Discrete Photonics in Waveguide Arrays

Mario I. Molina

**Departamento de Física, MSI-Nucleus on Advanced Optics,
and Center for Optics and Photonics (CEFOP), Facultad de Ciencias,
Universidad de Chile, Santiago, Chile**

http://fisica.ciencias.uchile.cl/nonopt/NLOG.html
http://www.cefop.cl//

Why study physics of discrete systems?

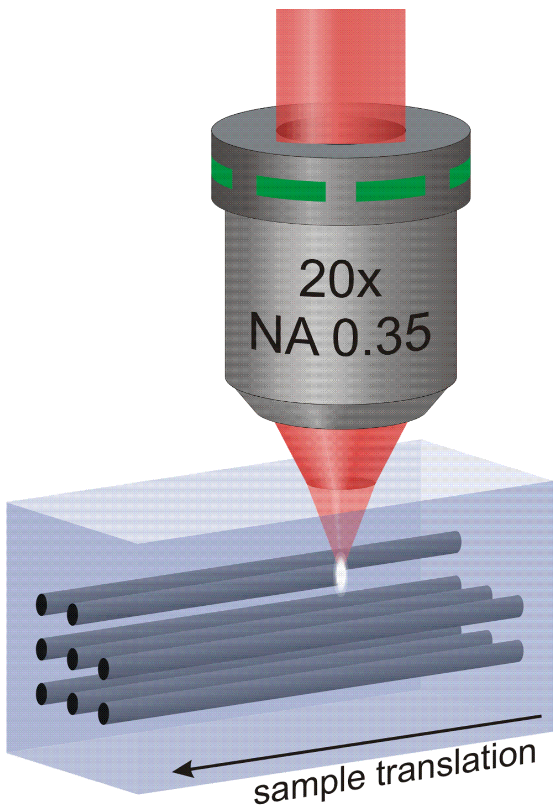
Testbed to test general phenomenology

Richer physics than continuous counterpart

Greater potential for applications

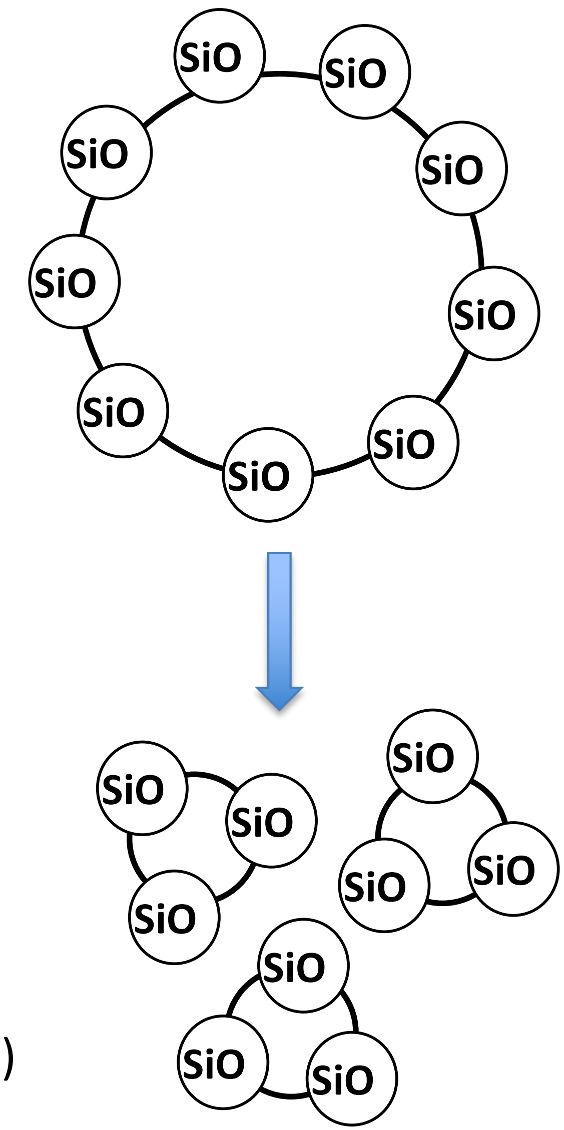
Waveguides in fused silica

A. Szameit et al, Opt. Express 13,10552 (2005).

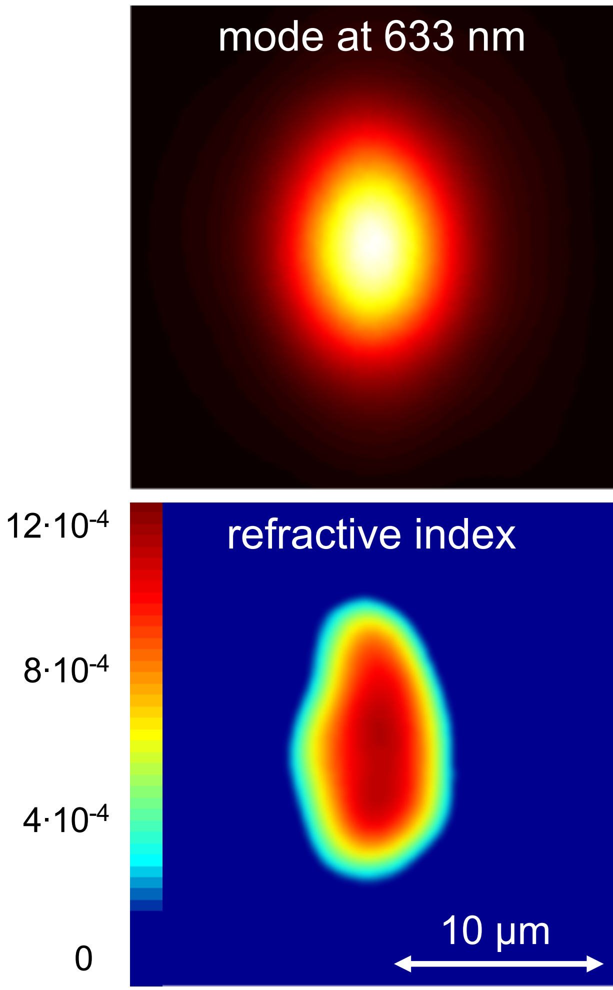


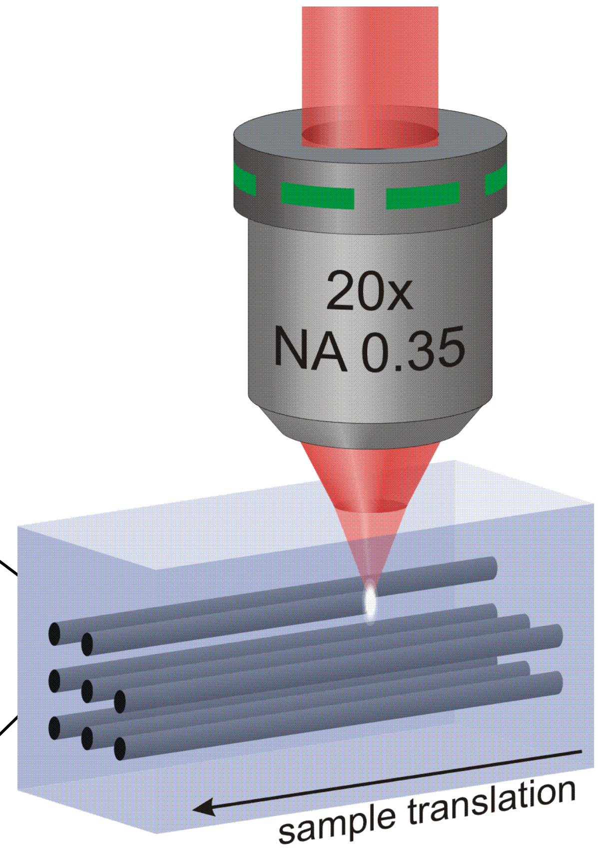
9-ring structure

3-ring structures
(densification,
refractive index increase)



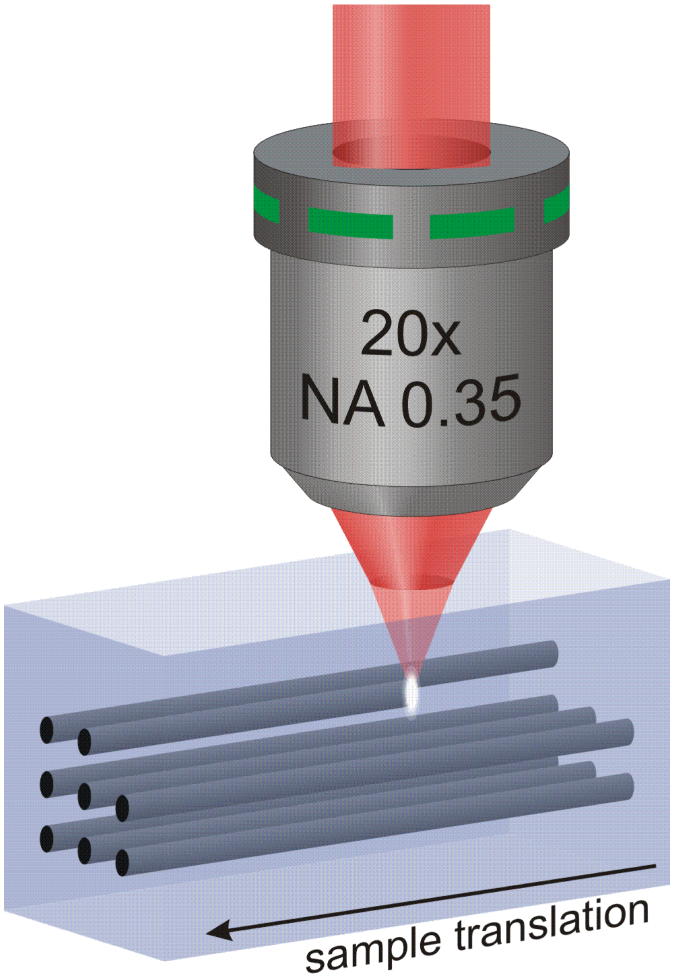
Waveguides in fused silica

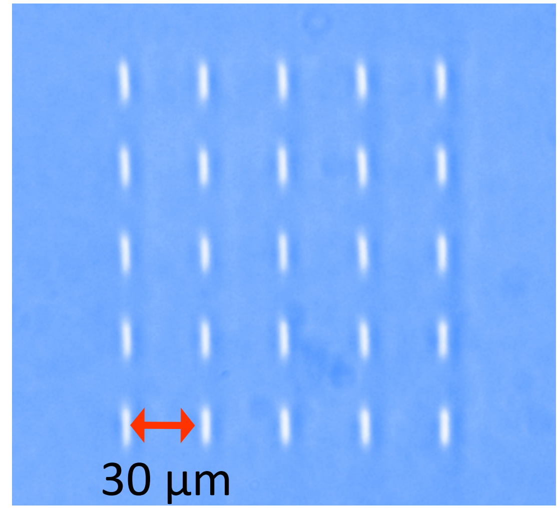


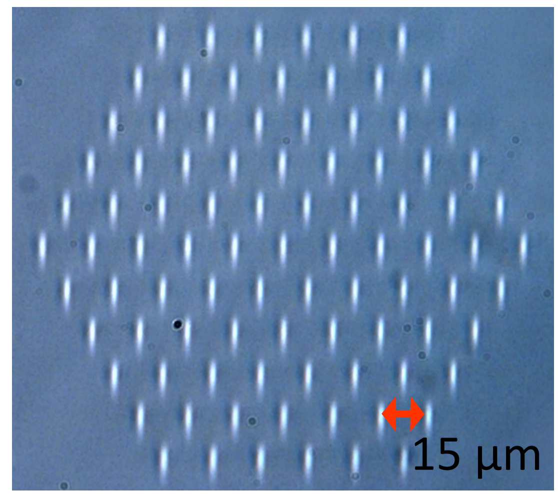


typical $\Delta n = 10^{-3}$

Waveguides in fused silica



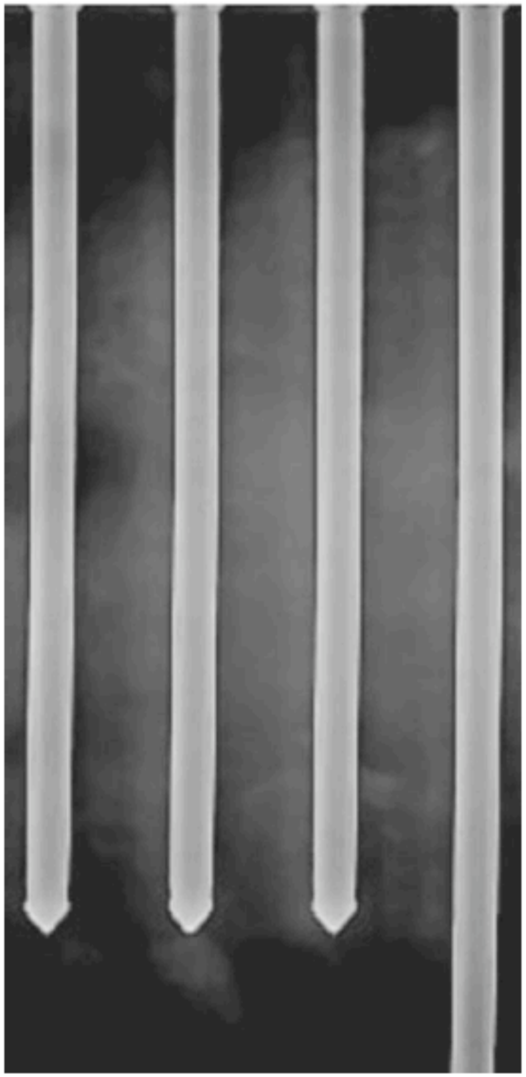


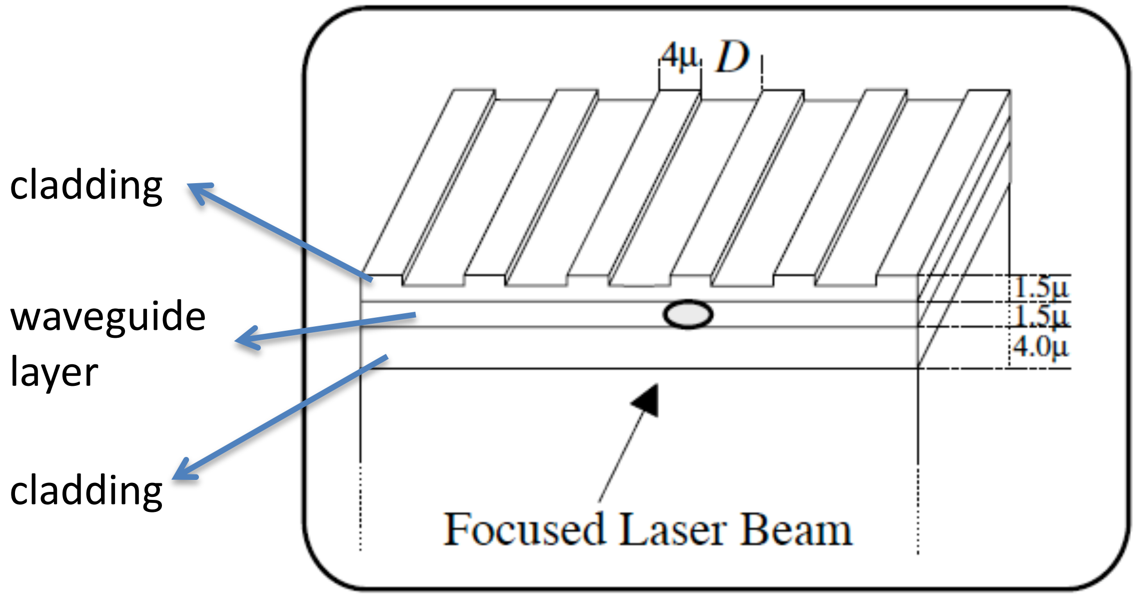


AS et al., Opt. Lett. **33**, 663 (2008).
AS et al., Appl. Phys. B **82**, 507 (2006).

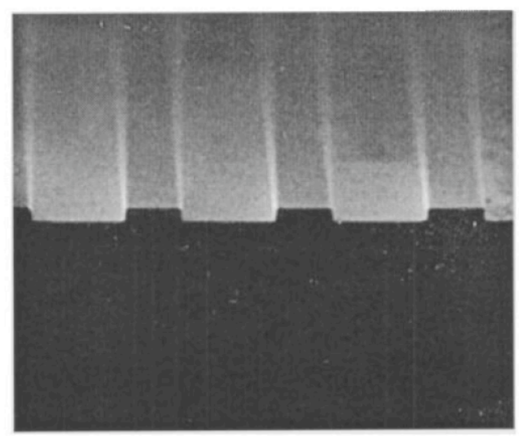
Semiconductor Waveguides

P. Millar, J.S. Aitchson,J.U. Kang, G.I. Stegeman, J. Opt. Soc. Am. B 14, 3224 (1997).

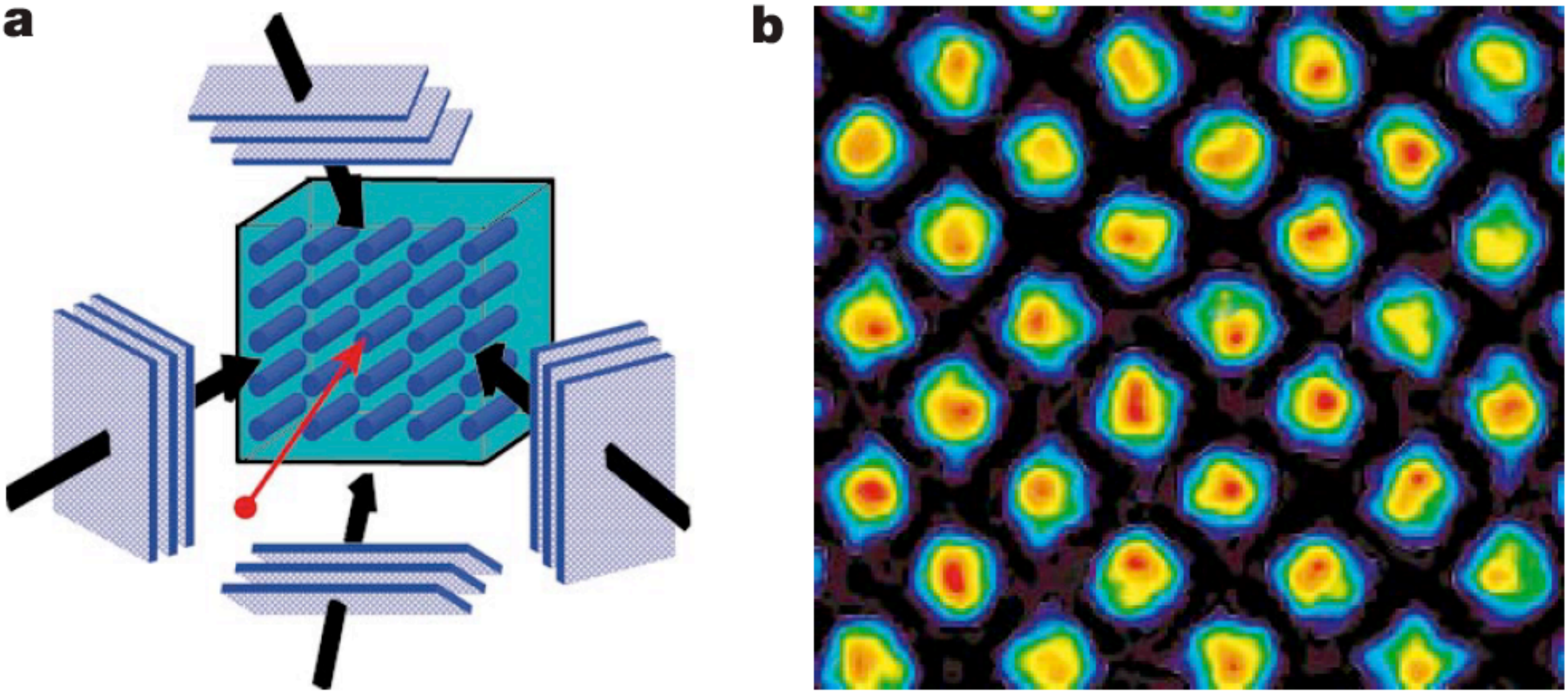




Substrate: Ga As
Cladding: $Al_{0.24}Ga_{0.76}As$
Waveguide layer:$Al_{0.18}Ga_{0.82}As$



Photorefractive Waveguides



Light→releases electrons→drift→local E fields→electro-optic effect→distribution of refractive indices

Coupled-modes theory

Maxwell:

$$\vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

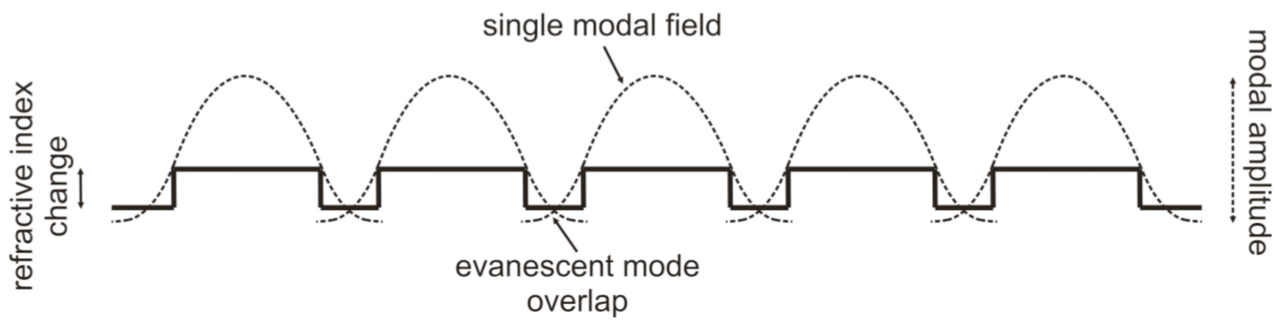
$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial}{\partial t}\vec{D} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{D} = \vec{E} + \vec{P} \qquad \vec{H} = \vec{B} + \vec{M}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

$$\vec{P} = \chi^{(1)}\vec{E} + \chi^{(3)}|\vec{E}|^2\vec{E}$$

Teoría de modos acoplados



$$E(x,z) = \sum_{n=-\infty}^{\infty} C_n(z)\phi(x - x_n)$$

$$\left|\frac{d^2C_n}{dz^2}\right| \ll k_0 \left|\frac{dC_n}{dz}\right| \qquad n = n_0 + n_2|E|^2$$ Kerr

$$i\frac{dC_n}{dz} + V(C_{n+1} + C_{n-1}) + \gamma|C_n|^2 C_n = 0$$

Discrete nonlinear Schrodinger (DNLS) equation

Coupled-modes theory

$$P = \sum_n |C_n|^2$$

$$H = \sum_n \{V(C_n C^*_{n+1} + C^*_n C_{n+1}) + (\gamma/2)|C_n|^4\}$$

Conserved quantities

$$q_n = C_n; \qquad p_n = i\ C^*_n$$

$$(d/dt)q_n = \partial H/\partial p_n \qquad (d/dt)p_n = -\partial H/\partial q_n$$

Hamiltonian system

$$C_n = u_n \exp(i\beta z)$$

Stationary mode

$$-\beta u_n + (u_{n+1} + u_{n-1}) + \chi|u_n|^2 u_n = 0$$

Nonlinear eigenvalue equation

Coupled-modes theory

Finding the localized nonlinear mode

$$-EC_n + V(C_{n+1} + C_{n-1}) + \chi |C_n|^2 C_n = 0$$

$$\lambda \equiv E/V, \qquad \phi_n \equiv \sqrt{\chi/V}\, C_n$$

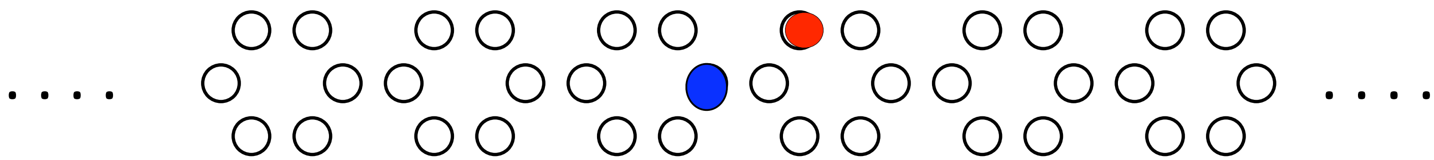
$$-\lambda \phi_n + (\phi_{n+1} + \phi_{n-1}) + |\phi_n|^2 \phi_n = 0$$

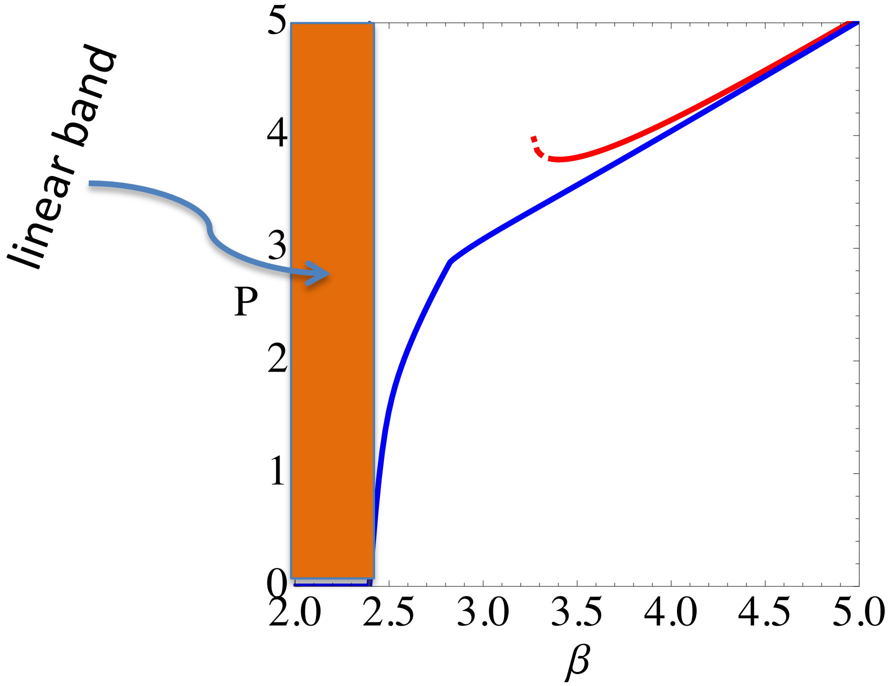
$$\vec{F}(\vec{\phi}) = 0 \quad \text{use Newton-Raphson}$$

Need good seed (anticontinuous limit)
Find many solution families (characterized by power
vs prop.const. curve

Coupled-modes theory

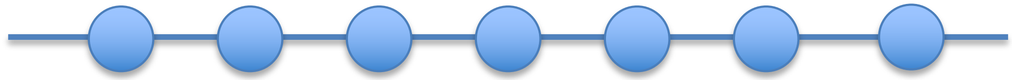
Example: Graphene ribbon



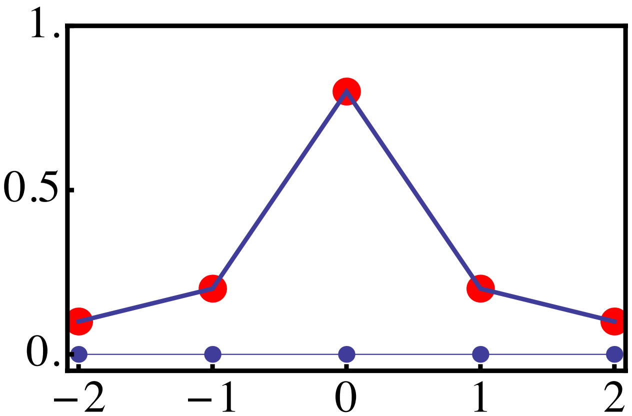


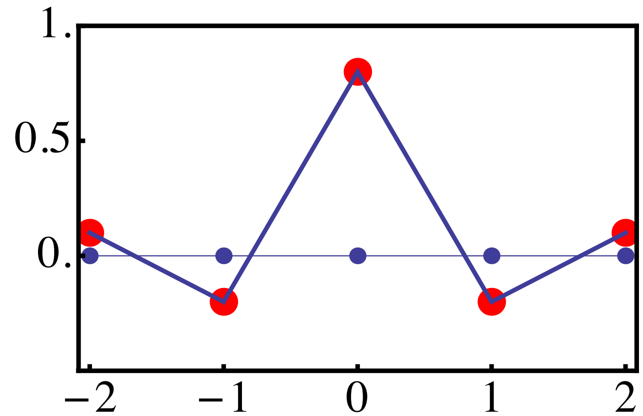
Coupled-modes theory

Pure 1D CASE



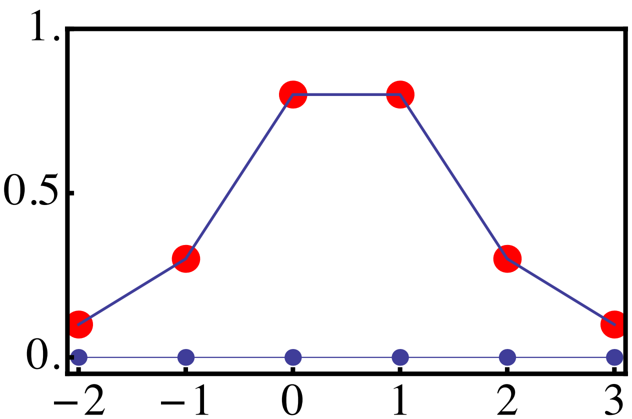
odd unstaggered

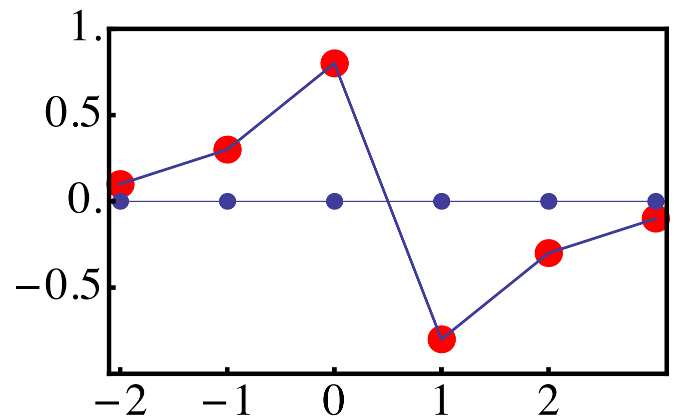




odd staggered

even





twisted

Linear stability

$$C_n(z) = \phi_n e^{-i\lambda z} \qquad \text{sol. of DNLS}$$

$$C_n(z) \to (\phi_n + \delta\phi_n)e^{-i\lambda z}, \qquad |\delta\phi_n/\phi_n| \ll 1$$

$$\Longrightarrow \quad \text{Equation for } \ \delta\phi_n = \delta u_n + i\delta v_n$$

$$\text{define } \delta\vec{u} = (\delta u_1, \delta u_2, ..., \delta u_N), \qquad \delta\vec{v} = (\delta v_1., \delta v_2, ..., \delta v_N)$$

$$\mathcal{A}_{nm} = \delta_{n,m+1} + \delta_{n,m-1} + (\lambda + \phi_n^2)\delta_{n,m}$$

$$\mathcal{B}_{nm} = \delta_{n,m+1} + \delta_{n,m-1} + (\lambda + 3\phi_n^2)\delta_{n,m}$$

$$\boxed{\delta\ddot{\vec{U}} + \mathcal{B}\mathcal{A}\ \delta\vec{U} = 0 \ \text{ and } \ \delta\ddot{\vec{V}} + \mathcal{A}\mathcal{B}\ \delta\vec{V} = 0}$$

Coupled-modes theory

$$\{\mathrm{m}\}=\text{eigenvalues of } \mathcal{A}\mathcal{B} = \text{eigenvalues of } \mathcal{B}\mathcal{A}$$

instability gain

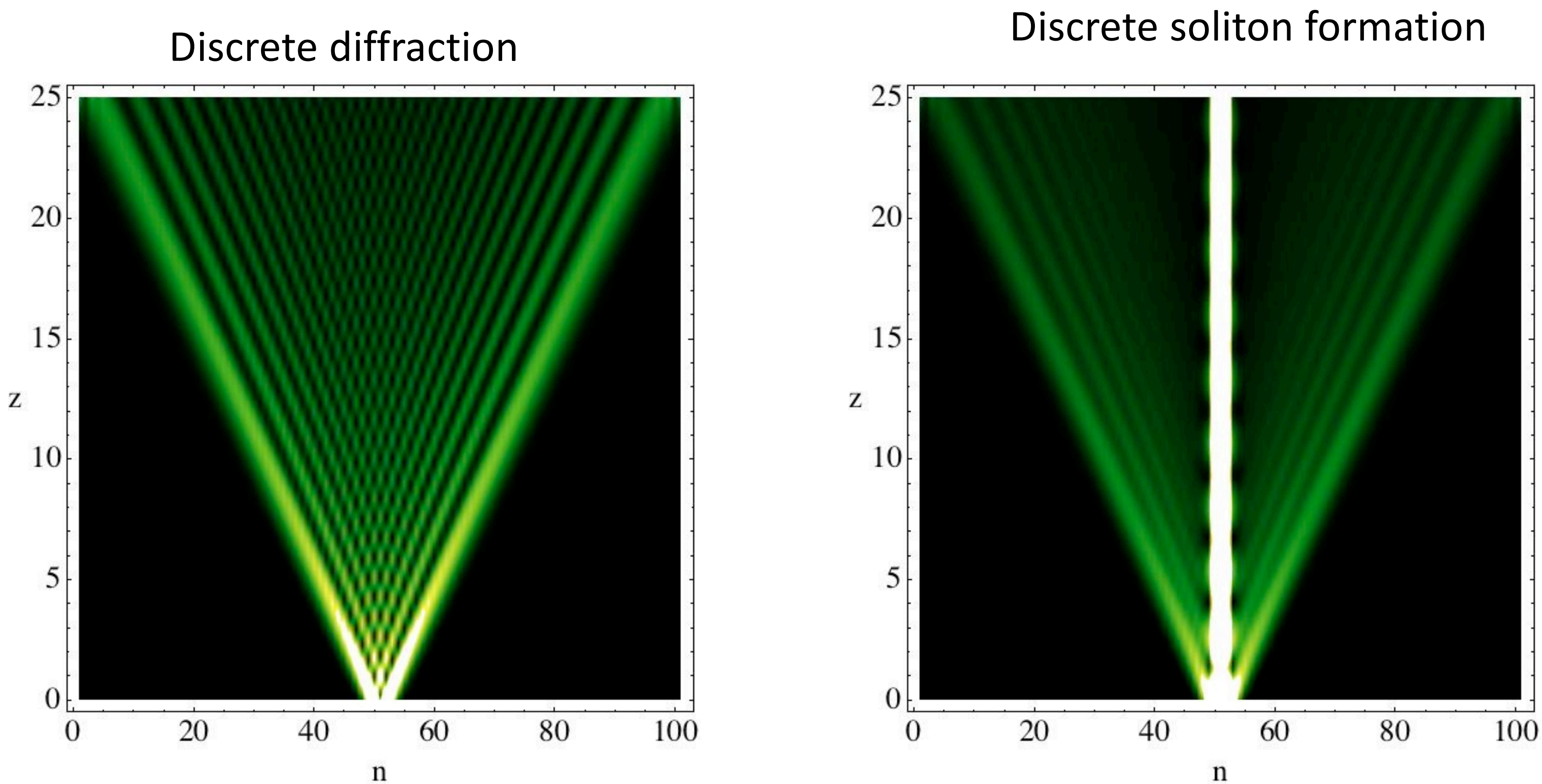
$$G^* = \mathrm{Max}\left\{\sqrt{(1/2)(-\mathrm{Re}[\mathrm{m}] + \sqrt{\mathrm{Re}[\mathrm{m}]^2 + \mathrm{Im}[\mathrm{m}]^2})}\right\}$$

$$G^* = 0 \quad \text{stable}$$

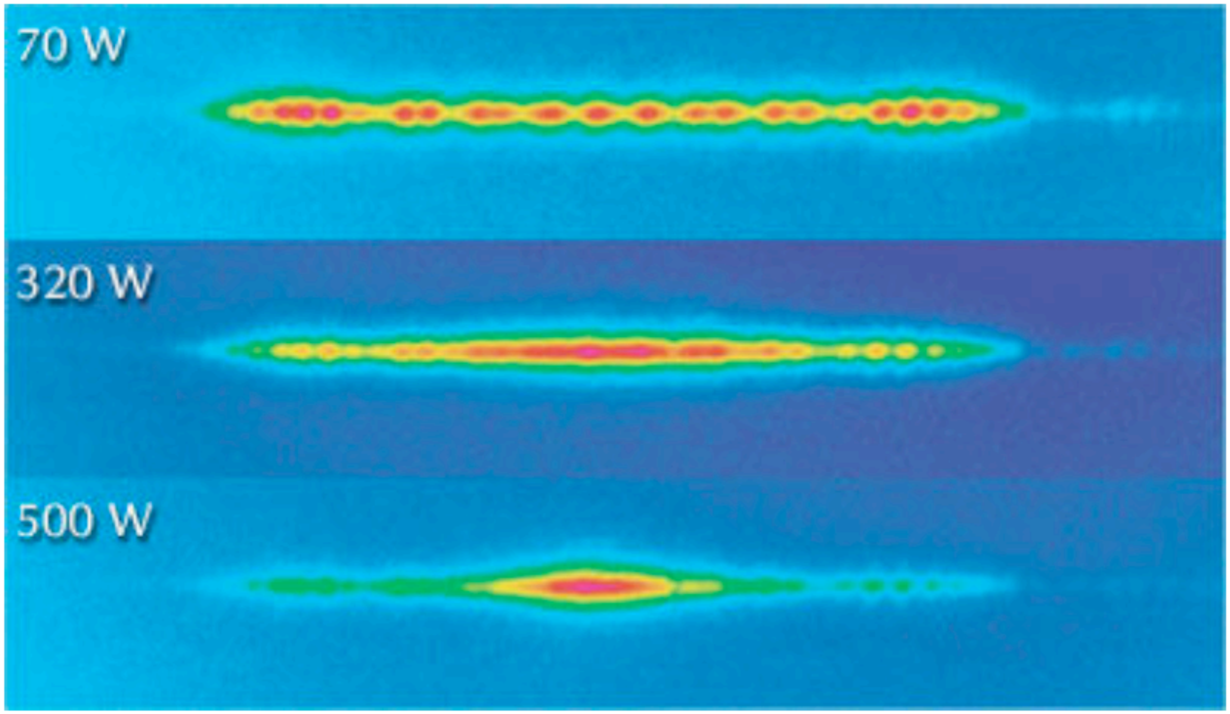
$$G^* > 0 \quad \text{unstable}$$

Coupled-modes theory

Numerical propagation



First experimental observation of discrete soliton



Difraccion discreta

Soliton discreto

H. Eisenberg at al, PRL 81, 3383 (1998).