

Prob. 26 (Setway) →

Two spaceships A and B are moving along the x-axis. Spacecraft A has length L and spacecraft B has length $3L$.

Both spaceships are moving with the same speed v .

Spacecraft A is moving to the right with velocity v_A . What is the velocity v_B of spacecraft B?

The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of $0.35c$, determine the speed of the faster spaceship.

$\gamma = \frac{c}{v}$ $\gamma_A = \frac{c}{v_A}$ $\gamma_B = \frac{c}{v_B}$

$$\textcircled{S} \quad L_A = \frac{L}{\gamma_A}$$

$$L_B = \frac{3L}{\gamma_B}$$

$$\text{but } L_A = L_B \Rightarrow 1 = \frac{\frac{L}{\gamma_A}}{\frac{3L}{\gamma_B}} = \frac{\gamma_B}{\gamma_A \cdot 3} = \frac{\gamma_B}{3\gamma_A}$$

$$\Rightarrow \gamma_B = 3\gamma_A \Rightarrow \frac{1}{\gamma_B} = \frac{1}{3\gamma_A} \Rightarrow \sqrt{1 - \left(\frac{v_B}{c}\right)^2} = \frac{1}{3} \sqrt{1 - \left(\frac{v_A}{c}\right)^2}$$

$$1 - \left(\frac{v_B}{c}\right)^2 = \frac{1}{9} \left(1 - \left(\frac{v_A}{c}\right)^2\right) = \frac{1}{9} - \frac{1}{9} \left(\frac{v_A}{c}\right)^2$$

$$\frac{8}{9} - \left(\frac{v_B}{c}\right)^2 = \frac{1}{9} \left(\frac{v_A}{c}\right)^2 \Rightarrow \left(\frac{v_B}{c}\right)^2 = \frac{8}{9} + \frac{1}{9} \left(\frac{v_A}{c}\right)^2$$

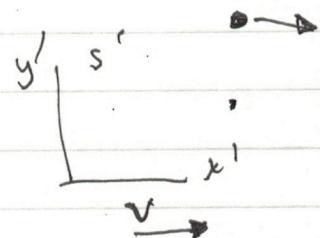
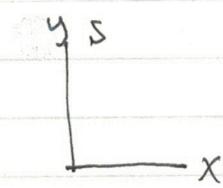
$$\frac{v_B}{c} = \sqrt{\frac{8}{9} + \frac{1}{9} \left(\frac{v_A}{c}\right)^2} < 1$$

$$= 0.95$$

Movimientos Acelerados

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} ; \quad u_y = \frac{u_y' / \gamma}{1 + \frac{v u_x'}{c^2}}$$

$$t = \gamma (t' + \frac{u_x' v}{c^2})$$



$$\begin{aligned} du_x &= \frac{du_x'}{1 + \frac{v u_x'}{c^2}} - \left[\frac{(u_x' + v)}{\left(1 + \frac{v u_x'}{c^2}\right)^2} \cdot \frac{v}{c^2} du_x' \right] \\ &= \frac{1 + \frac{v u_x'}{c^2} - u_x' \cancel{\frac{v}{c^2}} - \left(\frac{v}{c}\right)^2}{\left(1 + \frac{v u_x'}{c^2}\right)^2} du_x' = \frac{\left(1 - \left(\frac{v}{c}\right)^2\right) du_x'}{\left(1 + \frac{u_x' v}{c^2}\right)^2} \end{aligned}$$

teniendo $dt = \gamma (dt' + \frac{v}{c^2} dx')$ $\Rightarrow dt' = \gamma dt (1 + \frac{v u_x'}{c^2})$

$$\Rightarrow a_x = \frac{du_x}{dt} = \frac{\left(du_x'/dt'\right)}{\gamma^3 \left(1 + \frac{v u_x'}{c^2}\right)^3}$$

$$\therefore a_x = \frac{a_x'}{\gamma^3 \left[1 + \frac{v u_x'}{c^2}\right]^3}$$

similarmente [Ejercicio]

$$a_y = \frac{a_y'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^2} - \frac{(v u_y' / c^2) a_x'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^3}$$

Ejemplo cinemática relativista

(S)



$$Q_x^1 = g$$

$$Q_x^1 = 0$$

$$Q_x = \frac{Q_x^1}{\gamma^3(1 + \gamma u_x^1)^3} = \frac{g}{\gamma^3(u_x)}$$

$$\frac{du_x}{dt} = g \left(1 - \left(\frac{u_x}{c}\right)^2\right)^{3/2}$$

$$\int_{0}^{u_x} \frac{du_x}{\left(1 - \left(\frac{u_x}{c}\right)^2\right)^{3/2}} = \int_0^t g dt$$

$$\underbrace{\frac{u_x}{\sqrt{1 - \left(\frac{u_x}{c}\right)^2}}} = gt$$

$$\Rightarrow \frac{u^2}{1 - \frac{u^2}{c^2}} = (gt)^2 \Rightarrow u^2 = (gt)^2 - (gt)^2 \frac{u^2}{c^2}$$

$$u^2 \left(1 + \frac{(gt)^2}{c^2}\right) = (gt)^2$$

$$u^2 = \frac{(gt)^2}{1 + \frac{(gt)^2}{c^2}} \Rightarrow \boxed{u = \frac{gt}{\sqrt{1 + (gt/c)^2}}} \quad (*)$$

$$\text{Si } u = c/2$$

$$\Rightarrow \left(\frac{c}{2}\right)^2 \left(1 + \frac{(gt)^2}{c^2}\right) = (gt)^2 \Rightarrow (c/2)^2 = (gt)^2 - \left(\frac{gt}{2}\right)^2 = \frac{3}{4}(gt)^2$$

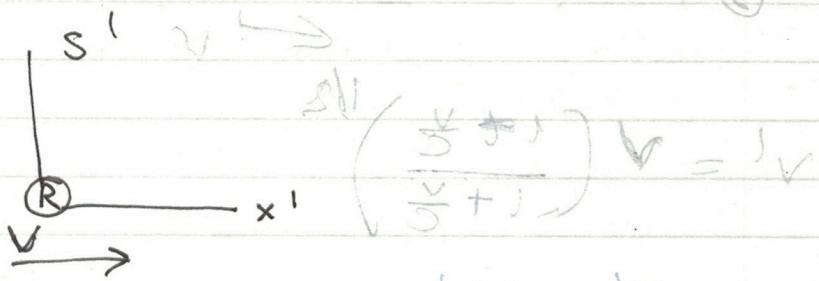
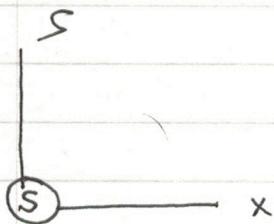
$$c^2 = 3(gt)^2 \Rightarrow c = \sqrt{3}gt$$

$$(*) \Rightarrow x(t) = \frac{c^2}{g} \left[-1 + \sqrt{1 + (gt/c)^2} \right]$$

$$\text{si } c \gg 1, x \rightarrow \frac{1}{2}gt^2$$

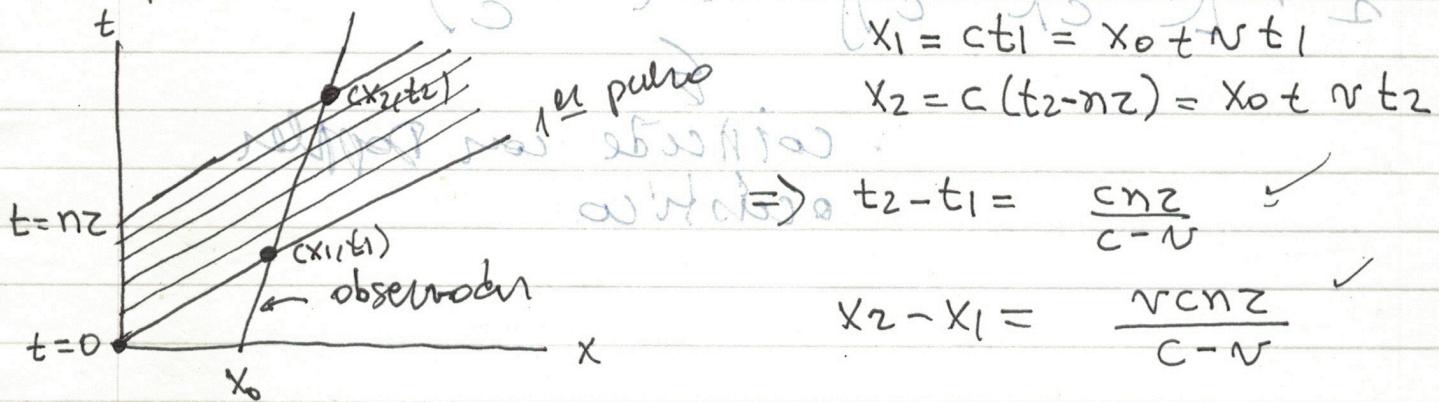
$$\Rightarrow \boxed{t = \frac{c}{\sqrt{3}g}} = 6.8 \text{ months}$$

Efecto Doppler relativo



Fuente (S) emite pulso de luz en $t = nz \Rightarrow$ en S' , la freq. medida es $\nu = 1/(z + (v/c))$ $\nu \approx 1/v$

El 1^{er} pulso \rightarrow en el punto en $t = 0$, cuando el receptor (S') está en $x = (x_0(v-1)/c) + x_0$ $v = ((v-1)(v-1)/c) v$



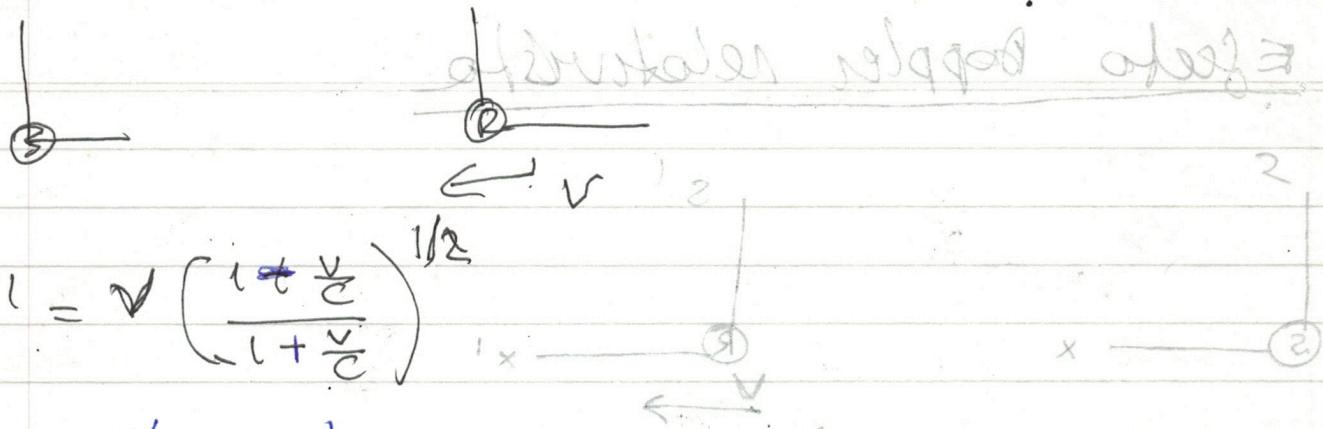
$$\text{En } S': t_2' - t_1' = \gamma [(t_2 - t_1) - \frac{v}{c^2} (x_2 - x_1)]$$

$$= \gamma \left(\frac{cnz}{c-v} - \frac{v}{c^2} \cdot \frac{vcnz}{c-v} \right) = \frac{\gamma cnz}{c-v} \left(1 - \frac{v^2}{c^2} \right)$$

$$\Rightarrow z' = \frac{\gamma cnz}{c-v} \left(1 - \frac{v^2}{c^2} \right) = \frac{\gamma z}{1 - (\frac{v}{c})} \left(1 - \frac{v^2}{c^2} \right) = \frac{z}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \left(1 - \frac{v^2}{c^2} \right) \cdot \frac{1}{1 - \frac{v}{c}}$$

$$= \frac{z \sqrt{1 - \frac{v^2}{c^2}}}{1 - \gamma} = z \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \Rightarrow \boxed{\nu' = \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2} \nu}$$

Doppler longitudinal



$$v' = v \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2}$$

$v/c \ll 1$

$$v' \approx v \left[\left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} + \left(\frac{v}{c} \right)^2 \right)^{-1/2} \right]$$

(2) At time t , the source is at position $x - vt$. The distance to the source is $s = c(t)$.

$$s = v \left[\left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} + \left(\frac{v}{c} \right)^2 \right)^{-1/2} \right] = v \left(1 - \frac{v}{c} \right)$$

$$st + vt = (ct - vt) \Rightarrow s = ct$$

coincide con Doppler

$$\frac{s}{c-v} = \frac{ct}{c-v}$$

$$\frac{s}{c-v} = \frac{ct}{c-v}$$

$$[(ct - vt) / (c - v)] - [(ct - vt) / (c - v)] = 1 \quad \text{m/s}$$

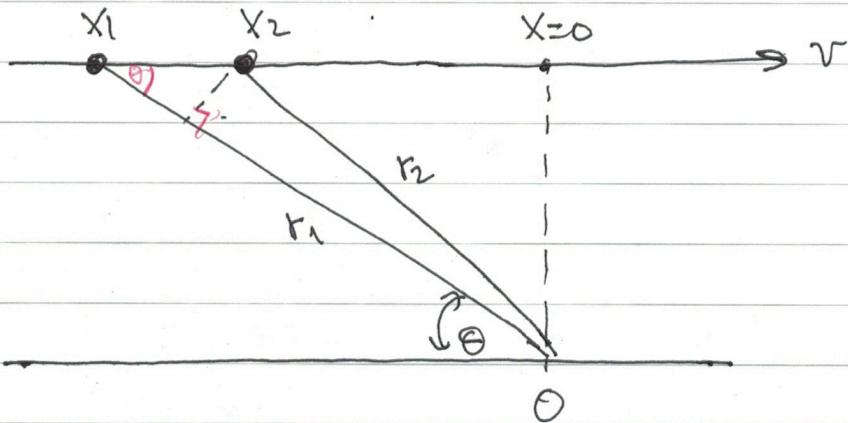
$$= \left(\frac{c-v}{c} - \frac{c-v}{c} \cdot \frac{v}{c} \right) = \frac{v}{c} \cos(1 - \alpha_s)$$

$$\frac{1 - \left(\frac{v}{c} - 1 \right)}{\frac{c-v}{c}} \cdot \frac{v}{\frac{c-v}{c}} = \left(\frac{v}{c} - 1 \right) \cdot \frac{cv}{c-v} = \left(\frac{v}{c} - 1 \right) \frac{cv}{c-v} = 1 \quad \Leftarrow$$

$$\boxed{v \left(\frac{\frac{v}{c} - 1}{\frac{v}{c} + 1} \right) = 1} \quad \Leftarrow \quad \boxed{\frac{\frac{v}{c} + 1}{\frac{v}{c} - 1} v = \frac{1 + v/c - 1}{v/c - 1} v =}$$

Dopplerfrequenz

Doppler Transversal



2 pulsos sucesivos son emitidos en $x=x_1$ y $x=x_2$ en los instantes $t=t_1$ y $t=t_2$.

En el sist. en reposo c/n el satélite, el intervalo entre pulsos es τ . $\Rightarrow t_2-t_1 = \gamma \tau$ (por dilatación temporal)

El pulso #1 demora r_1/c en llegar a O
 " " #2 " " r_2/c " " " "

$$\Rightarrow \text{Intervalo entre pulsos: } \tau' = t_2 + \frac{r_2}{c} - (t_1 + \frac{r_1}{c})$$

$$\text{Si } |x_2 - x_1| \ll r_1 \Rightarrow r_1 - r_2 \approx (x_2 - x_1) \cos \theta$$

$$= (vt_2 - vt_1) \cos \theta = v(t_2 - t_1) \cos \theta = v \gamma \tau \cos \theta$$

$$\therefore \tau' = (t_2 - t_1) + \frac{1}{c} (r_1 - r_2) = \gamma \tau - \frac{v}{c} \gamma \tau \cos \theta = \gamma \tau \left(1 - \frac{v}{c} \cos \theta \right)$$

$$\Rightarrow v' = \frac{v}{\left(1 - \frac{v}{c} \cos \theta \right)} = \boxed{\frac{v \left(1 - \left(\frac{v}{c} \right)^2 \cos^2 \theta \right)^{1/2}}{\left(1 - \left(\frac{v}{c} \right)^2 \cos^2 \theta \right)}}$$