political theory dating from Aristotle holds that political participation appeals to the social nature of people. According to this tradition, people express themselves by performing civic duties, and self-expression is intrinsically satisfying.[9] Deliberative theories of democracy stress the satisfaction that people take in exercising the responsibilities of citizenship, such as voting.

By voting rather than not voting, I increase the probability that people who agree with my politics will like the election's results. So people who agree with my politics will say that I ought to vote. The fact that citizens often praise voters and criticize nonvoters indicates the existence of a social norm. Besides self-expression, people may vote to obtain praise or avoid criticism from others.

To represent the influence of civil duty, let v_i denote the value to i of fulfilling i's civic duty, where v_i is large for some people and small for others. According to the civic virtue theory, everyone votes whose value v_i outweighs the net cost $C_i - p_iB_i$. Thus citizens vote when $v_i \geq C - p_iB_p$.[10] This formula encapsulates

---

[7] To illustrate, in a vote between a Republican and Democratic candidate, the benefit B_i of a Republican voter i equals $u^i(x_r) - u^i(x_d)$, where "u" is willingness to pay.

[8] See discussion in Hasen 1996. Using a different method of calculation, Romer 1996 concludes that the probability of a tie in a U.S. presidential election in which fifty million people vote is approximately 10^{-4} (p. 200).

[9] Expressive voting theory is explored in Brennan and Lomasky 1993.

[10] Let $f(v, b)$ denote the density function representing the distribution of social value v and material benefit b among citizens. The total number of voters in an election, according to this theory, equals the sum of all the voters for whom v exceeds $C - pB$, or voter participation $= \int_{C-pB} f$.

a *mixed-motive theory of voting*, which combines self-interest and civic duty. The mixed-motive theory has testable implications.[11]

Questions

1. Use the concept of the "power of a vote" to explain why self-interested people would not allow voter participation rates to approach zero.

2. Predict the conditions under which a social norm requiring voter participation would be effective.

3. "By not voting I increase the power of everyone else's vote. Therefore, not voting is a kindness to others that should be encouraged. There is no civic duty to vote." Is anything wrong with this argument?

Ignorance and Abstention

Sometimes a rational person abstains from voting even though participation costs nothing. Rational abstention depends on who knows what. To understand rational abstention, assume that you are a member of a law faculty that must vote on whether or not to offer a job to a particular applicant. Your faculty follows a procedure of majority rule, with the chairman breaking ties. You ask yourself whether you should vote or abstain. If you vote, your vote will be either indecisive or decisive. If your vote will be indecisive, then voting or abstaining does not affect the outcome. For example, if six colleagues vote "yes" and four colleagues vote "no," then your vote will be indecisive.

If your vote will be decisive, then you will determine the outcome by voting, or, by abstaining you will allow the chairman to determine the outcome. For example, if five colleagues vote "yes" and five colleagues vote "no," then your vote will break the tie or your abstention will permit the chairman to break the tie. So you should decide whether to vote or abstain by asking whether you prefer to decide the outcome yourself or have the chairman decide it.

Two considerations should guide this decision: information and values. If you know more than your chairman knows about the issue, then you should vote. If your chairman knows more than you know about the issue, and if your chairman has the same values as you, then you should abstain. The hard choice comes when your chairman knows more than you know about the issue, and your chairman's values differ substantially from yours. Here you must balance information and values in deciding whether you prefer that the chairman or you determine the outcome of the vote.

To illustrate a hard choice, assume that your chairman can evaluate the job candidate's scholarship in constitutional law better than you. If the only issue

[11] As with self-interested theory, mixed-motive theory predicts that voter participation should increase when the power of a vote p increases, the private material benefit B_i from winning the election increases, or the opportunity cost of voting C_i decreases. In addition, the mixed-motive theory predicts that voter participation increases when the value of conforming to the social norm v_i increases. It might increase because more people internalize civic virtue, the social advantage from political participation increases, or the social cost from not voting decreases.

were constitutional scholarship, then you would abstain. Another issue, however, is ethnic diversity on the faculty. Everyone on the faculty has the same information about the ethnicity of job candidates, but colleagues disagree about its importance. Compared to the chairman, you put more weight on ethnic diversity and less weight on constitutional scholarship. If the only issue were ethnic diversity, then you would vote. Here you must balance the chairman's superior information against your disagreement with the chairman over values.

This analysis shows why ignorance about candidates or issues may cause rational voters not to participate in elections. The logic of nonparticipation follows the logic of abstention. If the citizen's vote will be indecisive, then voting or abstaining does not affect the outcome. If the citizen's vote will be decisive, then the citizen will determine the outcome by voting. Alternatively, by abstaining the citizens will make another voter decisive in determining the outcome. Call this person the *next decisive* voter. A rational citizen will decide whether or not to vote by asking whether he prefers to decide the outcome or have the next decisive voter decide.

The next-decisive-voter theory explains why rational, civic-minded citizens might not participate in elections. The case for rational nonparticipation by a citizen is strongest when the next decisive voter has similar values and better information. The next-decisive-voter theory predicts that participation rates will fall as values become more homogeneous (the distribution of values compacts) or information becomes more heterogeneous (the distribution of information spreads). The next-decisive-voter theory also predicts that people who abstain have less political information on average than people who vote.

In the past, many citizens could not choose to vote because incipient democracies restricted voting by gender, race, class, and property ownership. In modern democratic states, however, every adult citizen typically enjoys the right to vote. Outside of the state, voting restrictions remain important in some organizations. For example, corporations typically allocate votes to owners in proportion to their shares, cooperatives typically allow one vote per member, and homeowners associations or business improvement districts typically restrict voting to property owners (Ellickson 1998). I will develop a theory of elections that predicts outcomes from the preferences of voters, regardless of the extent of the franchise.

Questions

Feral cats prompt your town to elect a Cat-Catcher. When you come to vote in the general election, you scan the list of candidates for Cat-Catcher and realize that you know little about them. Describe how you might rationally decide whether to vote or abstain. Why might a rational citizen prefer to cast a *blank* ballot in an election instead of not participating?

Representing a Voter's Preferences

Imagine a simple electoral contest with two viable candidates, say, the nominees of the democratic and republican parties. In the election campaign, each

candidate announces a platform that describes his position on the major issues. The platform encompasses the candidate's general ideology and specific policies on such matters as subsidies, tax relief, and regulations. In response, each citizen votes for the candidate whose platform conforms closest to his political preferences. The candidates understand these facts. Consequently, each candidate tries to find a platform that will command a majority of votes against the opposition's platform.

Some notation facilitates discussing this model. Specifically, let x_r denote the platform announced by the first candidate (republican), and let x_d denote the platform announced by the second candidate (democratic). Let n denote the number of citizens who will vote in the election. Each citizen ranks the possible platforms from best to worst. The ranking of platforms by any individual, say the ith individual, is indicated by a utility function. Higher values of the utility function indicate a higher ranking for the political platform. Thus, the utility value of platform x_1 to citizen i is $u^i(x_1)$, and the utility value of platform x_2 to citizen i is $u^i(x_2)$. If citizen i prefers x_1 to x_2, then the utility value of the former exceeds the utility value of the latter: $u^i(x_1) > u^i(x_2)$.

Each citizen is assumed to vote for the candidate with the preferred platform. To illustrate, many U.S. elections offer a choice between a Democratic and a Republican candidate. The utility $u^i(x_r)$ is the one that the ith citizen expects to enjoy by electing the Republican, and $u^i(x_d)$ is the utility that he expects from electing the Democrat. Here is the ith citizen's voting rule:

$$u^i(x_r) > u^i(x_d) \Rightarrow \text{citizen i votes Republican}$$

$$u^i(x_r) < u^i(x_d) \Rightarrow \text{citizen i votes Democratic}$$

$$u^i(x_r) = u^i(x_d) \Rightarrow \text{citizen i votes by flipping a coin.}$$

In deciding how to vote, all n citizens follow the same procedure as citizen i, except the utility functions are different for different people.

In this model, each side announces its program to the public and the winner in the election imposes its political platform upon everyone. In contrast, a consumer in the grocery store fills his shopping cart with goods for his own private use. Political platforms especially concern public goods such as expenditures on parks and the military, not private goods such as expenditures on ice cream and carrots. For public goods, the state supplies one quantity for everyone. This fact about public goods has consequences that I discuss several times in this book, especially in chapter 5.

A public good is indicated mathematically when the same variable enters the utility functions of different people. To illustrate, assume that two individuals, denoted i and j, have utilities u^i and u^j. If the variable x enters both their utility functions in the same quantity, as in $u^i(x)$ and $u^j(x)$, then x has the mathematical character of a public good. To illustrate, x might denote state expenditures on military defense. If, however, each person enjoys different quantities of the variable x, which can be denoted $u^i(x_i)$ and $u^j(x_j)$, then x has the mathematical character of a private good. To illustrate, $u^i(x_i)$ might denote person i's utility from consuming x_i pints of ice cream.

In this sketch of a voting model, political positions determine votes. In addition to political positions, a candidate's appearance, personality, or other personal attributes often sway voters. Furthermore, the candidates have to communicate with voters, which involves costly advertising and raises problems of credibility.[12] Besides positions on issues, the framing of issues also influences voting. To illustrate, framing affirmative action as a racial preference or nondiscrimination affects the response of Californians to it. Similarly, framing social security benefits as welfare or entitlement affects the response of citizens to it. These complications are best omitted in the initial stage of developing a theory of elections, which is my next topic.

AGGREGATING VOTES

Having analyzed how citizens decide whether to vote and how to vote, the next step is to explain how candidates choose their platforms to win elections. I begin with a simple, intuitive model of the central tendency in democratic elections.

Median Rule

Under certain conditions, electoral competition causes party platforms to converge toward the center of the distribution of political sentiment. To be more precise, the winning platform in certain conditions is the one favored by the citizen who is the median in the statistical distribution of political sentiment (Black 1958). This conclusion corresponds to the familiar fact that the candidates in U.S. presidential elections tend to adopt moderate positions on the political spectrum (Downs 1957).

The median tendency in democracy can be illustrated by using an example in which there are three voters, denoted A, B, and C. Figure 2-1 depicts their preferences, possibly as determined by a poll. Consider the change in A's utility, denoted $u^a(x)$, when moving from left to right in figure 2-1. A's utility increases when moving right in the direction of x_a^*. After passing x_a^*, A's utility decreases when moving further to the right. A prefers x_a^* the most because the utility curve $u^a(x)$ achieves its highest point when the platform is x_a^*. Similarly, the most preferred platform for voters A, B, and C are denoted x_a^*, x_b^*, and x_c^*, respectively.

Assume that two candidates compete for votes of the three citizens. To keep the analysis simple, I assume that no one abstains and all three voters have complete information. Each candidate must choose a political platform, and then each citizen votes for the candidate whose platform yields higher utility. It is not hard to see that in this three-voter example, the platform x_b^* will beat any other platform.

To see why, assume that x_b^* is chosen by the democrat and the republican chooses any platform located a little farther to the right. Voters A and B will get

---

[12] For an interesting model of political signaling through advertising, see Dharmapala 1998.
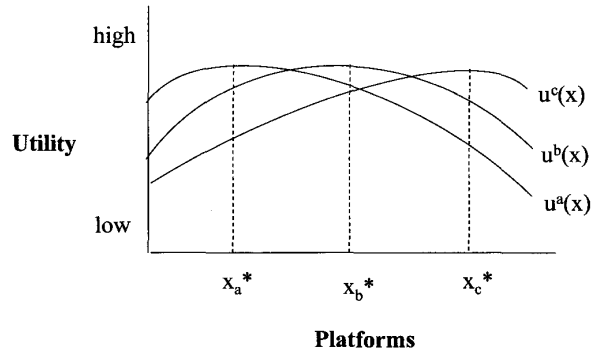
Platforms

Fig. 2-1 Platforms and Preferences

more utility from the democratic platform than from the republican platform, whereas C will get more utility from the republican platform, so the democrat will win by a 2 to 1 majority.

Reversing the example, assume that the republican chooses the platform x_b^* and the democrat chooses any platform a little further to the left. B and C will get more utility from the republican platform, whereas C will get more utility from the democratic platform, so the republican will win by a 2 to 1 majority. Thus, the party that discovers and announces platform x_b^* is unbeatable in the election.

In this three-person example, the winning platform x_b^* is the one most preferred by B. Notice that B is in the middle of the distribution of preferences in the sense that one voter's most preferred point lies to the right and one voter's most preferred point lies to the left. In general, when there are many individuals, rather than just three, the median is defined as the individual with an equal number above and below. For any odd number of n individuals, the median has $(n - 1)/2$ voters to the left and $(n - 1)/2$ voters to the right. When voters have preferences like those in figure 2-1, the winning platform is the one most preferred by the median voter. Since this platform defeats every alternative, it is the political equilibrium in the electoral competition. The actual winner in the election depends on which candidate has the information and opportunity to choose the equilibrium platform.[13]

The median rule explains the central tendency in some political systems. For example, many Americans can locate themselves along a simple left-right continuum, with "liberal Democrat" at one end and "conservative Republican" at

---

[13] If the candidates must commit to a platform with imperfect information, each one will make a guess about the dominant platform. If the candidates know the dominant platform and one candidate chooses the platform before the other (e.g., the incumbent chooses first), then the candidate who chooses first will win. If both candidates know the dominant platform and they choose simultaneously, both will choose the same platform, in which case voters will be indifferent over who wins.

the other. A common pattern in U.S. presidential campaigns is for the Republican candidate to take a position on the right wing in the primary elections when seeking the nomination and, once nominated, to move nearer to the middle of the political spectrum. The initial right-wing position appeals to the median voter in the Republican Party, as required to secure the party's nomination, and the moderate position appeals to the median voter among all the citizens, as required to win the general election. Similarly, Democratic Party candidates often start from the left in the primaries and move toward the middle after nomination.

Note that the median rule assumes that voters disagree, but it makes no assumptions about the cause of the disagreement. Some citizens try to advance their self-interest and others may try to advance a conception of the public interest. Their reasons for ranking political alternatives do not affect the scope of the median rule.

The median-voter model leaves out important features of real elections, such as party loyalty, voter ignorance, campaign spending, and personal appeal of candidates. Despite these omissions, the median rule is a useful starting point for a theory of electoral competition.

Questions

1. Suppose that left-wing voters become so filled with righteous anger at their political choices that they boycott a general election and do not vote. In which direction will their behavior shift the winning platform?

2. Explain why the median rule assumes disagreement, but not necessarily self-interested voting.

3. There are three voters (A, B, and C) and three alternatives (x_1, x_2, x_3). The voters rank the alternative from 1 to 3, with "3" indicating the most preferred alternative and "1" indicating the least preferred:

$$\text{person A: } 3 = u^a(x_1), \quad 2 = u^a(x_2), \quad 1 = u^a(x_3)$$
$$\text{person B: } 3 = u^b(x_2), \quad 2 = u^b(x_1), \quad 1 = u^b(x_3)$$
$$\text{person C: } 3 = u^c(x_3), \quad 2 = u^c(x_2), \quad 1 = u^c(x_1).$$

Which alternative wins a majority in paired voting against both of the others? Who is the median voter?

4. Three voters have the following utility functions:

$$\text{person A: } u^a = 2 + x$$
$$\text{person B: } u^b = 2 + 2x - x^2$$
$$\text{person C: } u^c = 3 - .5x.$$

Apply the median rule to find the value of x that represents a voting equilibrium.

5. Majority rule allegedly increases the government's legitimacy and intimidates a rebellious opposition by demonstrating publicly that more citizens support

the government's policies than oppose them. Defend or criticize this proposition by using the median rule.

Legislatures and Committees

The preceding analysis of political platforms concerns a general election in which candidates try to choose the winning platform. The analysis applies equally well to legislatures or committees that follow majority rule. In any such governing body, there will be some set of policies representing the status quo. From time to time a member will make a new proposal. After debate, the body will vote on the new proposal. If the new proposal fails to gain a majority, the status quo will persist. If the new proposal gains a majority, the group abandons the old status quo and the winning proposal becomes the new status quo. Future proposals may challenge the new status quo.

For purposes of formal analysis, there is an exact translation from platforms in a general election to proposals in a legislature or committee. Each proposal is pitted against the status quo. If the preferences of the legislators satisfy conditions prescribed in the median rule, the proposal most preferred by the median legislator will prevail.

Duverger's Law

In many countries like Britain and the United States, two major parties dominate important elected offices. In other countries, many parties win important offices. In addition to culture and history, the electoral procedure determines the number of parties. According to the "winner-take-all-plurality" procedure, the candidate who receives the most votes in a single election wins the office. To illustrate, if votes were divided among three candidates in the proportions 40 percent, 29 percent, and 31 percent, then the candidate receiving 40 percent wins the office. In plurality rule, there is no run-off between the two top contenders, no point voting, and no proportional representation.

Countries with the winner-take-all-plurality rule tend to have two dominant parties. This proposition is sufficiently true to be called a "law," although it is not an iron law.[14] "Duverger's Law" asserts that winner-take-all-plurality rule tends to eliminate small parties and create a two-party system. Collective-choice theory explains why rational voting produces this outcome (Riker 1982b). With several candidates, citizens vote strategically. To illustrate, if a voter's most preferred candidate has little chance of victory, the voter may opt for his second choice. Under the winner-take-all-plurality rule, citizens tend to vote for candidates whom they think others will vote for,[15] and this behavior compresses the number of viable parties to two. The equilibrium share of a third-party vote must be small in an election with many voters (Palfrey 1989).

[14] Canada and India are exceptions.

[15] Such an election resembles the beauty contest proposed by Keynes, in which the judges receive rewards for picking the winning contestant.

Coalition theory reaches the same conclusion. To see why, assume that the electorate falls into three groups of equal size called Left, Middle, and Right. If each group supports its own candidate, the probability of any one winning under the winner-take-all-plurality rule is one-third. However, if some Middle voters can be coaxed into the Left, then their combined strength will enable Left-Middle to win all the elections. Knowing this, Right will respond by coaxing some Middle voters into Right-Middle. At the end of this process, two large parties compete for the middle voters. Thus the winner-take-all-plurality rule tends to produce two dominant, evenly matched parties located near the center of the political spectrum.[16] Once established, this situation is very stable, because a vote for a third party has no probability of influencing the outcome, whereas a vote for one of the major parties could be decisive. In single-dimension voting, two parties will occupy the space of alternatives so as to preclude the entry of a successful third party (Palfrey 1984).

To make this argument precise, recall that the "power" of a vote equals the probability that it will be decisive. If the Republican and Democratic Parties are equally matched in a given election, then a change of one vote in either direction could tip the election. In contrast, a vote for a third party (say, the Libertarians or the Greens) has no prospect of changing the outcome. Thus a vote for one of the major parties has power, whereas a vote for a third party has no power.

I have explained how competition in winner-take-all elections tends to eliminate third parties. What keeps the two competing parties from merging into one grand coalition? If the parties remain separate, the winning party enjoys the spoils of power (offices, contracts, grants, etc.). If the parties merge, they must share the spoils of power with each other. Thus the desire to concentrate the spoils of power usually prevents mergers between the two dominant parties. The analysis of political coalitions in the next chapter develops this idea in detail.

In Japan, however, the desire to concentrate the spoils of power did not produce effective multiparty competition. Instead, one party (the Liberal Democratic Party or LDP) has held power during most of the second half of the twentieth century. This hegemonic party, however, contains powerful factions within it, which compete for power. The reasons why a single party dominates Japanese democracy are uncertain. Perhaps the citizens perceive that a single party can better impose political control on an exceptionally powerful administrative bureaucracy. Or perhaps a national coalition assuaged persistent fears of communism during the cold war. Or perhaps the explanation lies in Japan's special electoral rules.[17]

· [16] My informal "proof" of Duverger's Law assumes the existence of a uniquely stable equilibrium in two-party competition. A sophisticated defense of Duverger's Law is found in Palfrey 1989. A discussion of the prospects of third parties in U.S. elections is in Gardner 1980.

[17] Until electoral changes were made in 1994, each electoral district in Japan returned several representatives to the House of Representatives, but the citizens could vote for only one of them. For example, if a district had three seats, the three candidates enjoying the most votes won, and each citizen residing in the district could vote for only one candidate. Instead of favoring a single hegemonic party, however, these rules seem to favor smaller parties. See Christensen 1994 and Cox 1994. Thanks to Tom Ginsburg for these facts and citations.

Questions

1. In the United States the two major parties choose a presidential candidate by primary elections in each state, which lead up to a national convention. The Democratic Party approximately follows the winner-take-all-plurality rule in each state, meaning the candidate who gets the most votes in the state's primary gets all the state's votes at the national convention. The Republican Party, in contrast, follows a rule closer to proportional representation, meaning that the votes at the national convention are divided in proportion to the votes the candidates received in the primary election. If you were a candidate with a small group of loyal followers who wanted to influence your party's nomination, but had little chance of actually winning it for yourself, would you rather be a Republican or Democrat?

2. In the United States, Duverger's Law seems to work at the national level but not at the local level. What might explain the ineffectiveness of Duverger's Law at the local level?

Alternative Voting Rules

Winner-take-all is one great family of voting rules used by democratic states. The other great family of voting rules is proportional representation, in which each political party receives seats in the legislature in proportion to the number of votes it receives in the election. Later I discuss proportional representation in detail. For now, note that winner-take-all consolidates parties and proportional representation fragments parties. Consolidation forces voters to choose the more preferred of the two parties, even if neither party closely reflects a voter's political preferences. In contrast, fragmentation permits each voter to choose a party closely matched to his own preferences. Conversely, consolidated parties tend toward stable government, whereas fragmented parties tend toward unstable government. Chapter 4 discusses this trade-off between representation and stability in more detail.

Besides these two great families of voting rules, a few governments and many private organizations use entirely different voting rules.[18] A survey by Jonathan Levin and Barry Nalebuff distinguishes sixteen types of voting rules, each with its own procedures, results, and intellectual champions (Levin and Nalebuff 1995). Examples are the single transferable vote,[19] various forms of

---

[18] For an empirical walkabout, see A. Wombat and I. Wallaby, "The Boomerang Effect in National Elections," *Central Australian Review of Law and Economics* 79 (1994): 114-647. Thanks to Geoff Brennan for this citation.

[19] With the single transferable vote, which is used to elect the Dail (Assembly) in Ireland and the Senate in Australia, each voter casts one vote and each candidate obtaining a prescribed quota of votes is elected. Votes in excess of the quota for a winning candidate are transferred to another candidate as designated by the voter. See Tideman 1995.

point voting inspired by Borda,[20] and approval voting.[21] What difference does the voting rule make to outcomes? Levin and Nalebuff conclude that different voting rules typically give the same results when a consensus exists among voters, whereas they give different results when a consensus does not exist.

Tinkering with voting rules can change the outcomes of close elections. Given that voting rules change outcomes, which voting rule is best? Scholars disagree widely about the standard for determining the best rule.[22] Chapter 4 briefly discusses several alternatives. As scholars identify the characteristics of different voting rules, some democratic organizations may take advantage of new knowledge to tailor their voting rules for desired results.

Questions

1. Why might African Americans benefit by changing U.S. electoral rules from plurality rule to proportional representation?

2. Compared to proportional representation, winner-take-all voting causes more corruption in drawing district boundaries ("gerrymandering"). Why?

EVALUATING EQUILIBRIA

The preceding section explained that under certain conditions, majority rule favors the platform preferred by the median voter. The location of the winning platform near the center of the political spectrum dampens the influence of extremists, which stabilizes democracy. Do other attributes make majority rule desirable? Economists evaluate public policies relative to a standard of efficiency. As defined in economic models, "efficiency" requires satisfying individual preferences. To satisfy preferences, governments supply public goods and the legal framework for markets to supply private goods. Is government efficient under the median rule?

The answer depends on the *type* of efficiency. I will distinguish among several types of efficiency that play a prominent role in policy analysis.

---

[20] A comparison of point voting based on Borda and plurality voting based on Condorcet is in Young 1995. Young favors plurality voting for two reasons, especially in a sophisticated form called maximum likelihood. First, given a right alternative and several wrong alternatives, plurality-type rules maximize the likelihood of a right decision. Second, plurality rules satisfy the independence of irrelevant alternatives (unlike Borda), as well as several other desirable axioms.

[21] In approval voting, each voter indicates on the ballot whether he "approves" or "disapproves" of each candidate. The candidate receiving the greatest number of approvals is the winner. When voters have good information about how others will vote, the outcome of approval voting captures all the information about the preferences of voters. See Weber 1995.

[22] One standard minimizes the probability that the collective choice will make factual errors in judgment. Another standard minimizes the error in representing the preferences of citizens in the legislature. Still another standard ensures that an alternative that can defeat any other alternative in paired voting will win the election. Yet another standard tries to reduce strategic voting in order to minimize the misrepresentation of preferences by voters.

Pareto Efficiency

Pareto efficiency is achieved when no change can make someone better-off without making someone else worse-off. For any Pareto-inefficient political platform, at least one alternative platform exists that some voters like better and no voter likes less. Given this fact, the Pareto-inefficient platform receives less votes than the alternative platform. Since the alternative platform defeats the Pareto-inefficient platform in a vote, the Pareto-inefficient platform is not a voting equilibrium.[23] Conversely, a voting equilibrium (if it exists) is normally Pareto efficient.

These conclusions apply to the median rule as depicted in figure 2-1. To find the set of Pareto-efficient points in figure 2-1, begin at the origin of the graph, which corresponds to an extreme left-wing program, and start moving to the right along the horizontal axis. At first, all three voters prefer the move to the right. However, once the point x_a^* is reached, which is the most preferred point by the most left-wing voter, any further moves to the right make voter A worse-off. Similarly, start from the extreme right side of the horizontal axis and start moving to the left. At first, all three voters prefer the move to the left; however, after reaching the point x_c^*, any further move to the left makes voter C worse-off. Thus the set of Pareto-efficient points contains all the platforms in the interval between x_a^* and x_c^*. The median platform necessarily lies in this interval, so the median rule is Pareto efficient.

Cost-Benefit Efficiency

Most laws make some people better-off and others worse-off. Pareto efficiency provides no basis for choosing among such laws. Guiding political choices requires a more definite and controversial standard. Unlike Pareto efficiency, cost-benefit analysis commends changes for which the gains to the winners exceed the losses to the losers.[24] For example, a move from x_a^* to x_b^* in figure 2-1 harms A and benefits B and C, so the change is not an improvement by the standard of Pareto efficiency. If, however, the harm to A is less than the sum of the benefit to B and C, then the change is an improvement by the cost-benefit standard.

The median rule is not generally efficient by the cost-benefit standard. To see why, assume that a three-person committee must decide a difficult issue by majority vote. The committee agrees that each person will write his or her vote on a slip of paper. When the slips of paper are collected, the chairman reports, "I have two slips marked 'Yes' and one marked 'No, No, oh please, please No!'"

---

[23] In complex models with strategic behavior, Pareto-inefficient voting equilibria can exist.

[24] Since the winners gain more than the losers lose, the former could compensate the latter in principle. Thus an improvement by the cost-benefit standard is also a "potential Pareto improvement." The change is not an actual Pareto improvement unless compensation is actually paid. The criterion of potential Pareto improvement is also called the "Kaldor-Hicks" criterion, after the two economists who developed the idea. For a discussion of these concepts with application to law and economics, see Coleman 1980.

high ┤

Utility

low ┤

Platforms

x_a^* x_b^* x_c^*

$u^c(x)$
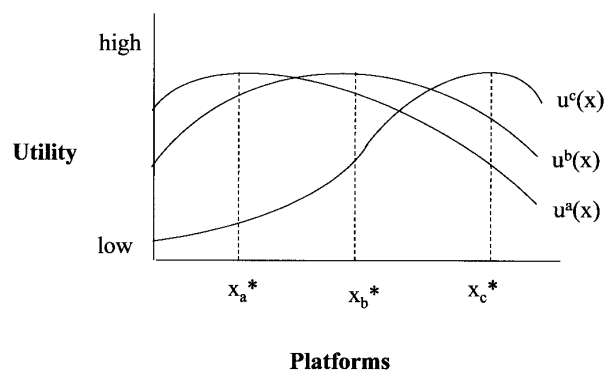
$u^b(x)$

$u^a(x)$

Platforms

Fig. 2-2 Intensity and Median Rule

Apparently two people favor the proposal and one person adamantly opposes it. In general, voting does not reflect the intensity voters feel toward issues. The intensity of feeling, however, influences the efficiency of the alternatives. The unresponsiveness of majority rule to the intensity of feeling about issues causes its inefficiency.

To illustrate the inefficiency graphically, assume that C's preferences in figure 2-1 shift down in the vicinity of x_a^* and x_b^* as depicted in figure 2-2. As a result of this change, C intensely dislikes left and moderate policies as depicted in figure 2-2. Unlike C, A and B have not changed their preferences. An efficient platform responds to shifts in sentiment, so efficiency requires the voter equilibrium to shift to the right.[25] However, the median platform, which commands a majority against any other platform, remains unchanged at x_b^* when passing from figure 2-1 to figure 2-2. Unlike the efficient platform, the median generally does not respond to changes in the intensity of sentiment in the wings of the distribution.

I have explained that the median rule is not generally cost-benefit efficient. Under a special assumption, however, it is. Majority rule counts voters, whereas cost-benefit analysis adds individual values. Counting voters gives the same result as adding individual values under the assumption of "strong symmetry." Under strong symmetry, each voter who gains from a change away from the

---

[25] This argument relies on the usual kind of marginalist reasoning found in economics. For an efficient platform, the benefits enjoyed by the winners from any small shift in the platform equals the harm suffered by the losers (marginal benefit = marginal cost). In comparing figures 2-1 and 2-2, the change in C's preferences, without any change in the sentiments of A and B, implies that the marginal benefit from shifting the platform a little to the right of x_b^* has increased, whereas the marginal cost remains unchanged. Therefore, the efficient platform must shift to the right as a result of the change in C's preferences. You can check this fact by sketching a curve equal to the sum of the utilities of the three people in figure 2-2.

median can be matched with at least one voter who loses, and the loser loses no less than the winner wins.[26]

To illustrate strong symmetry, consider a move from the median to the right. For each right-wing voter who gains, there must exist a left-wing voter who loses just as much. In figure 2-1, a move from x_b^* toward x_c^* benefits C and harms A. Strong symmetry implies that the loss to A is as large as the gain to C.

The requirement of symmetrical effects can be expressed in terms of the intensity of preferences. Democracy gives equal weight to all votes, regardless of how strongly the voters feel about the issues. From an efficiency perspective, however, more weight should be given to intensive preferences. When the distribution of political sentiment is strongly symmetrical, the intensity of right-wing feeling offsets the intensity of left-wing feeling, and vice versa.

Strong symmetry is rare in fact, but approximate symmetry is not so rare. To see why, consider the relationship between the total benefits and the mean benefits. By definition, the total benefits equal the mean benefits multiplied by the number of voters. Consequently, for a given number of voters, maximizing total benefits for all voters is equivalent to maximizing mean benefits.[27] In asymmetrical distributions, the mean and the median are different. In symmetrical distributions, such as the normal distribution, the mean and the median are identical. As the distribution of the voters' most preferred points becomes more symmetrical, the median approaches the mean. As the median approaches the mean, the voter equilibrium tends to become cost-benefit efficient. (Additional conditions are necessary to assure this result.[28])

Since many distributions are symmetrical, or nearly symmetrical, the mean and median are usually close together. Consequently, the winning platform in electoral competition is usually close to the efficient platform, at least in the stylized world discussed so far in this chapter. This fact provides a justification for majority rule as opposed to alternative procedures. *Super-majority rule* requires more than a majority—say, two-thirds—to enact a bill. Conversely, *sub-majority rule* requires less than a majority—say, one-third. Assuming symmetrical effects,

---

[26] In notation, let x_m^* denote the point most preferred by the median voter. Consider any alternative x^*. Let J denote the set of individuals who (strongly) prefer x_m^* to x^*, and let K denote the set of individuals who (weakly) prefer x^* to x_m^*. By strong symmetry, for each k in K there exists a j in J such that $u_j(x_m^*) - u_j(x^*) \geq u_k(x^*) - u_k(x_m^*)$. This fact implies

$$\sum_{i \in J \cup K} u_i(x_m^*) \geq \sum_{i \in J \cup K} u_i(x^*).$$

[27] Here we use the proposition that if continuously differentiable concave function $f(x)$ achieves its maximum at a value x^*, then the function $k \cdot f(x)$, where k is a constant, also achieves its maximum at the same point x^*. (If you know calculus, proof this proposition by taking the derivative and setting it equal to zero.)

[28] The "distribution" in this paragraph refers to the most preferred points of the voters. "Strong symmetry" concerns the utility functions of each individual. The "additional conditions" concern the relationship between utilities and most preferred points. Specifically, the representation of utilities must reduce to the representation of most preferred points.

majority rule is more efficient by the cost-benefit standard than rule by a super-majority or a sub-majority.

This discussion of efficiency measures costs and benefits relative to voters. Some citizens do not vote. How does voter participation affect this conclusion? What happens to the result when some people do not vote? If voters are a representative sample of all citizens, then the electoral outcome remains the same. To illustrate, Flemish and French speakers in Belgium often disagree about politics. If voter participation rates are the same in both language groups, then election results will be the same when 60 percent of the citizens vote as when 100 percent vote.

Conversely, if participants are a biased sample of all citizens, then voter participation rates change outcomes. To illustrate, if voter participation rates are 65 percent among the Flemish and 60 percent among the French, then election results in Belgium favor the Flemish compared to a situation where 100 percent of the citizens voted. Under median rule, biased participation causes inefficiency. To illustrate by an extreme example, North Carolinians cannot vote for the governor of Virginia, so a Virginia governor might benefit Virginians even at a large cost to North Carolinians. More generally, legislators vote on bills in representative democracy, but the benefits of the bills to legislators do not necessarily align with the benefits to citizens.

Questions

1. Compare attitudes of citizens toward military expenditure and abortion. In which case are preferences more likely to be strongly symmetrical?

2. According to contemporary surveys, a right-wing minority of American voters wants to outlaw abortion, and a left-wing a minority wants to outlaw the death penalty. Assume that each minority has very intense feelings. On cost-benefit grounds, would it be better for the minority to get its way on both issues or for the majority to get its way on both issues?

3. Among philosophers, "imperfect duty" means a duty to do an act some-times but not always. For example, a person with money may have the duty to give to some poor beggars but not to all poor beggars. Compared to 100 percent voter participation, participation by a representative sample of voters leaves the outcome unchanged and imposes the burden of voting on fewer people. From this fact, make an argument that voting is an imperfect duty of citizens.

4. Suppose that a beach that fills up with sunbathers on a warm Sunday afternoon. The sunbathers space themselves evenly such that the density of people is about the same everywhere on the beach. Two vendors with ice cream carts appear at the beach. The beach is one unit long and each vendor wants to choose a location for her cart that will maximize sales. The Parks Commission sets the price of ice cream. The hot sun makes people want ice cream and it also makes them reluctant to walk far to get it. If the vendors

are strictly competitive with each other and do not cooperate together, where will they locate? Why is this location inefficient by the cost-benefit standard?

5. Recall this question from above:
"Three voters have the following utility functions:

$$\text{person A: } u^a = 2 + x$$
$$\text{person B: } u^b = 2 + 2x - x^2$$
$$\text{person C: } u^c = 3 - .5x.$$

Apply the median rule to find the value of x that represents a voting equilibrium." Now suppose that C acquires an intensive dislike for large values of x, so that C's revised utility function becomes $u^c = 3 - x$. What is the voting equilibrium?

Welfare Analysis

Cost-benefit analysis gives equal weight to net benefits for everyone, regardless of income or wealth. Now I turn to another concept of efficiency that gives different weight to the net benefits of different people. When evaluating investment projects, the World Bank sometimes gives extra weight to the net benefits of very poor people. Weighting net benefits inversely by the income or wealth of the recipients is sometimes called "welfare analysis." The rationale underlying welfare analysis is that an extra dollar spent by the rich on opera tickets increases welfare by a smaller amount than an extra dollar spent by the poor on bread. (So why do many countries subsidize opera and not bread? I discuss the answer in chapter 10.)

The citizens in democratic countries vigorously debate whether or not the state should redistribute income from the rich to the poor. Libertarians typically oppose redistribution and socialists typically favor it. Consequently, libertarians often approve the policies favored by Pareto efficiency or cost-benefit analysis, whereas socialists often approve the policies favored by welfare analysis.

I characterized conditions under which majority rule maximizes the net benefits of voters. When does majority rule maximize the *welfare* of voters? I have no precise answer to this question. A democracy presumably creates a welfare state when the median voter believes that he will gain from it. Perhaps majority rule maximizes welfare when the median voter tries to maximize his welfare.

I distinguished three types of efficiency—Pareto, cost-benefit, and welfare. Applying each standard requires different amounts of information. Applying the standard of Pareto efficiency requires information about the preference orderings of individuals. Political polls provide reliable information of this kind. Cost-benefit analysis requires information about each person's willingness to pay for public goods. Cost-benefit techniques can often extract the necessary information from different kinds of data, but extraction is often difficult. Welfare analysis requires a set of weights for each class of people. Besides problems

of information, choosing a set of weights provokes disputes about social values. I clarify these three differences in the appendix to this chapter where I explain three types of utility functions.

Questions

1. Economic efficiency can mean Pareto efficiency, maximizing net benefits, or maximizing welfare. Is one concept more scientific than the others, or are all three equally scientific?

2. There are three voters (A, B, C) and three alternatives (x_1, x_2, x_3). The voters rank the alternatives from 1 to 3, with "3" indicating the most preferred alternative and "1" indicating the least preferred:

$$\text{person A:} \ 3 = u^a(x_1), \quad 2 = u^a(x_2), \quad 1 = u^a(x_3)$$
$$\text{person B:} \ 3 = u^b(x_2), \quad 2 = u^b(x_1), \quad 1 = u^b(x_3)$$
$$\text{person C:} \ 3 = u^c(x_3), \quad 2 = u^c(x_1), \quad 1 = u^c(x_2).$$

   a. Which alternative is the voter equilibrium in paired voting?
   b. Which alternatives are Pareto efficient?
   c. Which alternative yields the highest sum of utilities?

3. Assume that B in the preceding problem acquires an intensive dislike for alternatives x_1 and x_2. To indicate this fact, rescale his utility as follows:

$$\text{person B:} \ 3 = u^b(x_2), \quad .2 = u^b(x_1), \quad .1 = u^b(x_3).$$

   a. Which alternative is the voter equilibrium?
   b. Which alternatives are Pareto efficient?
   c. Which alternative maximizes the sum of utilities?

No Equilibrium

My discussion of the median rule depicted electoral competition with a unique, stable equilibrium. A situation can arise, however, in which a political equilibrium does not exist. To appreciate intransitive cycles, the reader may recall a childhood game called "rock, paper, scissors." In this game, two players simultaneously thrust forward one hand in the shape of a rock (fist), a piece of paper (flat hand), or scissors (two fingers extended). The rules of the game are "rock breaks scissors," "scissors cut paper," and "paper covers rock." Each choice defeats one alternative and loses to the other. The best strategy for each player, assuming his opponent is fully rational, is to choose randomly among the three alternatives. Chance decides the game's outcome.

Like the child's game, there is sometimes no equilibrium in electoral competition. When there is no equilibrium, politics spins its wheels. Each time new officials are elected they undo the policies of their predecessors.
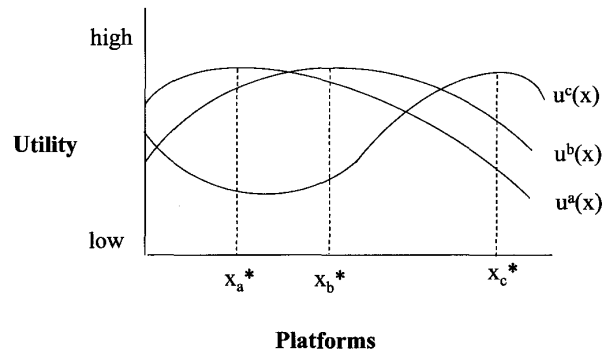
Platforms

Fig. 2-3 Intransitive Preferences

Spinning Wheels

To illustrate the absence of equilibrium, the preferences of C in figure 2-1 have been modified to yield figure 2-3. The preferences of A and B remain unchanged. The preferences of the three voters in figure 2-3 can be summarized as follows, where ">" means "preferred":

$$A: x_a > x_b > x_c$$
$$B:\quad x_b > x_c > x_a$$
$$C:\quad\quad x_c > x_a > x_b$$

Voting among these three alternatives yields the following outcomes:

$$x_a \text{ defeats } x_b$$
$$x_b \text{ defeats } x_c$$
$$x_c \text{ defeats } x_a.$$

Majority voting thus runs in a circle.

Examples of intransitive political preferences are easily constructed. To illustrate, consider these alternative levels of expenditures on public schools:

$$x_a^* = \text{low}$$
$$x_b^* = \text{moderate}$$
$$x_c^* = \text{high.}$$

There are three groups of voters of equal size. The conservative group prefers less expenditure on public schools rather than more. The moderate group prefers an intermediate level of expenditure. Finally, a third group of voters—call them

the "aspiring-to-be rich" (known in the United States as yuppies, or young urban professionals)—have more complicated preferences. They would most prefer a high level of expenditure, in which case they will send their children to public school, but, if the level is not high, they would prefer it to be low, in which case they will send their children to private school. The worst alternative for them is a moderate level of expenditure on public schools. Letting ">" indicate "preferred," the preference rankings of the three groups are:

$$
\begin{aligned}
\text{conservative}: \quad & x_a^* > x_b^* > x_c^* \\
\text{moderate}: \quad & x_b^* > x_c^* > x_a^* \\
\text{yuppy}: \quad & x_c^* > x_a^* > x_b^*.
\end{aligned}
$$

In a majority vote, x_a^* defeats x_b^*, x_b^* defeats x_c^*, and x_c^* defeats x_a^*, so the outcome is intransitive.[29]

The cause of intransitivity can be clarified with the help of some technical terms. In figure 2-3, the conservative corresponds to A, the moderate corresponds to B, and the yuppies correspond to C. For the conservative and moderate, the graph forms a hill with a single peak. The preferences of the yuppy, however, resemble a valley with the bottom at x_b^* and with peaks at x_a^* and x_c^*. The sides of a valley are higher than its interior, so these preferences have a double peak. The median rule applies whenever preferences have a single peak, but not necessarily when they have a double peak. Strictly speaking, a sufficient condition for the most preferred point of the median voter to be a unique equilibrium in majority voting over paired alternatives is that everyone's preferences have a single peak, whereas a necessary condition for intransitivity is the presence of preferences with multiple peaks.

Consider the application of these results to elections for the legislature. Assume that two candidates for the same seat must pick a platform, and assume the preferences of voters form an intransitive cycle over platforms under majority rule. The two candidates are, in effect, playing rock, paper, and scissors. If they choose platforms simultaneously, luck determines the outcome. If one chooses before the other, the party who chooses second will always win. This fact can disadvantage the incumbent.[30]

---

[29] In this example, majority rule is "intranstive." A relation R is transitive by definition if, for any three variables x, y, and z, the following condition holds:

$$xRy \ \& \ yRz \Rightarrow xRz.$$

An intransitive relation is one that is not transitive, i.e., there exist three variables x, y, and z such that

$$xRy \ \& \ yRz \ \& \ zRx.$$

[30] The incumbent's platform may be known from his past acts. If preferences are intransitive, a platform exists that voters would prefer to the incumbent's platform. The challenger, who is free to make a fresh choice, can adopt one of these winning programs.

Questions

1. An election pits an incumbent against a challenger. Assume that the preferences of voters form an intransitive cycle under majority rule. Neither candidate is committed to a program at the commencement of the campaign. Would you advise your candidate to profess platitudes or take a firm stand on the issues?

2. Recall the beach example: A beach fills up with sunbathers on a warm Sunday afternoon. The sunbathers space themselves evenly such that the density of people is about the same everywhere on the beach. The hot sun makes people want ice cream and it also makes them reluctant to walk far to get it. Now suppose that three vendors with ice cream carts appear at the beach. The beach is one unit long and each vendor wants to choose a location that will maximize sales. If the vendors are strictly competitive with each other and do not cooperate together, where will they locate? (Hint: Intransitivity gives the answer.)

3. There are three voters (A, B, C) and three alternatives (x_1, x_2, x_3). The voters rank the alternatives from 1 to 3, with "3" indicating the most preferred alternative, and "1" indicating the least preferred alternative:

$$\text{person A: } 3 = u^a(x_1), \quad 2 = u^a(x_2), \quad 1 = u^a(x_3)$$
$$\text{person B: } 3 = u^b(x_2), \quad 2 = u^b(x_3), \quad 1 = u^b(x_1)$$
$$\text{person C: } 3 = u^c(x_3), \quad 2 = u^c(x_1), \quad 1 = u^c(x_2).$$

    a. Is there a voting equilibrium?
    b. Draw a bar graph with x_1, x_2, and x_3 arranged in that order on the horizontal axis and the preference ranking of each voter shown on the vertical axis. Which voter's preferences have two peaks?

Domination and the Core

Before proceeding, I want to restate the results about voting cycles in the language of cooperative game theory, which introduces concepts used later in this book. For given rules of collective choice, a *decisive* coalition gets its way when its members agree. To be precise, a coalition is *decisive* for a given pair of alternatives if, whenever everyone in the coalition prefers one alternative to the other alternative, the coalition can obtain its preference. Thus, when a state faces a choice between x and y, a coalition C is decisive over the choice if, when everyone in C prefers x to y, the state chooses x over y.

As explained, a decisive coalition gets its way when its members agree. If the members of a decisive coalition agree that they prefer one alternative over another, then the more preferred alternative *dominates* the less preferred alternative. To be precise, alternative x dominates alternative y if a decisive coalition C exists in which everyone prefers x to y.

Domination is important to stability. Whenever collective choice selects a dominated alternative, a coalition can form to replace it with the dominant alternative. A dominated alternative is, consequently, unstable. Intransitivity implies that *every* alternative is dominated by another alternative. Any alternative in an intransitive cycle is, consequently, unstable.

The phrase "Condorcet winner" refers to an alternative that can defeat any other alternative in paired voting.[31] A Condorcet winner is <u>un</u>dominated, which means that no decisive coalition can form whose members prefer an alternative to a Condorcet winner. An undominated alternative is, consequently, stable.

Game theorists call the set of *un*dominated alternatives the game's *core*. When a game is formulated mathematically, theorists ascertain whether or not its core is "empty." An intransitive set has an empty core (no undominated alternative exists), whereas a Condorcet winner is "in the core" (undominated alternative), so the core is not empty when a Condorcet winner exists.

Questions

1. Explain why any point outside the game's core is unstable.

2. What does it mean to say that the equilibrium price in a perfectly competitive market is in the "core" of the game played by firms and consumers?

Is Intransitivity Bad?

Voting intransitivities often occur. Are they bad? It is easy to see why intransitive preferences are irrational for individuals. Suppose that a student takes his desk lamp—call it lamp A—to the flea market to trade for another. The student sees lamp B, which he prefers to lamp A, and he offers to trade lamp A and \$5 for lamp B. The vendor accepts the offer. The student is carrying lamp B when he sees lamp C, which he prefers to lamp B, so he offers to trade lamp B and \$5 for lamp C. The vendor accepts. Now the student turns to leave the flea market and on the way out he passes the stall where lamp A is being offered for resale. Since he has intransitive preferences, he likes lamp A better than lamp C, so he offers to swap lamp C and \$5 for lamp A. The vendor accepts and the student goes home with lamp A (the same lamp he brought to the flea market) and he is \$15 poorer. The intransitive buyer is a "money pump" for sellers.

There is a long philosophical tradition holding that a rational person can rank states of the world from bad to good.[32] Without such an ordering, a person has no concept of a better world to strive for. Intransitive preferences do not yield a ranking from bad to good because they run in a circle. The intransitive student did not have a vision of a better lamp. The objection to intransitive preferences is that they reveal no vision of a better world on the part of the actor.

---

[31] The term is named in honor of an eighteenth-century French mathematician and politician who defined the concept and used it in an early study of voting rules (Condorcet 1976).

[32] This requirement of rational ethics, which is implicit in the utilitarian tradition, was first formulated in a forceful, sustained argument in Sidgwick 1966.

This characterization of individuals also applies to the state. Given intransitive voting, the state lacks coherent goals. Instead of rejecting worse states of the world in favor of better states, intransitive voting goes in a circle. Circular politics does not reveal the goal of a better world to be achieved by collective choice.

Political philosophy typically justifies laws enacted in a democracy on the grounds that they represent the "will of the majority" or the "intent of the people's representatives." Given intransitive voting, however, these phrases make no sense. Intransitive voters have no collective "will" because they contradict themselves. Intransitive voting thus creates a problem in justifying democracy. The next chapter, which concerns bargaining, will explain the problem's solution.

Questions

1. If a person has intransitive preferences, would you rather trade with him or employ him in your company?

2. Contrast the "will of the majority" or the "intent of the legislature" under the median rule and intransitivity.

Impossibility

Students who first encounter voter intransitivity are inclined to minimize its importance. You might suppose, for example, that voter preferences are typically single-peaked like those in figure 2-1, which result in a voter equilibrium, and seldom double-peaked like those of voter C in figure 2-3, where no equilibrium exists. This supposition is a mistake. I have used figures depicting a single dimension of choice such as the size of total government expenditures. In a single dimension of choice, single-peaked preferences are apparently more common than double-peaked preferences. Collective choice, however, often involves multidimensional choices, such as expenditures on schools, police, and roads. Voter preferences often form intransitive cycles when political choices occur in multiple dimensions.

Voters' preferences may be single-peaked in one dimension of choice (x-axis), and also single-peaked in another dimension of choice (y-axis). The same voters' preferences, however, may be double-peaked on a curve in two-dimensional space. The voters with ordinary economic preferences ("convex indifference sets") often produce cyclical majorities in choices involving several dimensions.[33]

[33] To illustrate, consider an allocation of public funds to schools, roads, and police. Let x_a^* denote an allocation with large expenditures on schools, modest expenditures on roads, and little expenditures on police. Let x_b^* denote an allocation with large expenditures on roads, modest expenditures on police, and little expenditures on schools. Let x_c^* denote an allocation with large expenditures on police, modest expenditures on schools, and little expenditures on roads. Three voters with ordinary, convex preferences might rank the three possible allocations as follows:

person A (commuter with children): $x_a^* > x_b^* > x_c^*$.

person B (childless commuter): $x_b^* > x_c^* > x_a^*$

person C (fearful grandmother): $x_c^* > x_a^* > x_b^*$.

The three voters form an intransitive cycle under majority rule.

To state these facts more precisely, list all the logically possible ways to rank a small number of alternatives. Now consider the logically possible ways to assign these rankings to a small numbers of voters. Some assignments result in voting intransitivities and others result in Condorcet winners. The proportion of logically possible assignments that result in voting intransitivity increases with the number of alternatives and voters.[34] The so-called chaos theorem asserts that intransitivities are so frequent in multidimensional choice that almost any outcome could be reached by an appropriate sequence of votes (McKelvey 1979).[35]

Students often suppose that intransitive cycles result from particular voting procedures, such as voting over paired alternatives. Many variations in voting rules exist (run-off votes, point voting, super-majority rule, etc.). In a powerful generalization, Kenneth Arrow proved that no form of a democratic constitution can solve the problem of intransitivity in choosing public goods.[36] Tinkering with voting rules cannot solve the problem of intransitivities in democracy.

Agenda Setting

To prevent intransitive cycles from occurring, a democratic system must adopt specific rules and practices that have substantial costs. To illustrate, consider

[34] For example, the logically possible ways to rank the alternatives (x_1, x_2, x_3) are

$$R1: x_1 > x_2 > x_3$$
$$R2: x_2 > x_3 > x_1$$
$$R3: x_3 > x_1 > x_2$$
$$R4: x_1 > x_3 > x_2$$
$$R5: x_2 > x_1 > x_3$$
$$R6: x_3 > x_2 > x_1.$$

Let P_{123} denote the profile of preferences for three voters given by R_1, R_2, R_3. Some profiles yield voting cycles and other yield Condorcet winners. For example, P_{123} yields a voting cycle, whereas P_{124} yields the Condorcet winner x_1. As the number of alternatives increases and the number of voters increases, the proportion of logically possible profiles yielding voting cycles increases as a proportion of the total number of logically possible profiles (Riker 1982a). Riker remarks that political parties may reduce diversity in profiles, but politicians who want to manipulate outcomes may deliberately increase diversity (p. 122).

[35] Miller 1983 summarizes the relationship between diversity and intransitivity as follows:

The probabilistic literature on the paradox of voting has been concerned primarily with 1) calculating the likelihood that cyclical majorities arise in an impartial culture, i.e., a uniform distribution over all logically distinct individual orderings, and 2) determining how this likelihood changes as a culture deviates from impartiality. The basic conclusions are that the probability of cyclical majorities in an impartial culture increases as the number of alternatives, voters, or both increases. Moreover, as the number of alternatives increases, if majority rule fails to be transitive, the more likely it becomes that it will fail entirely and that one cycle will encompass all alternatives. Concerning departures from impartiality, the general thrust of conclusions is that greater social homogeneity (variously defined) with respect to preferences reduces the likelihood of cyclical majorities. (p. 126)

[36] This is just one interpretation of one of the most important theorems ever proven by an economist. See Arrow 1963, or chapters 3 and 3* of Sen 1970a.
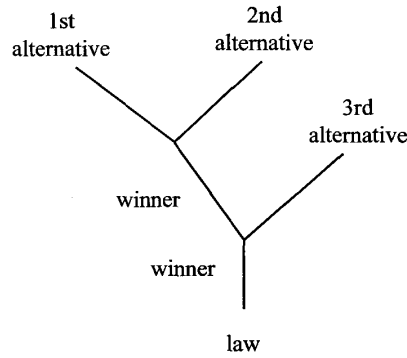
Fig. 2-4 Agenda

setting the legislative agenda. Voting in a legislature conforms to definite rules of procedure, often including a prohibition against reintroducing a defeated proposal. If defeated proposals cannot be reintroduced, an endless cycle of voting is impossible. Under these circumstances, the alternative that will prevail is the one that wins on the last vote. The alternative that will prevail on the last vote is usually predictable from the alternatives that prevail on the next-to-last vote. And the same relationship holds between the next-to-last vote and the vote preceding it. The agenda determines the order in which alternatives are considered. It is not hard to see that the final winner in the intransitive set can be determined by whoever sets the agenda. Thus, control of the legislative agenda avoids intransitivities by giving the agenda-setter the power to choose among intransitive alternatives.

To illustrate concretely, assume that the legislature considers three alternatives (x_a, x_b, x_c). The person controlling the agenda must fill in the "tree" in figure 2-4 that depicts the order of voting:

Assume that the three alternatives form the intransitive cycle,

$$x_a \text{ defeats } x_b$$
$$x_b \text{ defeats } x_c$$
$$x_c \text{ defeats } x_a.$$

Assume that the person who sets the agenda wants x_a to prevail. To assure the final victory of x_a, set the agenda so that the first vote pits x_c against x_b, and the final vote pits the winner of the first vote against x_a. Given this agenda, x_b defeats x_c in the first vote, and x_a defeats x_b in the final vote. Thus, the person who sets the agenda gets her most preferred outcome, as depicted in figure 2-5.

Alternatively, assume the person controlling the agenda wants x_b to prevail. To accomplish this end, set the agenda so that the first vote pits x_c against x_a, and the final vote pits the winner of the first vote against x_b. As a consequence, x_c defeats x_a in the first vote, and x_b defeats x_c in the final vote, as in figure 2-6.
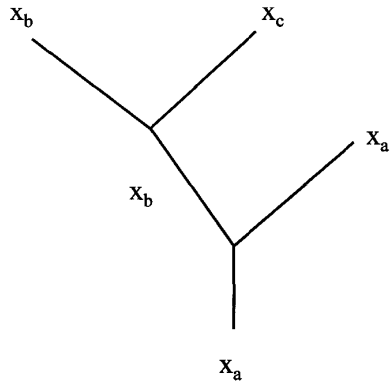
Fig. 2-5 Agenda Set for x_a to Win
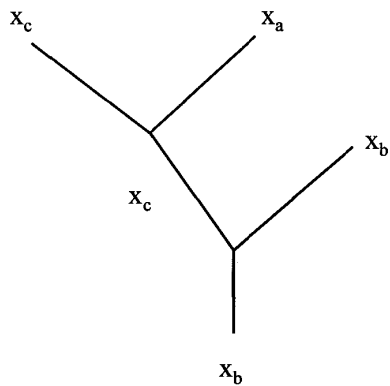
Fig. 2-6 Agenda Set for x_b to Win

In order for the person who sets the agenda to determine the outcome of voting over an intransitive cycle, he must think recursively. Specifically, he must figure out which alternative can be beaten by the one he most favors, pit them against each other in the last division, then repeat the same process of reasoning for the next-to-last division, and so forth back to the first division.

To avoid circular voting, legislatures characteristically adopt rules giving control over the agenda to particular officials, such as committee chairmen or the person presiding in the legislature. Empirical research concludes that the person who controls the agenda often determines the outcome in voting.[37] By choosing the agenda, the chairman in effect determines which majority will prevail. To

---

[37] Levine and Plott 1977.

illustrate by the preceding example, the chairman who sets the agenda determines whether the majority who prevails will be the one favoring x_a over x_b, x_b over x_c, or x_c over x_a.

Generalizing, democracy can avoid intransitive cycles by empowering someone to dictate which majority will prevail. Allowing a chairman to set the agenda achieves this end by one means. Other means also exist. Instead of giving control over the agenda to the chairman, the party leader can perform this role. To illustrate, the party of the British prime minister usually controls a majority of seats in Parliament. In important votes, the prime minister imposes strict discipline on members of the party. Consequently, the prime minister wins every important vote in Parliament. By dictating to the majority party, the British prime minister eliminates intransitive voting in the legislature. Unlike Britain, parliamentary systems without a majority party can cycle through coalitions, as has occurred in Italy in recent years.

Political commentators in the United States sometimes say that the president can use the media to "set the political agenda." To illustrate, assume the government must choose among x_a^*, x_b^*, and x_c^*, and the president wants x_b^* to win. To induce the majority of citizens to oppose the president, the opposition tries to frame the issue as a choice between x_a^* and x_b^*. The president, however, uses his command of the media to frame the issue as a choice between x_b^* and x_c^*, so that the majority of the public agrees with the president's choice.

Questions

1. To what extent can the following political actors set the agenda? (If you are not from the United States, substitute some similar offices from your country's government.)

> Speaker of the House of Representatives
>
> chief justice of the Supreme Court
>
> president
>
> committee chairmen in Congress
>
> director of an agency (e.g., chairman of the Federal Trade Commission).

2. Discuss the advantages and disadvantages of choosing someone to set an agenda rather than allowing politics to cycle.

3. A legislature with three voters (A, B, C) chooses among three alternatives (x_1, x_2, x_3). The voters rank the alternatives from 1 to 3, with "3" indicating the most preferred alternative and "1" indicating the least preferred:

$$\text{person A: } 3 = u^a(x_1), \quad 2 = u^a(x_2), \quad 1 = u^a(x_3)$$
$$\text{person B: } 3 = u^b(x_2), \quad 2 = u^b(x_3), \quad 1 = u^b(x_1)$$
$$\text{person C: } 3 = u^c(x_3), \quad 2 = u^c(x_1), \quad 1 = u^c(x_2).$$

The alternatives are to be pitted against each other in majority voting, and a defeated alternative cannot be reintroduced. Assume that C determines the order in which the alternatives are to be considered (agenda-setter). If each person votes for her preferred alternative in paired voting, describe the agenda that enables C to get her most preferred outcome.

4. Repeat the preceding question, but instead of assuming that each person votes for his preferred alternative in paired voting, assume that each person votes strategically on the first vote. For example, if the first vote pits x_1 against x_2, A foresees that voting for x_1 in the first vote will cause x_3 to win in the second vote. Since x_3 is the worst outcome for A, he decides to vote for x_2 instead of x_1 on the first vote. When the parties vote strategically, C can assure that her most preferred alternative is the final winner by setting the agenda so that her most preferred alternative is introduced on the first vote. Explain why.

CONCLUSION

This chapter analyzes majority rule as a method for satisfying the preferences of citizens for collective action. With single-peaked preferences, majority voting over paired alternatives reaches an equilibrium most preferred by the median voter. The equilibrium is always Pareto efficient, and it approaches cost-benefit efficiency as preferences approach strong symmetry. With multipeaked preferences, however, voting may not have an equilibrium. When voting cycles, outcomes are irrational or arbitrary, and the "will of the majority" has no clear meaning.

"Why didn't the dog bark?" Sometimes Sherlock Holmes or Miss Marple solves a mystery by asking why something that should have occurred did not occur. I have explained why intransitive cycles should occur. Like Sherlock Holmes or Miss Marple, you should ask why cycles do not occur in particular political systems. As you will see, the means by which a democracy avoids intransitive cycles often marks its character.

I already described one such device—agenda setting. Setting an agenda stops cycling by giving the agenda-setter power to choose the outcome within the intransitive set. A powerful agenda-setter dominates some political systems.

Single-peaked preferences are probable (but not certain) in a single dimension of choice, whereas cycling is probable (but not certain) in multiple dimensions of choice. As explained in chapter 5, some political systems avoid intransitivity by narrowing political choices to a single dimension.

The next chapter explains how democracy provides a framework for efficient bargaining over public goods, much like markets provide a framework for efficient bargaining over private goods. Political bargaining is the most fundamental means to avoid voting intransitivity. Whereas majority voting can lead to inefficient or irrational results, bargaining theory supplies a more affirmative vision