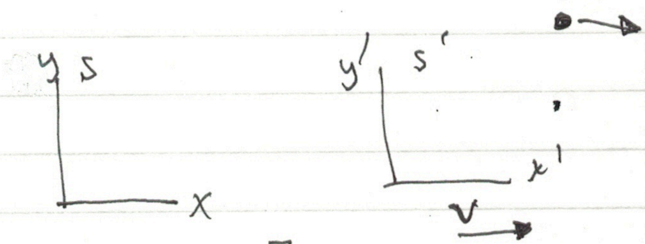
Movimientos Acelerados

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} \quad ; \quad u_y = \frac{u_y'/\gamma}{1 + \frac{v u_x'}{c^2}}$$

$$t = \gamma\left(t' + \frac{v x'}{c^2}\right)$$



$$du_x = \frac{du_x'}{1 + \frac{v u_x'}{c^2}} - \left[\frac{(u_x' + v)}{\left(1 + \frac{v u_x'}{c^2}\right)^2} \cdot \frac{v}{c^2} du_x'\right]$$

$$= \frac{1 + \cancel{\frac{v u_x'}{c^2}} - \cancel{u_x' \frac{v}{c^2}} - \left(\frac{v}{c}\right)^2}{\left(1 + \frac{v u_x'}{c^2}\right)^2} du_x' = \frac{\left[1 - \left(\frac{v}{c}\right)^2\right] du_x'}{\left(1 + \frac{u_x' v}{c^2}\right)^2}$$

también $dt = \gamma\left(dt' + \frac{v}{c^2} dx'\right) = \gamma\, dt'\left(1 + \frac{v u_x'}{c^2}\right)$

$$\Rightarrow a_x = \frac{du_x}{dt} = \frac{(du_x'/dt')}{\gamma^3 \left(1 + \frac{v u_x'}{c^2}\right)^3}$$

$$\therefore \boxed{a_x = \frac{a_x'}{\gamma^3 \left[1 + \frac{v u_x'}{c^2}\right]^3}}$$

similarmente [Ejercicio]

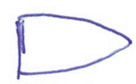
$$a_y = \frac{a_y'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^2} - \frac{(v u_y'/c^2)\, a_x'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^3}.$$

Ejemplo cinemática relativista

S' $\qquad a_{x'} = g$

$u_{x'} = 0$

S



v

$$a_x = \frac{a_{x'}}{\gamma^3\left(1 + \frac{v u_{x'}}{c^2}\right)^3} = \frac{g}{\gamma^3(u_x)}$$

$$\frac{du_x}{dt} = g\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}$$

$$\underbrace{\int_0^{u_x} \frac{du_x}{\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}}} = \int_0^t g\,dt$$

$$\frac{u_x}{\sqrt{1-\left(\frac{u_x}{c}\right)^2}} = gt$$

$$\Rightarrow \frac{u^2}{1-\frac{u^2}{c^2}} = (gt)^2 \Rightarrow u^2 = (gt)^2 - (gt)^2\frac{u^2}{c^2}$$

$$u^2\left(1+\frac{(gt)^2}{c^2}\right) = (gt)^2$$

$$u^2 = \frac{(gt)^2}{1+\left(\frac{gt}{c}\right)^2} \Rightarrow \boxed{u = \frac{gt}{\sqrt{1+(gt/c)^2}}} \qquad (*)$$

for $u = c/2$

$$\Rightarrow \left(\frac{c}{2}\right)^2\left(1+\left(\frac{gt}{c}\right)^2\right) = (gt)^2 \Rightarrow (c/2)^2 = (gt)^2 - \left(\frac{gt}{2}\right)^2 = \frac{3}{4}(gt)^2$$

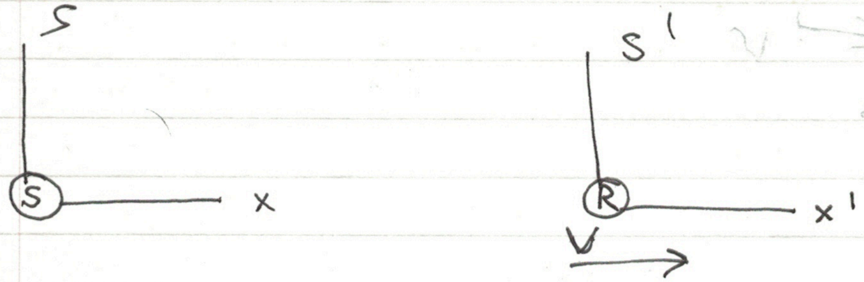
$$c^2 = 3(gt)^2 \Rightarrow c = \sqrt{3}\,gt$$

$$\Rightarrow \boxed{t = \frac{c}{\sqrt{3}\,g}} = 6.8 \text{ months}$$

$$(*) \Rightarrow x(t) = \frac{c^2}{g}\left[-1+\sqrt{1+(gt/c)^2}\right]$$

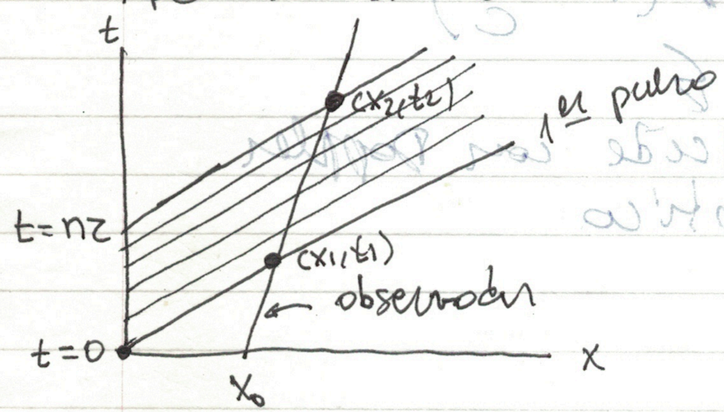
si $c \gg 1$, $x \rightarrow \frac{1}{2}gt^2$

Efecto Doppler relativista



Fuente (S) emite pulsos de luz en $t = n\tau$ $\Rightarrow$ en S, la frec. medida es $\nu = 1/\tau$

El 1er pulso es emitido en $t=0$, cuando el receptor (S') está en $x = x_0$



$$x_1 = ct_1 = x_0 + vt_1$$

$$x_2 = c(t_2 - n\tau) = x_0 + vt_2$$

$$\Rightarrow t_2 - t_1 = \frac{cn\tau}{c-v}$$

$$x_2 - x_1 = \frac{vcn\tau}{c-v}$$

En S': $$t_2' - t_1' = \gamma\left[(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)\right]$$

$$= \gamma\left(\frac{cn\tau}{c-v} - \frac{v}{c^2}\cdot\frac{vcn\tau}{c-v}\right) = \frac{\gamma c\tau n}{c-v}\left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \tau' = \frac{\gamma c\tau}{c-v}\left(1 - \frac{v^2}{c^2}\right) = \frac{\gamma\tau}{1-\left(\frac{v}{c}\right)}\left(1 - \frac{v^2}{c^2}\right) = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}\cdot\left(1-\frac{v^2}{c^2}\right)\cdot\frac{1}{1-\frac{v}{c}}$$

$$= \frac{\tau\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \tau\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \Rightarrow \boxed{\nu' = \left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}\nu}$$

Doppler longitudinal $\longrightarrow$

$$\nu' = \nu\left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}$$

como $v/c \ll 1$

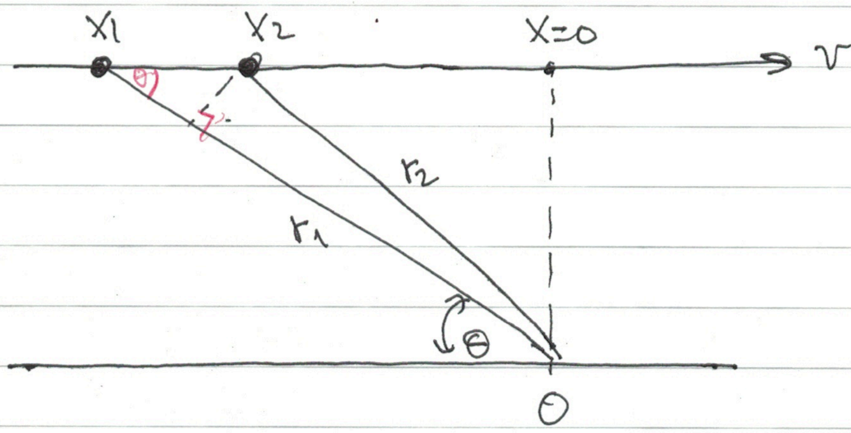
$$\nu' \approx \nu\left[\left(1-\frac{v}{c}\right)\left(1-\frac{v}{c}+\left(\frac{v}{c}\right)^2-\dots\right)\right]^{1/2}$$

$$\approx \nu\left[\left(1-\frac{v}{c}\right)\left(1-\frac{v}{c}\right)\right]^{1/2} = \nu\left(1-\frac{v}{c}\right)$$

Σ

· coincide con Doppler acústico

Doppler Transversal



2 pulsos sucesivos son emitidos en $x=x_1$ y $x=x_2$ en los instantes $t=t_1$ y $t=t_2$.

En el sist. en reposo c/r al satélite, el intervalo entre pulsos es τ. $\Rightarrow t_2-t_1=\gamma\tau$ (por dilatación temporal)

El pulso #1 demora r_1/c en llegar a O

" " #2 " r_2/c " " " O

$\Rightarrow$ Intervalo entre pulsos: $\tau' = t_2 + \frac{r_2}{c} - (t_1 + r_1/c)$

Si $|x_2-x_1| \ll r_1 \Rightarrow r_1-r_2 \simeq (x_2-x_1)\cos\theta$

$= (v t_2 - v t_1)\cos\theta = v(t_2-t_1)\cos\theta = v\gamma\tau\cos\theta$

$\therefore \tau' = (t_2-t_1) - \frac{1}{c}(r_1-r_2) = \gamma\tau - \gamma\frac{v}{c}\tau\cos\theta = \gamma\tau\left(1-\frac{v}{c}\cos\theta\right)$

$$\Rightarrow \nu' = \frac{\nu}{\gamma\left(1-\frac{v}{c}\cos\theta\right)} = \boxed{\frac{\nu\,\left(1-(v/c)^2\right)^{1/2}}{\left(1-(v/c)\cos\theta\right)}}$$