

Eventos

Evento 1: $x_1' = \gamma (x_1 - v t_1)$
 $t_1' = \gamma (t_1 - \frac{v}{c^2} x_1)$

$x_1 = \gamma (x_1' + v t_1')$
 $t_1 = \gamma (t_1' + \frac{v}{c^2} x_1')$

Evento 2: $x_2' = \gamma (x_2 - v t_2)$
 $t_2' = \gamma (t_2 - \frac{v}{c^2} x_2)$

$x_2 = \gamma (x_2' + v t_2')$
 $t_2 = \gamma (t_2' + \frac{v}{c^2} x_2')$

luego,

$$x_2' - x_1' = \gamma [(x_2 - x_1) - v(t_2 - t_1)]$$

$$t_2' - t_1' = \gamma [(t_2 - t_1) - \frac{v}{c^2} (x_2 - x_1)]$$

EX: sist. S' tiene veloc. v ch S . Se sincronizan los relojes
 en $t = t' = 0$ cuando $x = x' = 0$.

evento 1 ocurre en $x_1 = 10m$, $t_1 = 2 \times 10^{-7}$ segs ($y_1 = 0 = z_1$)

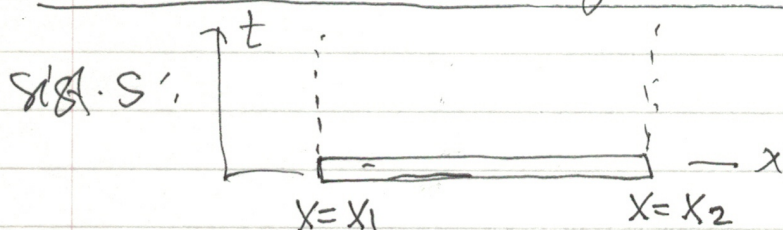
evento 2 ocurre en $x_2 = 50m$, $t_2 = 3 \times 10^{-7}$ segs, ($y_2 = 0 = z_2$)

¿cuál es la distancia entre ambos eventos, medidos en S' ?

$$(v/c)^2 = (9/25) \Rightarrow \gamma = (1 - (v/c)^2)^{1/2} = 5/4$$

$$\Rightarrow x_2' - x_1' = \frac{5}{4} [(50 - 10) - (\frac{3}{5}) 3 \times 10^8 (3 - 2) 10^{-7}] = 27.5 (m).$$

Contracción de longitudes



$L = x_2 - x_1$ (en el sist. en reposo?)

¿cuál es la long. de la barra
sist. S' ?

medida en otro

→ medir las posiciones de ambos extremos (x_1' y x_2')
al mismo tiempo t' (medido en S')

$$l' = x_2' - x_1'$$

observamos $x_1 = \gamma(x_1' + vt')$

$$x_2 = \gamma(x_2' + vt')$$

$$\Rightarrow \underbrace{x_2 - x_1}_l = \gamma \underbrace{(x_2' - x_1')}_{l'} \Rightarrow l' = \frac{l}{\gamma} = l \sqrt{1 - \left(\frac{v}{c}\right)^2} < l$$

Dilatación del tiempo: Sup. un reloj en reposo en
 $x = x_0$ en S .

evento 1: (x_0, t_1) "TIC"

evento 2: (x_0, t_2) "TAC"

¿cómo se ve esto desde S' ?

$$t_1' = \gamma \left(t_1 - \frac{vx_0}{c^2} \right)$$

$$t_2' = \gamma \left(t_2 - \frac{vx_0}{c^2} \right)$$

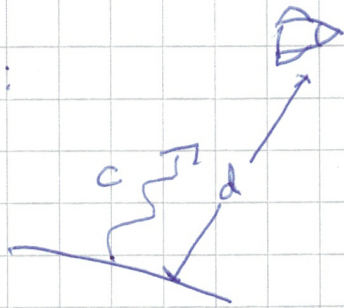
$$\} \Rightarrow \underbrace{t_2' - t_1'}_{\Delta t'} = \gamma (t_2 - t_1)$$

$$\Delta t' = \gamma \Delta t > \Delta t$$

⇒ reloj parece correr "más lento" al ser observado
desde S' .

una nave se aleja de la T a $v = 0.8c$. cuando se encuentra a una distancia $d = 6.66 \times 10^8 \text{ km}$, se le envía una señal de radio desde la T, ¿cuanto tarda en llegar la señal, medido en ambos sist. de referencia? ¿cual es la posición de la nave cuando recibe la señal, en ambos sist. de referencia?

Sistema tierra (s):



$$ct = d + vt$$

$$(c-v)t = d$$

$$t = \frac{d}{c-v}$$

y la posición sera $ct = \boxed{\frac{dc}{c-v}} = x$

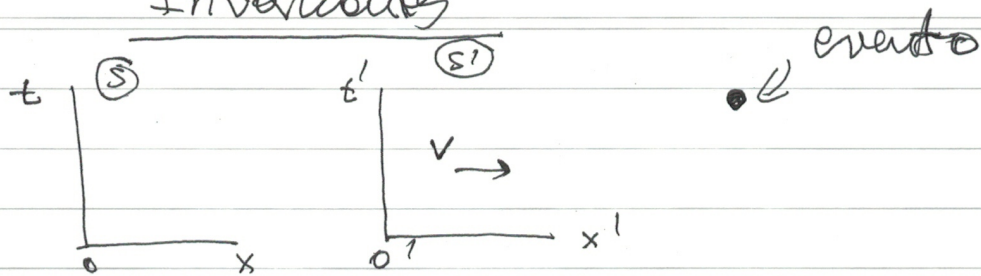
Sistema nave (s') : transformamos (x, t) evento $\rightarrow (x', t')$

$$t' = \gamma \left(t - \frac{vx}{c^2} \right) = \gamma \left[\frac{d}{c-v} - \frac{v \cdot dc}{c^2(c-v)} \right] = \frac{\gamma}{c-v} \left(d - \frac{v}{c}d \right)$$

$$= \frac{\gamma d (1 - \frac{v}{c})}{c-v} = \frac{\gamma d (1 - \frac{v}{c})}{c(1 - \frac{v}{c})} = \boxed{\frac{\gamma d}{c}}$$

$$x' = 0$$

Invariantes



$$x' = \gamma(x - vt)$$

$$t' = \gamma(t - vx/c^2)$$

Evaluemos: $(ct')^2 - (x')^2$

$$= \gamma^2(ct - vx/c)^2 - \gamma^2(x - vt)^2$$

$$= \gamma^2 \left[c^2t^2 + \frac{v^2x^2}{c^2} - 2vxt \right] - \gamma^2 \left[x^2 + v^2t^2 - 2xvt \right]$$

$$= \gamma^2 \left[c^2t^2 + \frac{v^2x^2}{c^2} - 2vxt - x^2 - v^2t^2 + 2xvt \right]$$

$$= \gamma^2 \left[(c^2 - v^2)t^2 - x^2 \left(1 - \frac{v^2}{c^2}\right) \right] = \gamma^2 \left[c^2 \left(1 - \frac{v^2}{c^2}\right) t^2 - x^2 \left(1 - \frac{v^2}{c^2}\right) \right]$$

$$= \cancel{\gamma^2 \left(1 - \frac{v^2}{c^2}\right)} [(ct)^2 - x^2] = (ct)^2 - x^2$$

$$\therefore s^2 \equiv (ct')^2 - (x')^2 = (ct)^2 - x^2$$

INVARIANTE
RELATIVISTA.

también: $E_0^2 \equiv E^2 - c^2 p^2$

$\downarrow$ $\downarrow$
 reposo total

An observer on Earth observes two spacecraft moving in the *same* direction toward the Earth. Spacecraft A appears to have a speed of $0.50c$, and spacecraft B appears to have a speed of $0.80c$. What is the speed of spacecraft A measured by an observer in spacecraft B?

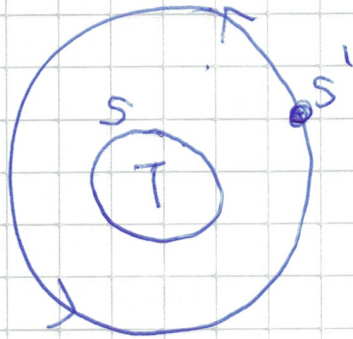


En general,
$$V_A = \frac{V_B + V_{A'}}{1 + \frac{\vec{V}_{A'} \cdot \vec{V}_B}{c^2}} \Rightarrow$$

$$V_{A'} = \frac{V_A - V_B}{1 - \frac{\vec{V}_A \cdot \vec{V}_B}{c^2}} \quad \text{In our case } \begin{aligned} \vec{V}_A &= -0.5c \hat{x} \\ \vec{V}_B &= -0.8c \hat{x} \end{aligned}$$

$$\Rightarrow V_{A'} = \frac{-0.5c - (-0.8c)}{1 - (-0.5)(-0.8)} = \boxed{0.5c}$$

In 1962, when Scott Carpenter orbited Earth 22 times, the press stated that for each orbit he aged 2 millionths of a second less than if he had remained on Earth. (a) Assuming that he was 160 km above Earth in an eastbound circular orbit, determine the time difference between someone on Earth and the orbiting astronaut for the 22 orbits. (b) Did the press report accurate information? Explain.



$$\Delta z = z - z' = z - (1/\gamma)z = (1 - 1/\gamma)z$$

$$z \approx \frac{2\pi R}{v} = 2\pi \sqrt{R/g} \quad (1)$$

$$\frac{GM}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad (0)$$

$$\gamma = (1 - (v/c)^2)^{-1/2} \approx 1 + \frac{1}{2}(v/c)^2$$

$$\Rightarrow \frac{1}{\gamma} \approx 1 - \frac{1}{2}(v/c)^2 \Rightarrow 1 - \frac{1}{8} = \frac{1}{2}(v/c)^2$$

$$\Rightarrow \Delta z = 2\pi \sqrt{R/g} \frac{1}{2} \left(\frac{v}{c}\right)^2 \stackrel{(0)}{=} \frac{\pi R g^{3/2}}{c^2} \quad (2)$$

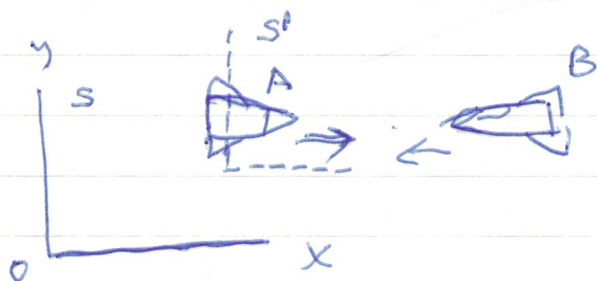
$$R = 6400 \text{ km} + 160$$

$$g = 9.8 \text{ m/s}^2$$

$$\Rightarrow \Delta z \approx 1.84 \text{ } \mu\text{s}$$

Ej. 2 naves A y B se mueven en direcciones opuestas

⑤ (tierra) $V_A = 0.750c$
 $V_B = 0.850c$



Hallar la veloc. de B c/n a A.

Solución tomar S' en la nave A, con $U = 0.750c$

La veloc. entre S y S' . La nave B se toma como un objeto moviéndose hacia la izquierda con velocidad $u_x = -0.850c$ c/n a la tierra (S)

$\Rightarrow$ la veloc. de B c/n a A (S') será

$$u_{x'} = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.977c$$