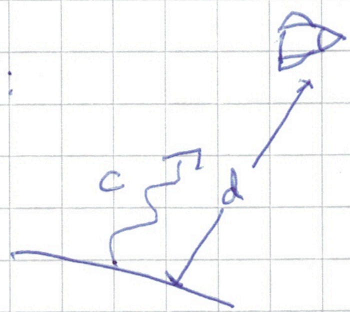
Una nave se aleja de la T a $v = 0.8c$. Cuando se encuentra a una distancia $d = 6{,}66 \times 10^8$ km, se le envía una señal de radio desde la T, ¿cuánto tarda en llegar la señal, medido en ambos sist. de referencia? ¿Cuál es la posición de la nave cuando recibe la señal, en ambos sist. de referencia?

Sistema Tierra (S):




$$ct = d + vt$$

$$(c-v)t = d$$

$$\boxed{t = \frac{d}{c-v}}$$

y la posición será $ct = \boxed{\frac{dc}{c-v}} = x$

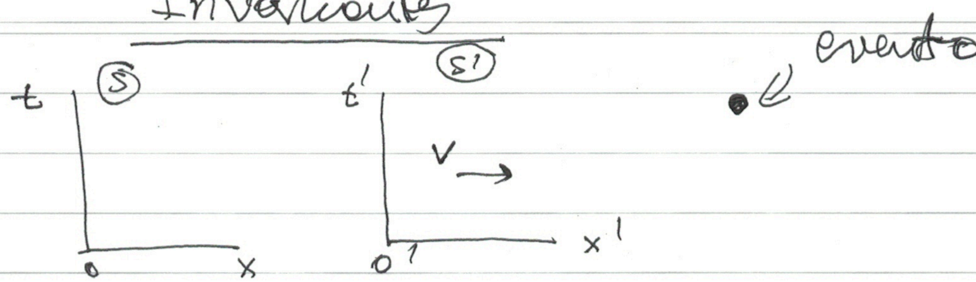
Sistema nave (S'): transformamos $\underbrace{(x, t)}_{\text{evento}} \rightarrow (x', t')$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \gamma\left[\frac{d}{c-v} - \frac{v \cdot dc}{c^2(c-v)}\right] = \frac{\gamma}{c-v}\left(d - \frac{v}{c}d\right)$$

$$= \frac{\gamma d}{c-v}\left(1 - \frac{v}{c}\right) = \frac{\gamma d}{c\cancel{(1-\frac{v}{c})}}\cancel{\left(1 - \frac{v}{c}\right)} = \boxed{\frac{\gamma d}{c}}$$

$$x' = 0$$

# Invariantes




$$x' = \gamma(x - vt)$$
$$t' = \gamma(t - vx/c^2)$$

Evaluemos: $(ct')^2 - (x')^2$

$$= \gamma^2\left(ct - \frac{vx}{c}\right)^2 - \gamma^2(x - vt)^2$$

$$= \gamma^2\left[c^2t^2 + \frac{v^2x^2}{c^2} - 2vxt\right] - \gamma^2\left[x^2 + v^2t^2 - 2xvt\right]$$

$$= \gamma^2\left[c^2t^2 + \frac{v^2x^2}{c^2} \cancel{- 2vxt} - x^2 - v^2t^2 \cancel{+ 2vxt}\right]$$

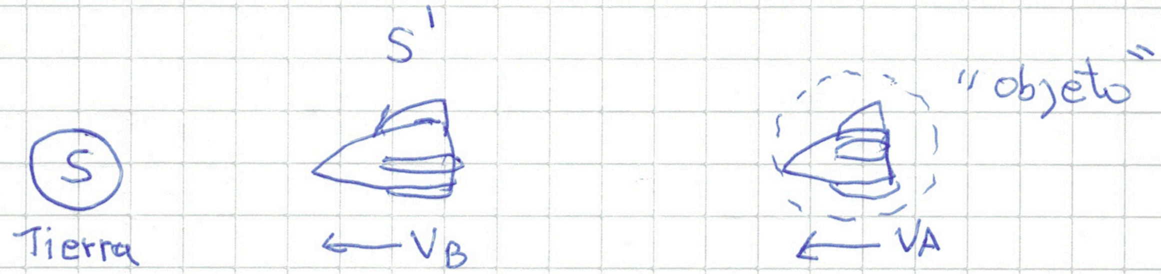
$$= \gamma^2\left[(c^2 - v^2)t^2 - x^2\left(1 - \left(\frac{v}{c}\right)^2\right)\right] = \gamma^2\left[c^2\left(1 - \left(\frac{v}{c}\right)^2\right)t^2 - x^2\left(1 - \left(\frac{v}{c}\right)^2\right)\right]$$

$$= \cancel{\gamma^2\left(1 - \left(\frac{v}{c}\right)^2\right)}\left[(ct)^2 - x^2\right] = (ct)^2 - x^2$$

$$\therefore \; s^2 \equiv (ct')^2 - (x')^2 = (ct)^2 - x^2$$ INVARIANTE RELATIVISTA.

también: $\underset{\text{reposo}}{E_0^2} \equiv \underset{\text{total}}{E^2} - c^2p^2$

An observer on Earth observes two spacecraft moving in the *same* direction toward the Earth. Spacecraft A appears to have a speed of $0.50c$, and spacecraft B appears to have a speed of $0.80c$. What is the speed of spacecraft A measured by an observer in spacecraft B?




En gral, $v_A = \dfrac{v_B + v_A'}{1 + \dfrac{\vec{v}_A' \cdot \vec{v}_B}{c^2}} \Rightarrow$

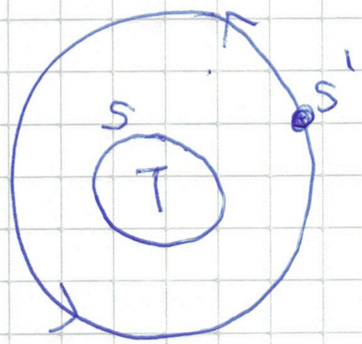
$$v_A' = \frac{v_A - v_B}{1 - \dfrac{\vec{v}_A \cdot \vec{v}_B}{c^2}} .$$

In our case $\vec{v}_A = -0.5c\,\hat{i}$

$\vec{v}_B = -0.8c\,\hat{i}$

$$\Rightarrow v_A' = \frac{-0.5c - (-0.8c)}{1 - (-0.5)(-0.8)} = \boxed{0.5c}$$

In 1962, when Scott Carpenter orbited Earth 22 times, the press stated that for each orbit he aged 2 millionths of a second less than if he had remained on Earth. (a) Assuming that he was 160 km above Earth in an eastbound circular orbit, determine the time difference between someone on Earth and the orbiting astronaut for the 22 orbits. (b) Did the press report accurate information? Explain.




$$\Delta\tau = \tau - \tau' = \tau - (1/\gamma)\tau = \left(1 - \frac{1}{\gamma}\right)\tau$$

$$\tau \approx \frac{2\pi R}{v} = 2\pi\sqrt{R/g} \qquad (1)$$

$$\frac{GmM}{R^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{GM}{R}} = \sqrt{gR} \qquad (0)$$

$$\gamma = \left(1 - (v/c)^2\right)^{-1/2} \simeq 1 + \frac{1}{2}(v/c)^2$$

$$\Rightarrow \frac{1}{\gamma} \simeq 1 - \frac{1}{2}(v/c)^2 \Rightarrow 1 - \frac{1}{\gamma} = \frac{1}{2}(v/c)^2$$

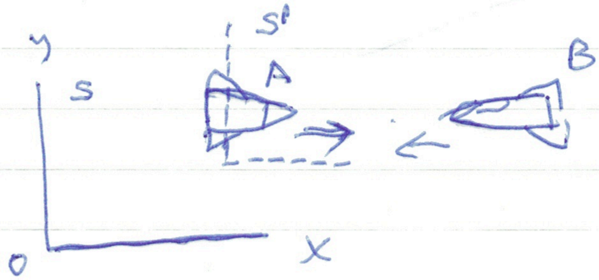
$$\Rightarrow \Delta\tau = \cancel{2}\pi\sqrt{R/g}\,\frac{1}{\cancel{2}}\left(\frac{v}{c}\right)^2 \overset{(0)}{=} \frac{\pi R^{3/2} g^{1/2}}{c^2} \qquad (2)$$

$R = 6400$ km +160
$g = 9.8$ m/s

$$\Rightarrow \boxed{\Delta\tau \approx 1.84 \text{ mu s}}$$

EJ: 2 naves A y B se mueven en direcciones opuestas

(S) (tierra) $V_A = 0.750c$
$V_B = 0.850c$



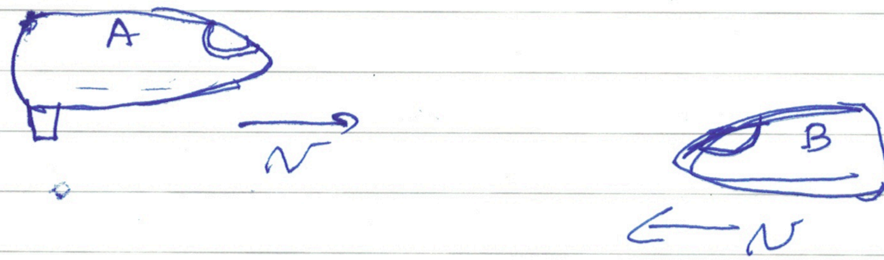

Hallar la veloc. de B c/r a A.

Solución: tomar S' en la nave A, con $v = 0.750c$ la veloc. entre S y S'. La nave B se toma como un objeto moviéndose hacia la izquierda con velocidad $u_x = -0.850c$ c/r a la tierra (S)

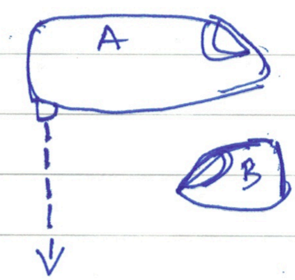
$\Rightarrow$ la veloc. de B c/r a A (S') será

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.9771c$$

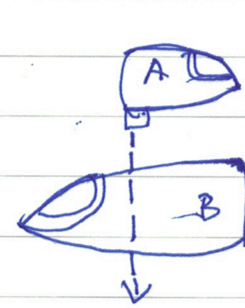
# Típica paradoja




(i)

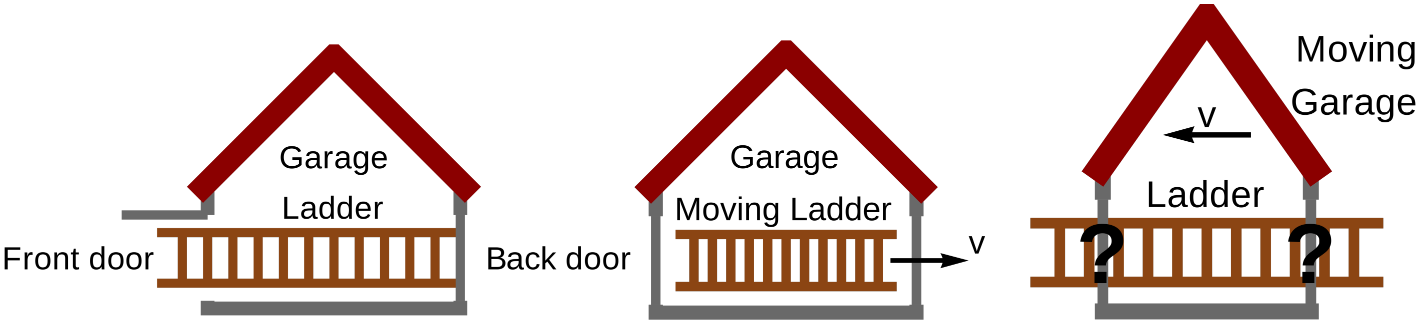


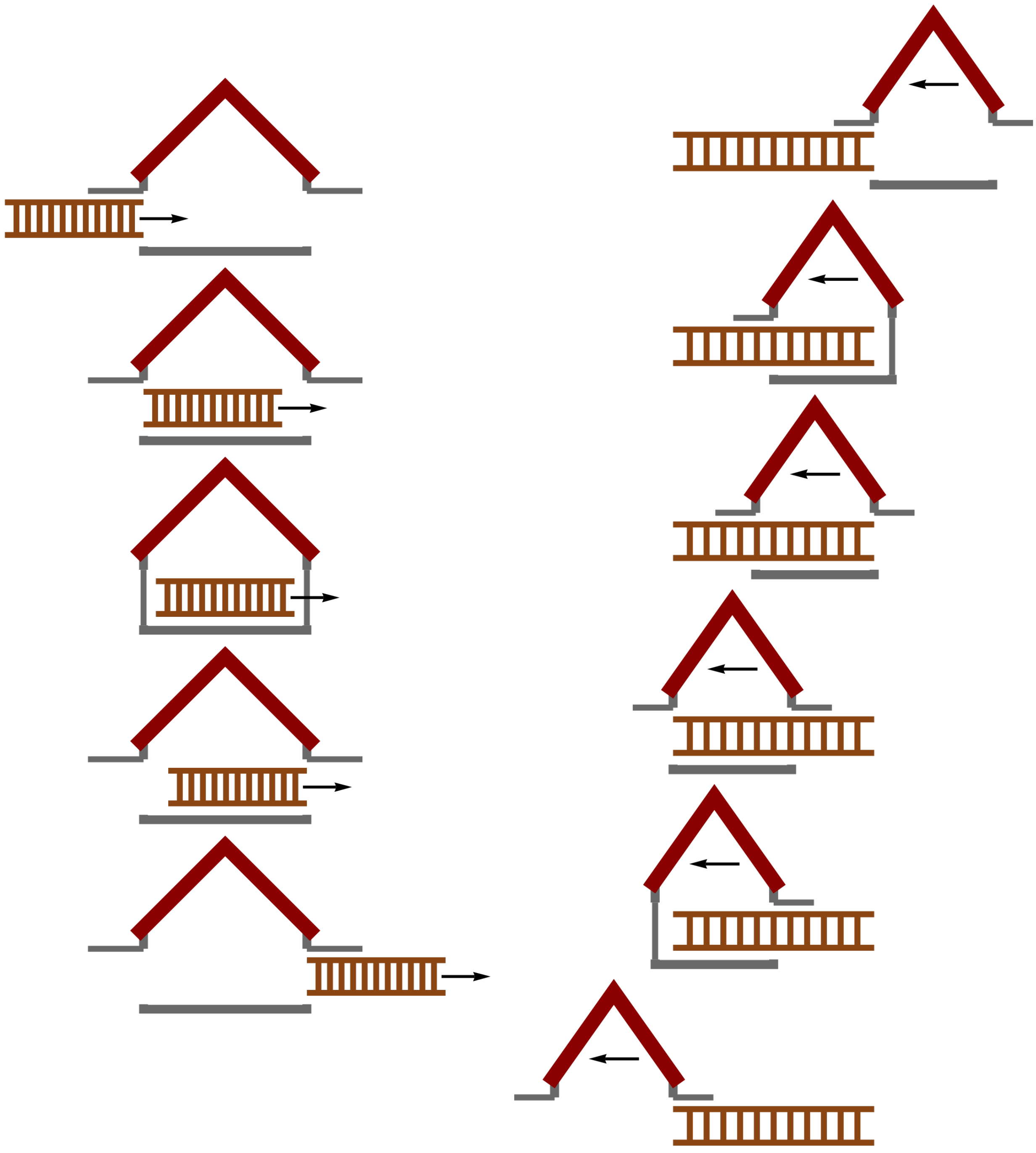

(ii)




ES DESTRUIDA LA NAVE B ?

# Paradoja de la escalera



**Simultaneidad !**

composición de velocidades


$$u_n = \frac{u_{n+1} + v}{1 + \frac{u_{n+1} v}{c^2}}$$

para $n \to \infty$, $u_n \to u$, $u_{n+1} \to u$

$$u = \frac{u + v}{1 + \frac{uv}{c^2}} \Rightarrow u\left(1 + \frac{uv}{c^2}\right) = u + v$$

$$\frac{u^2 v}{c^2} = v \Rightarrow u^2 = c^2$$

$$\boxed{u \to c}$$

## Muones

$$N(t) = N_0 e^{-t/\tau}$$

$\tau$ = vida media $\simeq 2\mu s$ (sist. reposo c/r al muón)
$v \simeq 0.998c$ y son creados en la alta atmosfera.

la distancia recorrida (desde S ) debería ser $h \simeq v\tau = 0.998c \cdot 2\mu s = 600$ m.

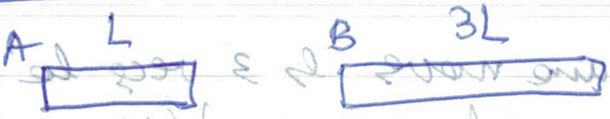
Desde la tierra, $\tau \to \gamma\tau \simeq 15 \times 2\mu s = 30\mu s$

$\Rightarrow h \simeq 9.000$ m

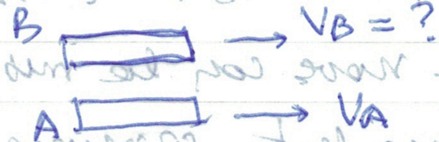
Desde el muón, $\tau$ no cambió, pero el suelo se aproxima a $0.998c \Rightarrow$ se contrae la altura y $9000\,m \to \frac{9000\,m}{\gamma} = \frac{9000}{15} = 600\,m$.

∴ El muón llega al suelo ya sea analizado desde S ó S'.

Prob. 26 (Serway) →




**The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of 0.35c, determine the speed of the faster spaceship.**




S ∴ $L_A = \dfrac{L}{\gamma_A}$

$L_B = \dfrac{3L}{\gamma_B}$

but $L_A = L_B \Rightarrow 1 = \dfrac{\frac{L}{\gamma_A}}{\frac{3L}{\gamma_B}} = \dfrac{\cancel{L}\gamma_B}{\gamma_A \cdot 3\cancel{L}} = \dfrac{\gamma_B}{3\gamma_A}$

$\Rightarrow \gamma_B = 3\gamma_A \Rightarrow \dfrac{1}{\gamma_B} = \dfrac{1}{3\gamma_A} \Rightarrow \sqrt{1-\left(\frac{V_B}{c}\right)^2} = \frac{1}{3}\sqrt{1-\left(\frac{V_A}{c}\right)^2}$

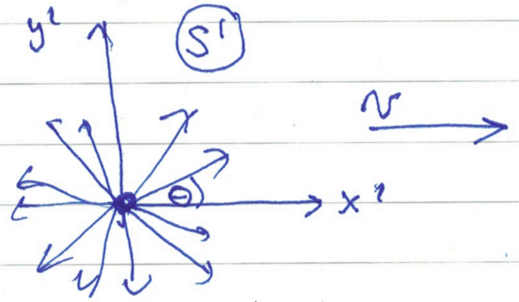
$1 - \left(\frac{V_B}{c}\right)^2 = \frac{1}{9}\left(1 - \left(\frac{V_A}{c}\right)^2\right) = \frac{1}{9} - \frac{1}{9}\left(\frac{V_A}{c}\right)^2$

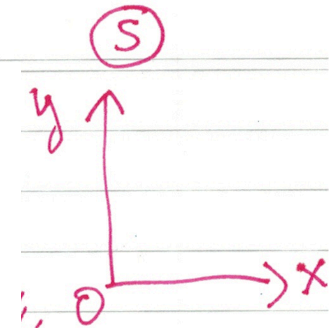
$\frac{8}{9} - \left(\frac{V_B}{c}\right)^2 = -\frac{1}{9}\left(\frac{V_A}{c}\right)^2 \Rightarrow \left(\frac{V_B}{c}\right)^2 = \frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2$

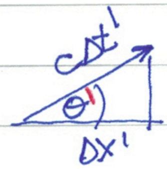
$\dfrac{V_B}{c} = \sqrt{\frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2} < 1$

$= 0.95$

# Haces de luz




$$\cos\theta' = \frac{\Delta x'}{c\Delta t'}$$

$$\cos\theta = \frac{\Delta x}{c\Delta t}$$

$$\cos\theta = \frac{\Delta x}{c\Delta t} = \frac{\cancel{\gamma}(\Delta x' + v\Delta t')}{c\cancel{\gamma}(\Delta t' + \frac{v}{c^2}\Delta x')} = \frac{\left(\frac{\Delta x'}{\Delta t'}\right) + v}{c\left(1 + \frac{v}{c^2}\frac{\Delta x'}{\Delta t'}\right)}$$

$$= \frac{\frac{\Delta x'}{c\Delta t'} + \frac{v}{c}}{1 + \frac{v}{c}\frac{\Delta x'}{c\Delta t'}} = \frac{\cos\theta' + v/c}{1 + \left(\frac{v}{c}\right)\cos\theta'}$$

$$\boxed{\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'}}$$