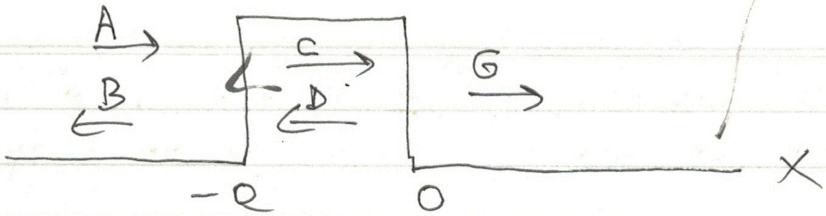


$$k=\sqrt{\frac{2mE}{\hbar^2}}$$

$u = Ae^{ikx} + Be^{-ikx}$ $\quad x<-a$

$u = Ce^{\alpha x} + De^{-\alpha x}$ $\quad -a<x<0$

$u = Ge^{ikx}$ $\quad x>0$

$$\alpha=\sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

cont. en $x=0$

u: $\quad C+D=G$

u': $\quad \alpha(C-D)=ikG$

cont. en $x=-a$

$Ae^{-ika}+Be^{ika}=Ce^{-\alpha a}+De^{\alpha a}$

$ik(Ae^{-ika}-Be^{ika})=\alpha(Ce^{-\alpha a}-De^{\alpha a})$

$$\left.\begin{aligned}\alpha C+\alpha D&=\alpha G\\ \alpha C-\alpha D&=ikG\end{aligned}\right| \qquad 2\alpha D=(\alpha-ik)G$$

$\Rightarrow \quad 2\alpha C=(\alpha+ik)G$

$C=\frac{(\alpha+ik)}{2\alpha}G \qquad D=\frac{(\alpha-ik)G}{2\alpha}$

$\Rightarrow Ce^{-\alpha a}+De^{\alpha a}=\left[\left(\frac{\alpha+ik}{2\alpha}\right)e^{-\alpha a}+\left(\frac{\alpha-ik}{2\alpha}\right)e^{\alpha a}\right]G=\left[\cosh(\alpha a)-\frac{ik}{\alpha}\sinh(\alpha a)\right]G$

y $Ce^{-\alpha a}-De^{\alpha a}=\left[\left(\frac{\alpha+ik}{2\alpha}\right)e^{-\alpha a}-\left(\frac{\alpha-ik}{2\alpha}\right)e^{\alpha a}\right]G=\left[\frac{ik}{\alpha}\cosh(\alpha a)-\sinh(\alpha a)\right]G$

$Ae^{-ika}+Be^{ika}=G\left[\cosh(\alpha a)-\frac{ik}{\alpha}\sinh(\alpha a)\right]$

$Ae^{-ika}-Be^{ika}=G\frac{\alpha}{ik}\left[\frac{ik}{\alpha}\cosh(\alpha a)-\sinh(\alpha a)\right]$

$\qquad =G\left[\cosh(\alpha a)-\frac{\alpha}{ik}\sinh(\alpha a)\right]$

next: Despejar B/A (relac. con el coefic. de reflexión)

$$\frac{Ae^{-ika}+Be^{ika}}{Ae^{-ika}-Be^{ika}} = \frac{\cosh(\alpha a)-\frac{ik}{\alpha}\sinh(\alpha a)}{\cosh(\alpha a)-\frac{\alpha}{ik}\sinh(\alpha a)}$$

$$\frac{\left(1+(B/A)e^{2ika}\right)\left(ch(\alpha a)-\frac{\alpha}{ik}sh(\alpha a)\right)}{\left(1-(B/A)e^{2ika}\right)\left(ch(\alpha a)-\frac{ik}{\alpha}sh(\alpha a)\right)} = 1$$

$$ch(\alpha)-\frac{\alpha}{ik}sh(\alpha a)+\left(\frac{B}{A}\right)e^{2ika}\left(ch(\alpha a)-\frac{\alpha}{ik}sh(\alpha a)\right) =$$

$$\left(ch(\alpha)-\frac{ik}{\alpha}sh(\alpha a)\right)-(B/A)e^{2ika}\left(ch(\alpha a)-\frac{ik}{\alpha}sh(\alpha a)\right)$$

$$\left(\frac{B}{A}\right)e^{2ika}\left[ch(\alpha a)-\frac{\alpha}{ik}sh(\alpha a)+ch(\alpha a)-\frac{ik}{\alpha}sh(\alpha a)\right] =$$

$$\cancel{ch(\alpha)}-\frac{ik}{\alpha}sh(\alpha a)-\cancel{ch(\alpha a)}+\frac{\alpha}{ik}sh(\alpha a)$$

$$e^{2ika}(B/A)\left[2ch(\alpha a)-\left(\frac{\alpha}{ik}+\frac{ik}{\alpha}\right)sh(\alpha a)\right] = \left(\frac{\alpha}{ik}-\frac{ik}{\alpha}\right)sh(\alpha a)$$

$$\frac{B}{A} = e^{-2ika}\frac{\left(\frac{\alpha}{ik}-\frac{ik}{\alpha}\right)sh(\alpha a)}{\left[2ch(\alpha a)-\left(\frac{\alpha}{ik}+\frac{ik}{\alpha}\right)sh(\alpha a)\right]} \quad \begin{matrix} ik\alpha \\ ik\alpha \end{matrix}$$

$$= e^{-2ika}\frac{(\alpha^2+k^2)\sinh(\alpha a)}{\left[2ik\alpha\cosh(\alpha a)+(k^2-\alpha^2)\sinh(\alpha a)\right]}$$

$$\Rightarrow R = \frac{|B|^2}{|A|^2} = \left[1+\frac{4(k\alpha)^2}{(k^2+\alpha^2)^2\sinh^2(\alpha a)}\right]^{-1}$$

para $k^2 = 2mE/\hbar^2$, $\alpha^2 = 2m(V_0-E)/\hbar^2$

$$R = \left[1 + \frac{4\left(\cancel{\frac{2m}{\hbar^2}}\right)E \cdot \cancel{\frac{2m}{\hbar^2}}(V_0-E)}{\cancel{\left(\frac{2m}{\hbar^2}\right)^2} V_0^2 \sinh^2(\alpha a)}\right] = \left(1 + \frac{4E(V_0-E)}{V_0^2 \sinh^2(\alpha a)}\right)$$

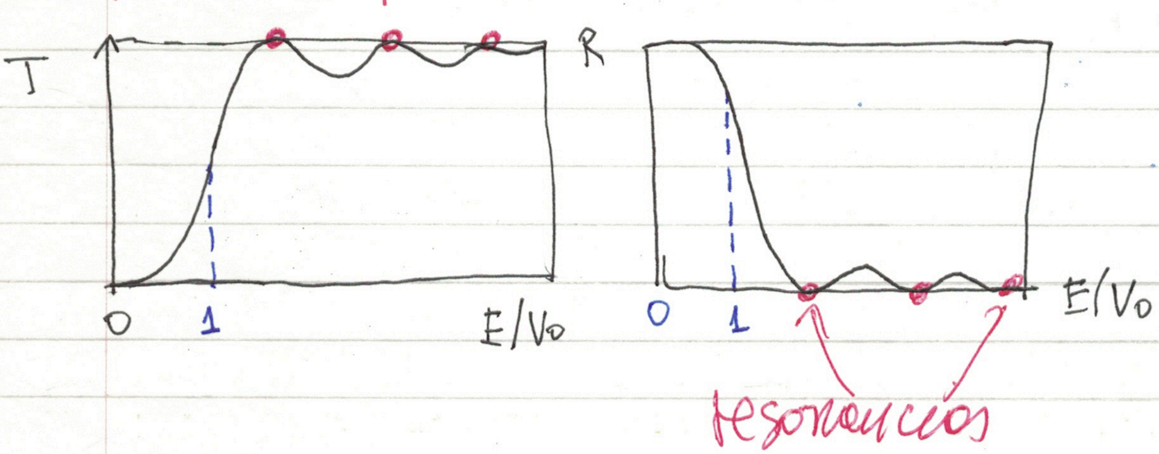
$$R = \left[1 + \frac{4(E/V_0)(1-(E/V_0))}{\sinh^2\left[\sqrt{\frac{2mV_0 a^2}{\hbar^2}\left(1-\frac{E}{V_0}\right)}\right]}\right]^{-1}$$

Graficar al para $V_0 = 8\hbar^2/ma^2 \Rightarrow R = \left[1 + \dfrac{4(E/V_0)(1-\frac{E}{V_0})}{\sinh^2\left[4\sqrt{1-\frac{E}{V_0}}\right]}\right]^{-1}$

Por su parte, $T = 1 - R$ (verdad?)

$$= \left[1 + \frac{\sinh^2\left[\sqrt{\frac{2mV_0 a^2}{\hbar^2}\left(1-\frac{E}{V_0}\right)}\right]}{4(E/V_0)(1-(E/V_0))}\right]^{-1}$$

También sirve para $E>V_0$: $(\sinh(\alpha i) = i\sin(\alpha) \Rightarrow \sinh^2(i\alpha) = -\sin^2(\alpha))$



Resonancias cuando $T = 1$
$R = 0$

$E > V_0$:
$$T = \left[1 + \frac{\sin^2\left(\sqrt{\frac{2ma^2V_0}{\hbar^2}\left(\frac{E}{V_0}-1\right)}\right)}{4(E/V_0)\left(\frac{E}{V_0}-1\right)}\right]^{-1}$$

$$T=1 \Rightarrow \sqrt{\frac{2ma^2V_0}{\hbar^2}\left(\frac{E}{V_0}-1\right)} = n\pi$$

$$\Rightarrow \boxed{E_n = V_0 + \frac{\pi^2\hbar^2}{2ma^2}\, n^2} \qquad n = 0, 1, 2, \ldots$$

ojo: No es cuantización de energías, son energías resonantes.

En general, $\langle \breve{\sigma} \rangle = \int_{-\infty}^{\infty} \psi^*(x)\, \sigma\, \psi(x)\, dx$

<u>Valores esperados e incerteza</u>

$$\Delta x \equiv x - \langle x \rangle$$

$$\delta x \equiv \sqrt{\langle (\Delta x)^2 \rangle} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad (3)$$

Igual'/. $\delta p \equiv \sqrt{\langle p_x^2 \rangle - \langle p_x \rangle^2}$ $\qquad \int_{-\infty}^{\infty} |\psi|^2 = 1 \qquad (4)$

Seja $\psi(x) = \frac{1}{\sqrt{\sigma\sqrt{2\pi}}}\, e^{-\frac{x^2}{4\sigma^2}} \Rightarrow |\psi|^2 = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}}$.

$$\Rightarrow \langle x^2 \rangle - \langle x \rangle^2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}} dx - \left[\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{x\, e^{-\frac{x^2}{2\sigma^2}} dx}_{0} \right]^2 = \sigma^2$$

$$\Rightarrow \boxed{\delta x = \sqrt{\sigma^2 - 0} = \sigma} \qquad (5)$$

$$\check{p}\psi = -i\hbar\frac{\partial\psi}{\partial x} = (-i\hbar)\left(\frac{-2x}{4\sigma^2}\right)\psi = \frac{i\hbar x}{2\sigma^2}\psi$$

$$\check{p}^2\psi = (-i\hbar)^2\frac{\partial^2\psi}{\partial x^2} = (-i\hbar)\left(\frac{i\hbar}{2\sigma^2}\right)\frac{\partial}{\partial x}(x\psi)$$

$$= \frac{\hbar^2}{2\sigma^2}\left[\psi + x\frac{\partial\psi}{\partial x}\right] = \frac{\hbar^2}{2\sigma^2}\left(\psi + x\left(\frac{-x}{2\sigma^2}\right)\psi\right)$$

$$= \frac{\hbar^2}{2\sigma^2}\left[1 - \frac{x^2}{2\sigma^2}\right]\psi$$

$$\langle p_x^2\rangle = \frac{\hbar^2}{2\sigma^2}\underbrace{\int_{-\infty}^{\infty}|\psi|^2dx}_{1} - \frac{\hbar^2}{(2\sigma^2)^2}\underbrace{\int_{-\infty}^{\infty}x^2|\psi|^2dx}_{\sigma^2}$$

$$= \frac{\hbar^2}{2\sigma^2} - \frac{\hbar^2}{4\sigma^2} = \frac{\hbar^2}{4\sigma^2}$$

$$\Rightarrow \boxed{\delta p_x = \frac{\hbar}{2\sigma}} \qquad (6)$$

$$\Rightarrow \delta x\,\delta p_x = \sigma\cdot\frac{\hbar}{2\sigma} = \frac{\hbar}{2} \; // \qquad (7)$$

[$\delta x\,\delta p_x > \hbar/2$ para qualq. ψ NO gaussiano]

Teo. de Ehrenfest

$$i\hbar \frac{\partial \Psi}{\partial t} = \check{H}\Psi \qquad (1)$$

con $\check{H} = \frac{\check{P}^2}{2m} + V \qquad (2)$

sea $\check{A}$ operador arbitrario

$$\frac{d}{dt}\langle A\rangle = \frac{d}{dt}\int \Psi^* A \Psi = \int \frac{\partial \Psi^*}{\partial t} \cdot A\Psi + \int \Psi^* \frac{\partial A}{\partial t}\Psi + \int \Psi^* A \frac{\partial \Psi}{\partial t}$$

$$(1) \rightarrow \frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar} H\Psi \;, \quad y \quad \frac{\partial \Psi^*}{\partial t} = \frac{i}{\hbar} H\Psi^* \qquad (3)$$

$$\Rightarrow \frac{d}{dt}\langle A\rangle = \frac{i}{\hbar}\langle H\Psi | A\Psi\rangle + \langle \Psi | \frac{\partial A}{\partial t}\cdot\Psi\rangle - \frac{i}{\hbar}\langle \Psi | AH\Psi\rangle$$

$$= \frac{i}{\hbar}\langle \Psi | [\check{H}, \check{A}]\Psi\rangle + \langle \frac{\partial A}{\partial t}\rangle \qquad (4)$$

$$\frac{d\langle \check{A}\rangle}{dt} = \frac{i}{\hbar}\langle [\check{H}, A]\rangle + \langle \frac{\partial A}{\partial t}\rangle \qquad (5)$$

($[A,B] = AB - BA$)

Aplicaciones: $\check{A} = \check{X}$

$$[H, X] = \left[\frac{P^2}{2m} + V, x\right] = \frac{1}{2m}[P^2, X] = \frac{1}{2m}(P^2X - XP^2)$$

$$(P^2X - XP^2)\Psi = -\hbar^2 \frac{d^2}{dx^2}(X\Psi) + \hbar^2 X \frac{d^2}{dx^2}\Psi$$

$$= -\hbar^2 \left(2\psi' + \cancel{x\psi''} - \cancel{x\psi''}\right) = -2\hbar^2 \frac{\partial \psi}{\partial x}$$

$$\therefore \ [H, x] = \frac{1}{2m} \cdot (-2)\hbar^2 \frac{\partial \psi}{\partial x} = -\frac{\hbar^2}{m} \frac{\partial \psi}{\partial x}$$

$$\Rightarrow \frac{d\langle x \rangle}{dt} = \left\langle \frac{i}{\hbar} \left(-\frac{\hbar^2}{m}\right) \frac{\partial \psi}{\partial x} \right\rangle = \frac{\langle p \rangle}{m}$$

$$\therefore \boxed{\frac{d\langle x \rangle}{dt} = \frac{\langle p \rangle}{m}} \qquad (6)$$

(2) $\check{A} = \check{P}$

$$[H, P] = \left[\frac{P^2}{2m} + V, P\right] = [V(x), P]$$

$$[V, P]\psi = (VP)\psi - (PV)\psi = V\left(-i\hbar \frac{\partial}{\partial x}\right)\psi + i\hbar \frac{\partial}{\partial x}(V\psi)$$

$$= -i\hbar(\partial \psi / \partial x)V + i\hbar(\partial V / \partial x)\psi + i\hbar V(\partial \psi / \partial x)$$

$$\Rightarrow [V, P] = i\hbar \frac{\partial V}{\partial x}$$

$$\Rightarrow \boxed{\frac{d\langle p \rangle}{dt} = \frac{i}{\cancel{\hbar}} \cdot i\cancel{\hbar} \frac{\partial V}{\partial x} = -\frac{\partial V}{\partial x} = F} \qquad (7)$$

$$(6), (7) \Rightarrow \boxed{F = m \frac{d^2\langle x \rangle}{dt^2}} \qquad (8)$$