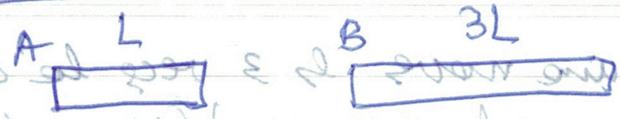
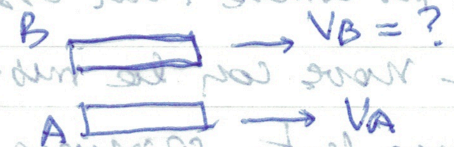
Prob. 26 (Serway) →



The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of 0.35c, determine the speed of the faster spaceship.



S :: $L_A = \frac{L}{\gamma_A}$

$$L_B = \frac{3L}{\gamma_B}$$

but $L_A = L_B \Rightarrow 1 = \frac{\frac{L}{\gamma_A}}{\frac{3L}{\gamma_B}} = \frac{\cancel{L}\gamma_B}{\gamma_A \cdot 3\cancel{L}} = \frac{\gamma_B}{3\gamma_A}$

$$\Rightarrow \gamma_B = 3\gamma_A \Rightarrow \frac{1}{\gamma_B} = \frac{1}{3\gamma_A} \Rightarrow \sqrt{1-\left(\frac{V_B}{c}\right)^2} = \frac{1}{3}\sqrt{1-\left(\frac{V_A}{c}\right)^2}$$

$$1 - \left(\frac{V_B}{c}\right)^2 = \frac{1}{9}\left(1-\left(\frac{V_A}{c}\right)^2\right) = \frac{1}{9} - \frac{1}{9}\left(\frac{V_A}{c}\right)^2$$

$$\frac{8}{9} - \left(\frac{V_B}{c}\right)^2 = -\frac{1}{9}\left(\frac{V_A}{c}\right)^2 \Rightarrow \left(\frac{V_B}{c}\right)^2 = \frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2$$

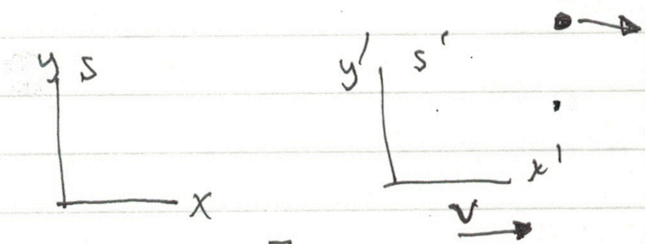
$$\frac{V_B}{c} = \sqrt{\frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2} < 1$$

$$= 0.95$$

Movimientos Acelerados

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} \quad ; \quad u_y = \frac{u_y'/\gamma}{1 + \frac{v u_x'}{c^2}}$$

$$t = \gamma\left(t' + \frac{v x'}{c^2}\right)$$



$$du_x = \frac{du_x'}{1 + \frac{v u_x'}{c^2}} - \left[\frac{(u_x' + v)}{\left(1 + \frac{v u_x'}{c^2}\right)^2} \cdot \frac{v}{c^2}\, du_x'\right]$$

$$= \frac{1 + \cancel{\frac{v u_x'}{c^2}} - \cancel{u_x'\frac{v}{c^2}} - \left(\frac{v}{c}\right)^2}{\left(1 + \frac{v u_x'}{c^2}\right)^2}\, du_x' = \frac{\left(1 - \left(\frac{v}{c}\right)^2\right) du_x'}{\left(1 + \frac{u_x' v}{c^2}\right)^2}$$

también $dt = \gamma\left(dt' + \frac{v}{c^2} dx'\right) = \gamma\, dt'\left(1 + \frac{v u_x'}{c^2}\right)$

$$\Rightarrow a_x = \frac{du_x}{dt} = \frac{(du_x'/dt')}{\gamma^3 \left(1 + \frac{v u_x'}{c^2}\right)^3}$$

$$\therefore \boxed{a_x = \frac{a_x'}{\gamma^3 \left[1 + \frac{v u_x'}{c^2}\right]^3}}$$

similarmente [Ejercicio]

$$a_y = \frac{a_y'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^2} - \frac{(v u_y'/c^2)\, a_x'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^3}.$$

Ejemplo cinemática relativista

S $\qquad$ S' $\qquad a_{x'} = g$, $\quad u_{x'} = 0$

$$a_x = \frac{a_{x'}}{\gamma^3\left(1+\frac{v u_{x'}}{c^2}\right)^3} = \frac{g}{\gamma^3(u_x)}$$

$$\frac{du_x}{dt} = g\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}$$

$$\underbrace{\int_0^{u_x} \frac{du_x}{\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}}} = \int_0^t g\,dt$$

$$\frac{u_x}{\sqrt{1-\left(\frac{u_x}{c}\right)^2}} = gt$$

$$\Rightarrow \frac{u^2}{1-\frac{u^2}{c^2}} = (gt)^2 \Rightarrow u^2 = (gt)^2 - (gt)^2\frac{u^2}{c^2}$$

$$u^2\left(1+\frac{(gt)^2}{c^2}\right) = (gt)^2$$

$$u^2 = \frac{(gt)^2}{1+\left(\frac{gt}{c}\right)^2} \Rightarrow \boxed{u = \frac{gt}{\sqrt{1+(gt/c)^2}}} \qquad (*)$$

si $u = c/2$

$$\Rightarrow \left(\frac{c}{2}\right)^2\left(1+\left(\frac{gt}{c}\right)^2\right) = (gt)^2 \Rightarrow (c/2)^2 = (gt)^2 - \left(\frac{gt}{2}\right)^2 = \frac{3}{4}(gt)^2$$

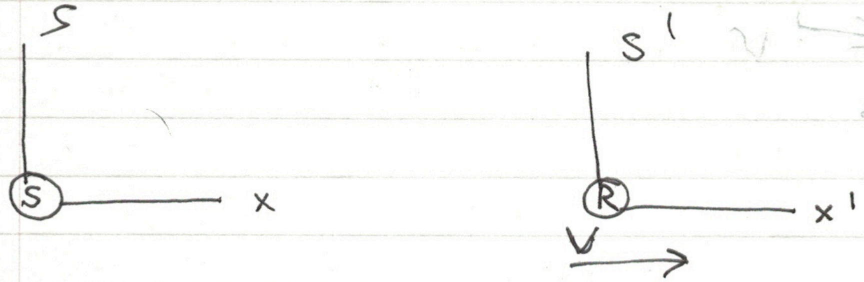
$$c^2 = 3(gt)^2 \Rightarrow c = \sqrt{3}\,gt$$

$$\Rightarrow \boxed{t = \frac{c}{\sqrt{3}\,g}} = 6.8 \text{ months}$$

$$(*) \Rightarrow x(t) = \frac{c^2}{g}\left[-1+\sqrt{1+(gt/c)^2}\right]$$

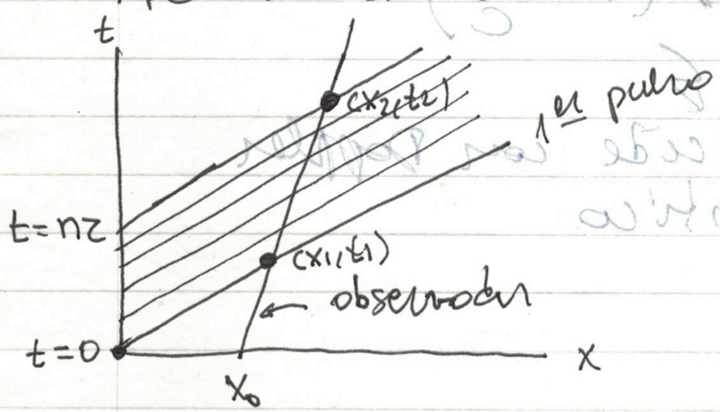
si $c \gg 1$, $x \to \frac{1}{2}gt^2$

Efecto Doppler relativista



Fuente (S) emite pulsos de luz en $t = n\tau$ ⇒ en S, la frec. medida es $\nu = 1/\tau$

El 1er pulso es emitido en $t=0$, cuando el receptor (S') está en $x = x_0$



$$x_1 = ct_1 = x_0 + vt_1$$

$$x_2 = c(t_2 - n\tau) = x_0 + vt_2$$

$$\Rightarrow t_2 - t_1 = \frac{cn\tau}{c-v}$$

$$x_2 - x_1 = \frac{vcn\tau}{c-v}$$

En S': $t_2' - t_1' = \gamma\left[(t_2-t_1) - \frac{v}{c^2}(x_2-x_1)\right]$

$$= \gamma\left(\frac{cn\tau}{c-v} - \frac{v}{c^2}\cdot\frac{vcn\tau}{c-v}\right) = \frac{\gamma c\tau n}{c-v}\left(1-\frac{v^2}{c^2}\right)$$

$$\Rightarrow \tau' = \frac{\gamma c\tau}{c-v}\left(1-\frac{v^2}{c^2}\right) = \frac{\gamma\tau}{1-\left(\frac{v}{c}\right)}\left(1-\frac{v^2}{c^2}\right) = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)\frac{1}{1-\frac{v}{c}}$$

$$= \frac{\tau\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \tau\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \Rightarrow \boxed{\nu' = \left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}\nu}$$

Doppler longitudinal →

$$\nu' = \nu \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2}$$

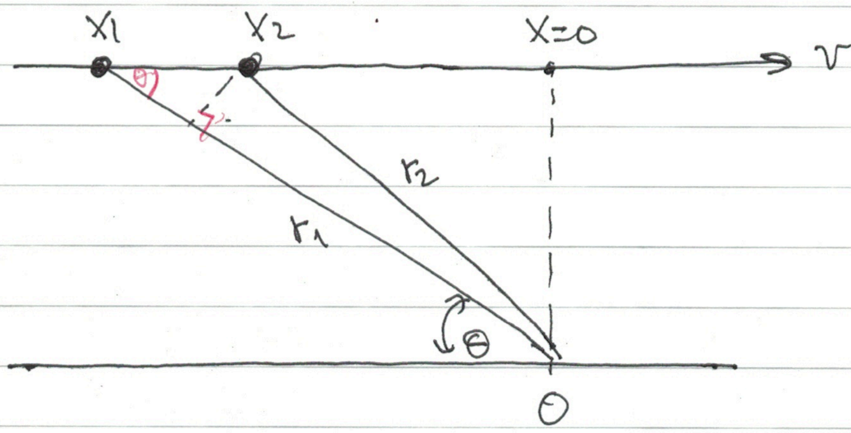
caso $v/c \ll 1$

$$\nu' \approx \nu \left[\left(1 - \frac{v}{c}\right) \left(1 - \frac{v}{c} + \left(\frac{v}{c}\right)^2 - \dots \right) \right]^{1/2}$$

$$\approx \nu \left[\left(1 - \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) \right]^{1/2} = \nu \left(1 - \frac{v}{c}\right)$$

· coincide con Doppler acústico

Doppler Transversal



2 pulsos sucesivos son emitidos en $x=x_1$ y $x=x_2$
en los instantes $t=t_1$ y $t=t_2$.
En el sist. en reposo c/r al satélite, el intervalo
entre pulsos es τ. ⇒ $t_2-t_1=\gamma\tau$ (por dilatación temporal)

El pulso #1 demora r_1/c en llegar a O
" " #2 " r_2/c " " " O

⇒ Intervalo entre pulsos : $\tau' = t_2 + \frac{r_2}{c} - (t_1 + r_1/c)$

Si $|x_2-x_1| \ll r_1$ ⇒ $r_1 - r_2 \simeq (x_2-x_1)\cos\theta$

$= (v t_2 - v t_1)\cos\theta = v(t_2-t_1)\cos\theta = v\gamma\tau\cos\theta$

∴ $\tau' = (t_2-t_1) + \frac{1}{c}(r_1-r_2) = \gamma\tau - \gamma\frac{v}{c}\tau\cos\theta = \gamma\tau(1-\frac{v}{c}\cos\theta)$

$$\Rightarrow \nu' = \frac{\nu}{\gamma(1-\frac{v}{c}\cos\theta)} = \boxed{\frac{\nu\,(1-(v/c)^2)^{1/2}}{(1-(v/c)\cos\theta)}}$$