PEMMF.
Estructuras algebraicas
Ayudantías.
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0.1 Integrales impropias

1. $\int_2^\infty \frac{x}{(x^2+1)^2}\,dx$

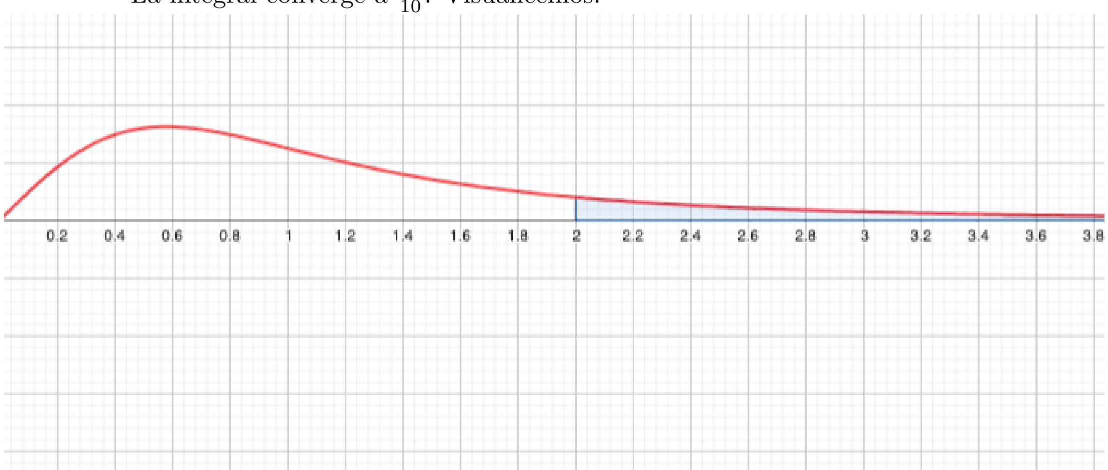
$$\int_2^\infty \frac{x}{(x^2+1)^2}\,dx = \lim_{b\to\infty}\int_2^b \frac{x}{(x^2+1)^2}\,dx$$

Usamos sustitución:

$$u = x^2+1 \quad \Rightarrow \quad du = 2xdx$$

$$\begin{aligned}
\lim_{b\to\infty}\frac{1}{2}\int_2^b \frac{2x}{(x^2+1)^2}\,dx &= \lim_{b\to\infty}\frac{1}{2}\int_{2^2+1}^{b^2+1} u^{-2}\,du \\
&= \lim_{b\to\infty}\frac{1}{2}\int_{5}^{b^2+1} u^{-2}\,du \\
&= \lim_{b\to\infty}\frac{1}{2}\left[-u^{-1}\right]_5^{b^2+1} \\
&= \lim_{b\to\infty}\frac{1}{2}\left(-\frac{1}{b^2+1}+\frac{1}{5}\right) \\
&= \frac{1}{2}\left(0+\frac{1}{5}\right) \\
&= \frac{1}{10}
\end{aligned}$$

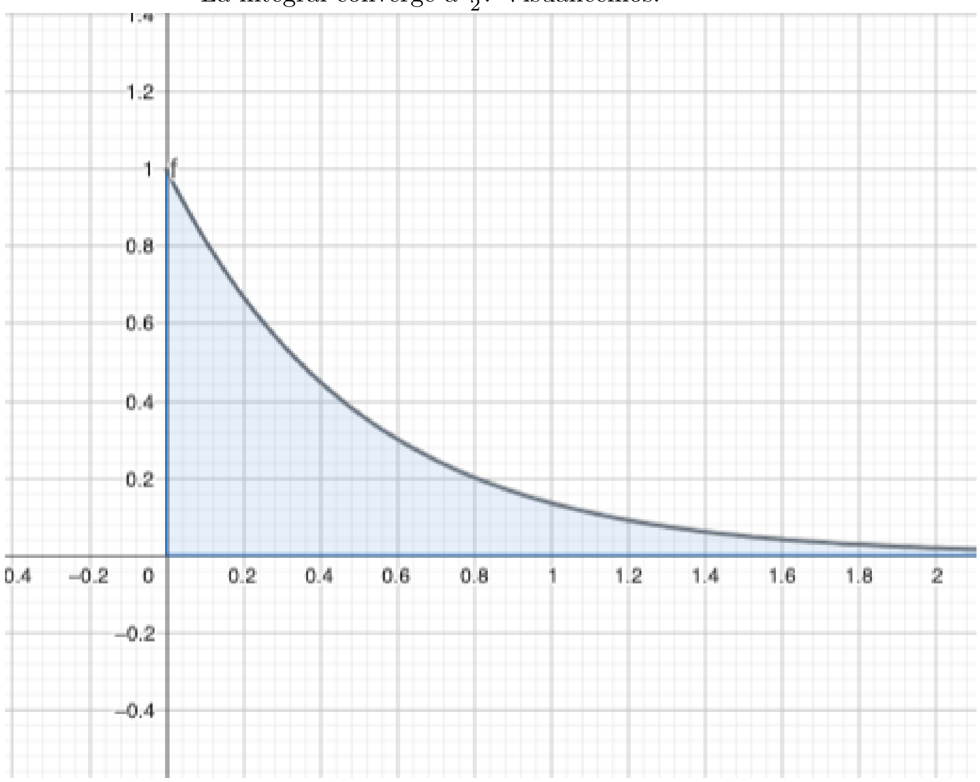
La integral converge a $\frac{1}{10}$. Visualicemos:



2.

$$\begin{aligned}\int_0^\infty e^{-2x}\,dx &= \lim_{b\to\infty}\int_0^b e^{-2x}\,dx\\ &= \lim_{b\to\infty}\left[-\frac{1}{2}e^{-2x}\right]_0^b\\ &= \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2b}+\frac{1}{2}e^{0}\right)\\ &= \lim_{b\to\infty}\left(-\frac{1}{2}e^{-2b}+\frac{1}{2}\right)\\ &= 0+\frac{1}{2}=\frac{1}{2}\end{aligned}$$

La integral converge a $\frac{1}{2}$. Visualicemos:



3.

$$\begin{aligned}
\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = \int_{-\infty}^{0} \frac{1}{1+x^2} dx + \int_{0}^{\infty} \frac{1}{1+x^2} dx \\
&= arctan(x)\Big|_{-\infty}^{0} + arctan(x)\Big|_{0}^{\infty} \\
&= (arctan(0) - arctan(-\infty)) + (arctan(\infty) - arctan(0)) \\
&= - -\frac{\pi}{2} + \frac{\pi}{2} \\
&= \pi
\end{aligned}$$