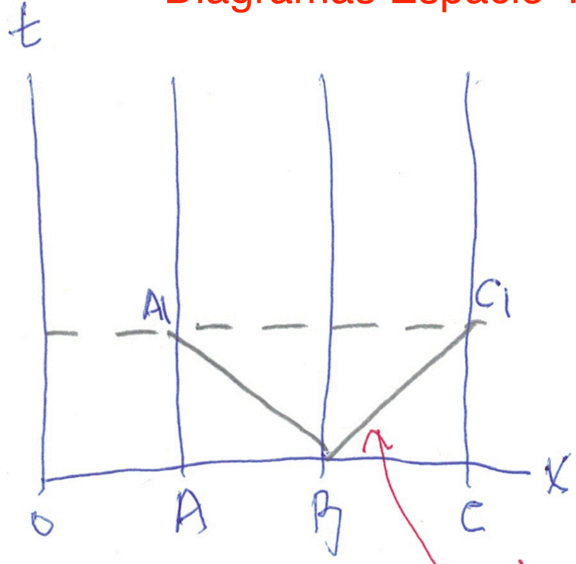
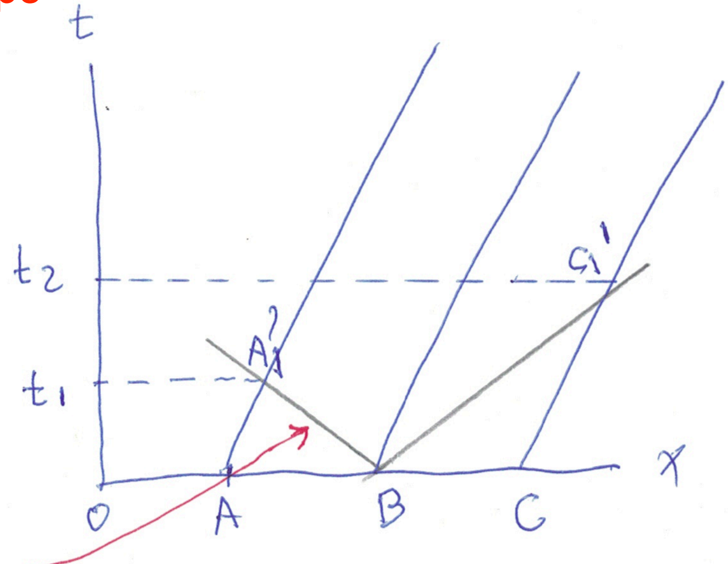
Diagramas Espacio-Tiempo





misma pendiente

A_1, C_1 eventos simultáneos

A_1', C_1' NO simultáneos en S

eq. entre sists. inerciales $\Rightarrow$ A_1' y C_1' son simultáneos en $\boxed{S'}$

simultaneidad depende del sist. de referencia usado

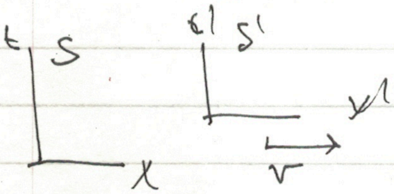
Ec. de transformación (entre S y S')

$x' = L_1(x,t)$
$t' = L_2(x,t)$

L_1, L_2 funcs. LINEALES
(Debido a que un obj. a veloc. cte. en S, debiera también tener veloc. cte. en S')

$$x = a x' + b t' \quad \text{con} \quad x' = a x - b t$$

debe reducirse al caso Galileano a bajas velocidades



$x = 0 \Rightarrow a x' + b t' = 0 \Rightarrow a x' + b t' = 0 \Rightarrow \frac{x'}{t'} = -\frac{b}{a} = \boxed{-v}$ ← vel de S

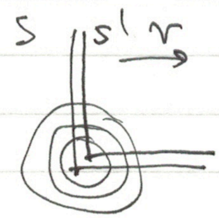
$x' = 0 \Rightarrow a x - b t = 0 \Rightarrow a x = b t \rightarrow \frac{x}{t} = \frac{b}{a} = \boxed{v}$

veloc. de S'

$$\boxed{v = b/a}$$

Si una señal de luz se origina en el origen de S: $x = ct$

Desde S': $x' = ct'$



$\Rightarrow \begin{cases} ct = act' + bt' \\ ct' = act - bt \end{cases} \Rightarrow ct' = c\left(a - \frac{b}{c}\right)t = ca\left(1 - \frac{v}{c}\right)t$

$ct = (ac+b)t' = (ac+b)a\left(1 - \frac{v}{c}\right)t = a^2(c+v)\left(1 - \frac{v}{c}\right)t$

$= a^2 c\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right)t = a^2 ct\left(1 - \left(\frac{v}{c}\right)^2\right)$

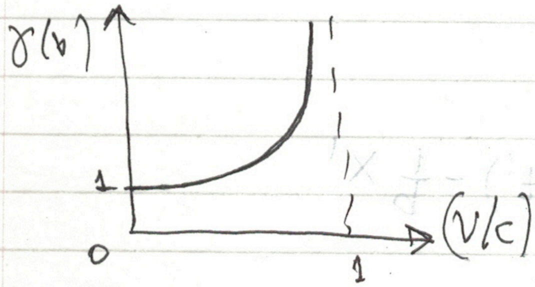
$$\Rightarrow a^2 = \frac{1}{1-(v/c)^2} \Rightarrow \boxed{a = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}}$$

$$\Rightarrow x = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}(x'+vt') = \gamma(x'+vt')$$

$$x' = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}(x-vt) = \gamma(x-vt)$$

Transf. de Lorentz

donde $\gamma(v) \equiv \left(1-(v/c)^2\right)^{-1/2}$



Ejercicio: Dadas las T. de L, obtenga

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right).$$

Para las demás coordenadas, $y' = y$
$z' = z$

T. de los tiempos

$$x = \gamma(x' + vt')$$

$$\Rightarrow vt' = x\left(\frac{1}{\gamma} - \gamma\right) + \gamma v t$$

pero, $\frac{1}{\gamma} - \gamma = \sqrt{1-\beta^2} - \frac{1}{\sqrt{1-\beta^2}}$ $(\beta \equiv v/c)$

$$= \frac{-\beta^2}{\sqrt{1-\beta^2}} = -\beta^2\gamma$$

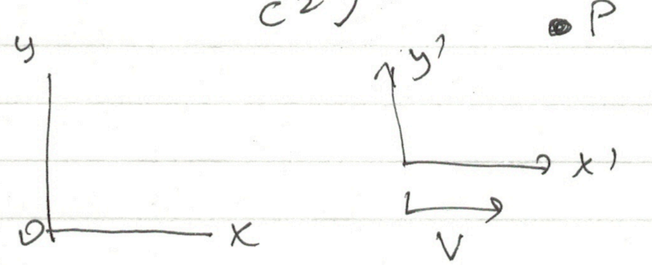
$$\Rightarrow vt' = \gamma v t - \beta^2 \gamma x = \gamma(vt - \beta^2 x)$$

$$t' = \gamma\left(t - \frac{1}{v}\left(\frac{v}{c}\right)^2 x\right)$$

$$\therefore \boxed{t' = \gamma\left(t - \frac{vx}{c^2}\right)}$$

Transf. de velocidade

$$x = \gamma(x' + vt') \quad ; \quad y = y'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$



em S': $\quad u_x' = \dfrac{dx'}{dt'} \quad ; \quad u_y' = \dfrac{dy'}{dt'}$

$$dx = \gamma(dx' + v\,dt')$$
$$dt = \gamma\left(dt' + \frac{v}{c^2}dx'\right)$$

$$\Rightarrow u_x = \frac{dx}{dt} = \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}dx'} = \frac{\cancel{dt'}(u_x' + v)}{\cancel{dt'}\left(1 + \frac{v u_x'}{c^2}\right)}$$

$$\therefore \boxed{u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}}} \Rightarrow \boxed{u_x' = \frac{u_x - v}{1 - \frac{v u_x}{c^2}}}$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{\gamma\left(dt' + \frac{v}{c^2}dx'\right)} = \boxed{\frac{u_y'/\gamma}{1 + \frac{v u_x'}{c^2}}}$$

$$\Rightarrow \boxed{u_y' = \frac{u_y/\gamma}{1 - \frac{v u_x}{c^2}}}$$

igual para u_z'

EX 1: $u_x' = v = 0.5c$

$$u_x = \frac{0.5c + 0.5c}{1 + (0.5)^2} = \frac{4}{5}c$$

En gral, si $v = \beta_1 c$ y $u_x' = \beta_2 c$ $\quad (0 < \beta_1, \beta_2 < 1)$

$$\Rightarrow \frac{u_x}{c} = \beta = \frac{\beta_1 + \beta_2}{1 + \beta_1\beta_2}$$

$$\Rightarrow 1 - \beta = 1 - \frac{(\beta_1 + \beta_2)}{1 + \beta_1\beta_2} = \frac{1 + \beta_1\beta_2 - \beta_1 - \beta_2}{1 + \beta_1\beta_2} = \frac{(1-\beta_1)(1-\beta_2)}{1 + \beta_1\beta_2}$$

o sea si $0 < \beta_1 < 1$ y $0 < \beta_2 < 1 \Rightarrow$ 1- beta > 0 —> beta <1

siempre.

En (S) se obs. q' un evento toma lugar en A sobre eje x y 10^{-6}s después otro evento ocurre en B, loc. a 900 m de A.
Hallar magnitud y dirección de (S') c/r a (S) tq. ambos eventos aparezcan simultáneos.

$A: t_A = 0, \; x_A = 0$
$B: t_B = 10^{-6} s, \; x_B = 900$ m

$$t_A' = \gamma\left(t_A - \frac{v x_A}{c^2}\right)$$

$$t_B' = \gamma\left(t_B - \frac{v x_B}{c^2}\right)$$

$$\underbrace{t_B' - t_A'}_{0} = \gamma(t_B - t_A) - \frac{\gamma v}{c^2}(x_B - x_A)$$

$$t_B - t_A = \frac{v}{c^2}(x_B - x_A) \Rightarrow \frac{v}{c} = \frac{c(t_B - t_A)}{x_B - x_A} = \frac{1}{3}$$