PEMMF.
Estructuras algebraicas
Ayudantías.
Rocío Acevedo

Pauta de ejercicios

1.

$$\begin{aligned}\int (a+bx^3)^2 dx =& \int \left(a^2 + 2abx^3 + b^2x^6\right) dx \\ =& a^2x + \frac{2abx^4}{4} + \frac{b^2x^7}{7} + C \\ =& a^2x + \frac{abx^4}{2} + \frac{b^2x^7}{7} + C.\end{aligned}$$

2.

$$\begin{aligned}\int \frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}} dx =& \int \frac{(a-2\sqrt{ax}+x)^2}{\sqrt{ax}} dx \\ =& \int \frac{a^2 - 4a\sqrt{ax} + 6ax - 4x\sqrt{ax} + x^2}{\sqrt{ax}} dx \\ =& \int \frac{a^2}{\sqrt{ax}} dx - 4a \int \frac{\sqrt{ax}}{\sqrt{ax}} dx + 6a \int \frac{x}{\sqrt{ax}} dx - 4 \int \frac{x\sqrt{ax}}{\sqrt{ax}} dx + \int \frac{x^2}{\sqrt{ax}} dx \\ =& a^{3/2} \int x^{1/2} dx - 4a \int 1 dx + 6a \int x^{1/2} dx - 4 \int x dx + \int x^{3/2} dx \\ =& 2a\sqrt{ax} + \frac{2x^3}{5\sqrt{ax}} - \frac{2\sqrt{a}x^{\frac{5}{2}}}{\sqrt{ax}} + \frac{4ax^2}{\sqrt{ax}} - \frac{4a^{\frac{3}{2}}x^{\frac{3}{2}}}{\sqrt{ax}}\end{aligned}$$

3.

$$\int \frac{dx}{a^2+x^2}$$

$$x = a\tan\theta \Longrightarrow dx = a\sec^2\theta d\theta$$

Sustituimos:

$$\begin{aligned}\int \frac{dx}{a^2+x^2} &= \int \frac{a\sec^2\theta}{a^2+a^2\tan^2\theta}d\theta \\ &= \int \frac{a\sec^2\theta}{a^2(1+\tan^2\theta)}\, d\theta \\ &= \int \frac{a\sec^2\theta}{a^2\sec^2\theta}\, d\theta \\ &= \int \frac{1}{a}\, d\theta \\ &= \frac{\theta}{a} + C \\ &= \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C.\end{aligned}$$

4.

$$\begin{aligned}\int \frac{dx}{2x^2+4} &= \int \frac{dx}{2x^2+4} \\ &= \frac{1}{2\sqrt{2}}\arctan(\frac{x}{\sqrt{2}}) + C.\end{aligned}$$

5.

$$\int_2^4 \frac{cos(x)}{1+sen(x)}$$

Sea $u = sen(x)+1 \Longrightarrow du = cos(x)dx.$

$$\begin{aligned}\int \frac{cos(x)}{1+sen(x)} &= \int \frac{cos(x)}{ucos(x)} \\ &= \int \frac{1}{u}du =\end{aligned}$$

$$\ln u = \ln 1 + sen(x)$$

Evaluado,

$$\ln 1 + sen(x) \mid_2^4 = \ln(1 + sen(4)) - \ln(1 + sen(2))$$

6.

$$\int_2^4 \frac{2x-1}{x^2+1} = \int_2^4 \frac{2x}{x^2+1} - \int_2^4 \frac{1}{x^2+1}$$

Resolvemos por separado. Tomamos $u = x^2 + 1 \Longrightarrow du = 2xdx$

$$\begin{aligned} \int \frac{2x}{x^2+1} &= 2 \int \frac{x}{x^2+1} \\ &= \int \frac{x}{u * 2x} du \\ &= \int \frac{1}{u} du = \ln(u) + C \end{aligned}$$

Evaluado,

$$\ln(x^2 + 1) \mid_2^4 = \ln(17) - \ln(5)$$

La otra tiene forma conocida, entonces:

$$-\int_2^4 \frac{1}{x^2+1} = -\arctan(4) - \arctan(2).$$

Así, queda:

$$\int_2^4 \frac{2x-1}{x^2+1} = \ln(17) - \ln(5) - \arctan(4) - \arctan(2) \approx 1.005.$$