

#2: Mov. de proyectil en Relatividad Especial.

→ lanzado con

Una partícula de masa m una veloc. inicial $v_0 \hat{x}$,
 en presencia de una $\vec{F}$ externa constante $-F \hat{y}$. Encuentra
 $v_x(t)$ y $v_y(t)$ explícito) como funciones de F, m y v_0 .

Demuestra analíticamente que

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} < c \text{ para cualquier instante.}$$

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \frac{d(m\gamma v_x)}{dt} = 0 \Rightarrow m\gamma v_x = \frac{mv_0}{\sqrt{1-(v_0/c)^2}} = p_0$$

$$\frac{d}{dt}(m\gamma v_y) = -F \Rightarrow m\gamma v_y = -Ft$$

$$\therefore \begin{aligned} m\gamma v_x &= p_0 \\ m\gamma v_y &= -Ft \end{aligned} \Rightarrow \frac{v_x}{v_y} = \frac{-p_0}{Ft} \Rightarrow \boxed{v_y = -\frac{Ft}{p_0} v_x}$$

$$\left. \begin{aligned} m^2 \gamma^2 v_x^2 &= p_0^2 \\ m^2 \gamma^2 v_y^2 &= (Ft)^2 \end{aligned} \right\} m^2 \gamma^2 v^2 = p_0^2 + (Ft)^2$$

$$\text{but } m\gamma v^2 = \frac{mv^2}{1-(v/c)^2} = (mc)^2(\gamma^2 - 1)$$

$$\Rightarrow (mc)^2(\gamma^2 - 1) = p_0^2 + (Ft)^2 \Rightarrow \boxed{\gamma^2 = 1 + \left(\frac{p_0}{mc}\right)^2 + \left(\frac{Ft}{mc}\right)^2}$$

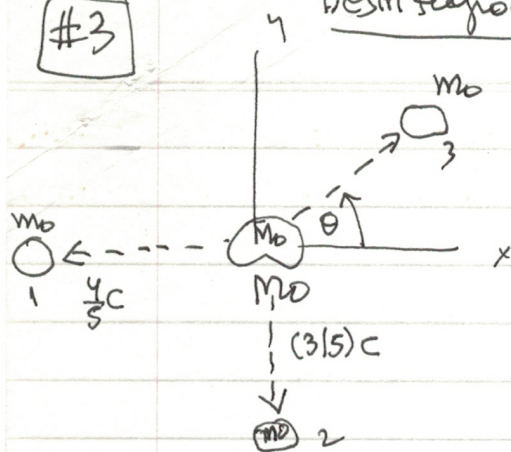
$$\text{then } v_x = \frac{p_0}{m\gamma} = \frac{p_0 c}{\sqrt{(mc)^2 + p_0^2 + (Ft)^2}}$$

$$v_y = \frac{-Fct}{\sqrt{(mc)^2 + p_0^2 + (Ft)^2}}$$

$$|\vec{v}| = c \left[\frac{p_0^2 + (Ft)^2}{(mc)^2 + p_0^2 + (Ft)^2} \right]^{1/2} < c, \text{ regardless of } t. \text{ indep. del signo de } F.$$

#3

Desintegración de una partícula



$$M_0 c^2 = m_0 \gamma_1 c^2 + m_0 \gamma_2 c^2 + m_0 \gamma_3 c^2$$

$$\frac{M_0}{m_0} = \gamma_1 + \gamma_2 + \gamma_3$$

$$\begin{aligned} m_0 \gamma_1 v_1 &= m_0 \gamma_3 v_3 \cos \theta \\ m_0 \gamma_2 v_2 &= m_0 \gamma_3 v_3 \sin \theta \end{aligned}$$

$$\Rightarrow \tan \theta = \frac{\gamma_2 v_2}{\gamma_1 v_1} = \frac{v_2}{v_1} \left(\frac{1 - (v_1/c)^2}{1 - (v_2/c)^2} \right)^{1/2}$$

$$\text{Also, } \gamma_1^2 v_1^2 + \gamma_2^2 v_2^2 = \gamma_3^2 v_3^2 = \frac{v_3^2}{1 - (v_3/c)^2} = -1 + \frac{1}{1 - (v_3/c)^2}$$

$$\Rightarrow 1 - (v_3/c)^2 = \frac{1}{1 + (\gamma_1 v_1/c)^2 + (\gamma_2 v_2/c)^2}$$

$$\Rightarrow \frac{v_3}{c} = \left[\frac{(\gamma_1 v_1/c)^2 + (\gamma_2 v_2/c)^2}{1 + (\gamma_1 v_1/c)^2 + (\gamma_2 v_2/c)^2} \right]^{1/2} < 1 \text{ always.}$$

$$\Rightarrow \gamma_3 = \sqrt{1 + (\gamma_1 v_1/c)^2 + (\gamma_2 v_2/c)^2}$$

$$\Rightarrow \frac{M}{m_0} = \gamma_1 + \gamma_2 + \left(1 + (\gamma_1 v_1/c)^2 + (\gamma_2 v_2/c)^2 \right)^{1/2}$$

$$\text{using } v_1 = \frac{4}{5}c; \quad v_2 = \frac{3}{5}c \quad \Rightarrow$$

$$\begin{aligned} \frac{M}{m_0} &= 4.74 \\ \frac{v_3}{c} &= 0.837 \\ \theta &= 29.4^\circ \end{aligned}$$

1. Demuestre rigurosamente que los siguientes procesos son imposibles:
 - (a) Un solo fotón choca contra un electrón estacionario y le transfiere toda su energía.
 - (b) Un solo fotón en el vacío es transformado en un electrón mas un positrón.
 - (c) Un positrón rápido y un electrón estacionario se aniquilan mutuamente, produciendo un solo fotón.
2. Considere el decaimiento de un núcleo radiactivo en reposo por emisión de partículas alfa

$$X \rightarrow Y + \alpha \tag{1}$$

Definimos la energ a de desintegraci n Q como $Q \equiv (M_X - M_Y - M_\alpha)c^2$. Esta energ a debe ser compartida entre la part cula α y el n cleo Y con el objeto de conservar la energ a y el momento del proceso de decaimiento. Encuentre una expresi n para la energ a cin tica K_α en t rminos de Q y las masas en reposo de los n cleos.



$$\left. \begin{aligned} E_\gamma + mc^2 &= m\gamma c^2 \\ \frac{E_\gamma}{c} &= m\gamma v \end{aligned} \right\}$$

$$\left. \begin{aligned} E_\gamma + mc^2 &= m\gamma c^2 \\ E_\gamma &= m\gamma cv \end{aligned} \right\}$$

$$mc^2 = m\gamma c^2 - m\gamma cv$$

$$\cancel{c^2} = \gamma \cancel{c^2} \left(1 - \frac{v}{c}\right)$$

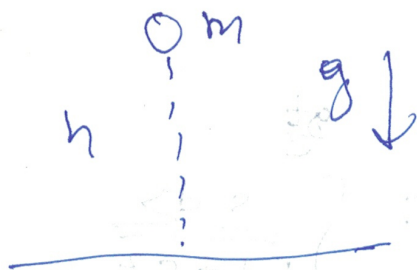
$$1 = \gamma \left(1 - \frac{v}{c}\right) = \frac{1}{\sqrt{1 - (v/c)^2}} \left(1 - (v/c)\right)$$

$$= \frac{1}{\sqrt{(1 + \frac{v}{c})(1 - \frac{v}{c})}} \sqrt{1 - (v/c)} \sqrt{1 - (v/c)}$$

$$1 = \sqrt{\frac{1 - (v/c)}{1 + (v/c)}} < 1$$

$\times$

#2



calcular tiempo de caída

$$F = mg = \frac{d}{dt}(m\gamma u) \Rightarrow g = \frac{d}{dt}(\gamma u) = \gamma \frac{du}{dt} + u \frac{d\gamma}{dt}$$

$$= \gamma \frac{du}{dt} + u \frac{d\gamma}{du} \frac{du}{dt}$$

$$g = \left(\frac{du}{dt}\right) \left(\gamma + u \frac{d\gamma}{du}\right)$$

$$\gamma = \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2} \Rightarrow \frac{d\gamma}{du} = \frac{1}{2} \left(1 - \left(\frac{u}{c}\right)^2\right)^{-3/2} \cdot \left(\frac{2u}{c^2}\right) = \frac{u}{c^2} \left(1 - \left(\frac{u}{c}\right)^2\right)^{-3/2}$$

$$g = \left(\frac{du}{dt}\right) \left[\gamma + \frac{(u/c)^2}{\left(1 - (u/c)^2\right)^{3/2}} \right] = \left(\frac{du}{dt}\right) \left(\frac{1 - (u/c)^2 + (u/c)^2}{\left(1 - (u/c)^2\right)^{3/2}} \right) =$$

$$g = \left(1 - \left(\frac{u}{c}\right)^2\right)^{-3/2} \left(\frac{du}{dt}\right)$$

$$\int_0^t g dt = \int_0^u du \left(1 - \left(\frac{u}{c}\right)^2\right)^{-3/2} = \frac{u}{\sqrt{1 - (u/c)^2}} \Big|_0^u = \frac{u}{\sqrt{1 - (u/c)^2}}$$

$$\therefore (gt)^2 = \frac{u^2}{1 - (u/c)^2} \Rightarrow (gt)^2 - (gt)^2 \left(\frac{u}{c}\right)^2 = u^2$$

$$(gt)^2 = u^2 \left(1 + \left(\frac{gt}{c}\right)^2\right)$$

$$\therefore u(t) = \frac{gt}{\sqrt{1 + \left(\frac{gt}{c}\right)^2}}$$

$$u(t) \xrightarrow{t \rightarrow \infty} c$$

límite no-relativista:
 $c \rightarrow \infty, u \rightarrow gt$

$$\frac{dy}{dt} = \frac{gt}{(1 + (\frac{gt}{c})^2)^{1/2}}$$

$$y(t) = h + \int_0^t \frac{gt \, dt}{(1 + (\frac{gt}{c})^2)^{1/2}} = \frac{c^2}{g} \int_0^{\frac{gt}{c}} \frac{s \, ds}{\sqrt{1+s^2}}$$

$\frac{d}{ds} \sqrt{1+s^2}$

$$= \frac{c^2}{g} \sqrt{1+s^2} \Big|_0^{\frac{gt}{c}} = \frac{c^2}{g} [\sqrt{1 + (\frac{gt}{c})^2} - 1]$$

$$\boxed{y(t) = h - \frac{c^2}{g} (1 + \sqrt{1 + (\frac{gt}{c})^2})}$$

$$y=0 \Rightarrow h = \frac{c^2}{g} (\dots) \Rightarrow (\frac{gh}{c^2}) = 1 + \sqrt{1 + (\frac{gt}{c})^2}$$

$$\sqrt{1 + (\frac{gt}{c})^2} = (\frac{gh}{c^2}) \Rightarrow 1 + (\frac{gt}{c})^2 = (\frac{gh}{c^2})^2$$

$$-1 + (1 + (\frac{gh}{c^2})^2) = (\frac{gt}{c})^2$$

$$\Rightarrow \boxed{t = \frac{c}{g} [-1 + (1 + \frac{gh^2}{c^2})]^{1/2}}$$

carga electrica en
campo uniforme

$$F = eE = \frac{dp}{dt}$$

$$\frac{d}{dt}(m \gamma u) = eE$$

$$\frac{d}{dt} \left(\frac{u}{(1 - (u/c)^2)^{1/2}} \right) = \frac{eE}{m}$$

$$\frac{\left(\frac{du}{dt} \right)}{\sqrt{1 - (u/c)^2}} + u \cdot \left(\frac{-1/2}{1 - (u/c)^2} \right) \cdot \left(\frac{-2u}{c^2} \right) \frac{du}{dt} = \frac{eE}{m}$$

$$\left(\frac{du}{dt} \right) \left[\frac{1}{(1 - (u/c)^2)^{1/2}} + \frac{(u/c)^2}{(1 - (u/c)^2)^{3/2}} \right] = \frac{eE}{m}$$

$$\frac{1 - (u/c)^2 + (u/c)^2}{(1 - (u/c)^2)^{3/2}} = \frac{1}{(1 - (u/c)^2)^{3/2}}$$

$$\left(\frac{du}{dt} \right) \cdot \frac{1}{(1 - (u/c)^2)^{3/2}} = \frac{eE}{m}$$

$$\boxed{\frac{du}{dt} = \frac{eE}{m} (1 - (u/c)^2)^{3/2}}$$

mas facil: $\frac{d}{dt}(m\gamma u) = qE$
 $m\gamma u = qEt$

$$\gamma u = \frac{qEt}{m}$$

$$\gamma^2 u^2 = \left(\frac{qEt}{m}\right)^2$$

$$c^2(\gamma^2 - 1) = \left(\frac{qEt}{m}\right)^2$$

$$\gamma^2 = 1 + \left(\frac{qEt}{mc}\right)^2 = \frac{1}{1 - \left(\frac{u}{c}\right)^2}$$

$$\Rightarrow 1 - \left(\frac{u}{c}\right)^2 = \frac{1}{1 + \left(\frac{qEt}{mc}\right)^2}$$

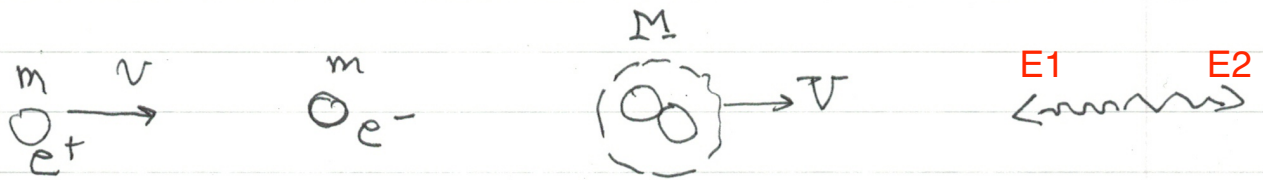
$$\left(\frac{u}{c}\right)^2 = 1 - \frac{1}{1 + \left(\frac{qEt}{mc}\right)^2} = \frac{1 + \left(\frac{qEt}{mc}\right)^2 - 1}{1 + \left(\frac{qEt}{mc}\right)^2} = \frac{\left(\frac{qEt}{mc}\right)^2}{1 + \left(\frac{qEt}{mc}\right)^2}$$

$$\boxed{\frac{u}{c} = \frac{\left(\frac{qEt}{mc}\right)}{\sqrt{1 + \left(\frac{qEt}{mc}\right)^2}}} = \frac{dx}{dt}$$

$$u = \frac{\left(\frac{qEt}{m}\right)}{\sqrt{1 + \left(\frac{qEt}{mc}\right)^2}} = \frac{dx}{dt}$$

$$X = \left(1 + \left(\frac{qEt}{mc}\right)^2\right)^{1/2} \frac{mc^2}{qE} - \frac{mc^2}{qE}$$

$$X(t) = \frac{mc^2}{qE} \left[\sqrt{1 + \left(\frac{qEt}{mc}\right)^2} - 1 \right]$$



$$m\gamma(v)v + 0 = M\gamma(v)V \quad (1)$$

$$m\gamma(v)c^2 + mc^2 = M\gamma(v)c^2 \quad (2)$$

$$\frac{m\gamma(v)v}{m\gamma(v)c^2 + mc^2} = \frac{V}{c^2}$$

$$V = \frac{\gamma(v)v}{\gamma(v) + 1} = \left(\frac{\gamma}{\gamma + 1} \right) v$$

luego decae
en 2 fotones:

$$\begin{aligned} M\gamma_2 c^2 &= E_1 + E_2 \\ M\gamma_2 V c &= E_1 - E_2 \end{aligned}$$

$$\Rightarrow \begin{aligned} E_1 &= \frac{1}{2} M\gamma_2 c^2 \left(1 + \frac{V}{c} \right) \\ E_2 &= \frac{1}{2} M\gamma_2 c^2 \left(1 - \frac{V}{c} \right) \end{aligned}$$

$$\text{but } M\gamma_2 \stackrel{(2)}{=} m\gamma_1 + m = m(\gamma_1 + 1)$$

$$E_1 = \frac{1}{2} mc^2 (1 + \gamma_1) \left(1 + \frac{V}{c} \right)$$

$$E_2 = \frac{1}{2} mc^2 (1 + \gamma_1) \left(1 - \frac{V}{c} \right)$$