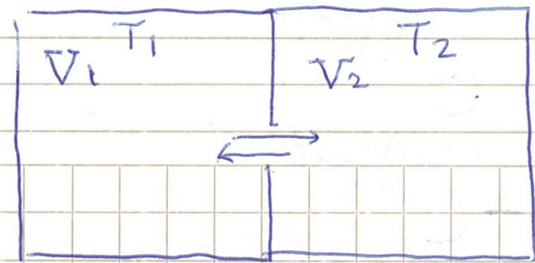


$$\frac{dN_1}{dt} = -A_1\left(\frac{N_1}{V_1}\right)\frac{\langle v\rangle_1}{4} + A_2\left(\frac{N_2}{V_2}\right)\frac{\langle v\rangle_2}{4}$$

$$\frac{dN_2}{dt} = -A_2\left(\frac{N_2}{V_2}\right)\frac{\langle v\rangle_2}{4} + A_1\left(\frac{N_1}{V_1}\right)\frac{\langle v\rangle_1}{4}$$

Sup: $A_1 = A_2$, $V_1 = V_2$ y $T_1 = T_2$

$$\Rightarrow \begin{array}{l} \dot{N}_1 = -\alpha N_1 + \alpha N_2 \\ \dot{N}_2 = \alpha N_1 - \alpha N_2 \end{array} \qquad \begin{array}{l} N_1(0) = N_0 \\ N_2(0) = 0 \end{array}$$

$$\Rightarrow \dot{N}_1 + \dot{N}_2 = 0 \Rightarrow \frac{d}{dt}(N_1 + N_2) = 0 \Rightarrow \boxed{N_1 + N_2 = N_0} \quad (1)$$

$$\Rightarrow N_2 = N_0 - N_1$$

$$\Rightarrow \dot{N}_1 = -\alpha N_1 + \alpha(N_0 - N_1)$$
$$= -2\alpha N_1 + \alpha N_0$$

$$\frac{d}{dt}N_1 = \alpha N_0 - 2\alpha N_1 \Rightarrow N_1 = \frac{N_0}{2} + A\,e^{-2\alpha t}$$

y usando $N_1(0) = N_0 \Rightarrow \boxed{N_1(t) = \frac{N_0}{2}\left(1 + e^{-2\alpha t}\right)}$

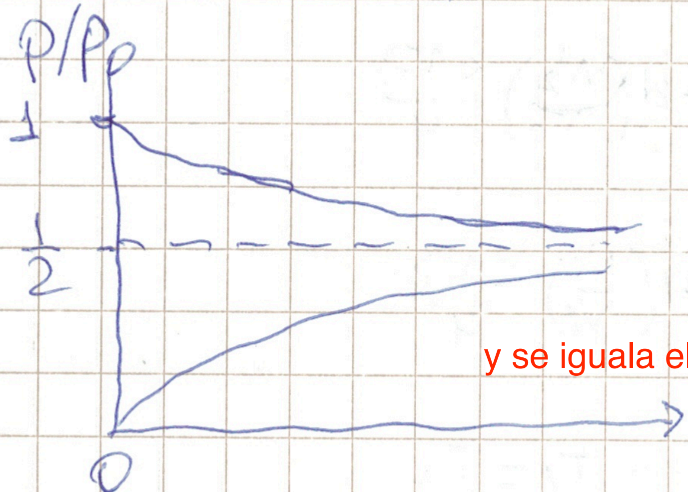
$$N_2 = N_0 - N_1 = N_0 - \frac{N_0}{2} - \frac{N_0}{2}e^{-2\alpha t} = \frac{N_0}{2} - \frac{N_0}{2}e^{-2\alpha t}$$

$$\Rightarrow \boxed{N_2(t) = \frac{N_0}{2}\left(1 - e^{-2\alpha t}\right)}$$

$\longrightarrow$

$$\Rightarrow P_1(t) = \frac{P_1(0)}{2}\left(1 + e^{-\frac{2A\langle v\rangle}{4V}t}\right)$$

$$P_2(t) = \frac{P_1(0)}{2}\left(1 + e^{-\frac{2A\langle v\rangle}{4V}t}\right)$$



se equilibran las presiones

y se iguala el numero de particulas en ambos containers

ejercicios

1. (a) Suponiendo que las moléculas de Hidrógeno tienen una velocidad cuadrática media (rms) de 1,270 m/s a 300 K, calcule la rms a 600 K.
   (b) Estime la temperatura a la cual la velocidad cuadrática media de las moléculas de nitrógeno en la atmósfera es igual a la velocidad de escape del campo gravitacional de la Tierra. (masa atómica Nitrógeno=14, radio Tierra=6,400 km).

2. Un sistema posee niveles de energía no-degenerados con energia $E = (n + 1/2)\hbar\omega$, donde $\hbar\omega = 8.625 \times 10^{-5}$ev, y $n = 0, 1, 2, 3, \ldots$.
   (a) Calcule la probabilidad P_{10} de que el sistema esté en el estado $n = 10$ si esta en contacto con un baño térmico a temperatura ambiente ($T = 300K$).
   (b) Cual será la probabilidad P_{10} en los casos límites de temperatura muy bajas y muy altas?
   (c) A que temperatura se maximizara P_{10}? Para tal temperatura calcule P_{10}.

[1] (a) $V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{3KT}{m}}$

$V_1 = \sqrt{\frac{3KT_1}{m}}$

$\Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{3KT_1}{\cancel{m}}} \cdot \sqrt{\frac{\cancel{m}}{3KT_2}} = \sqrt{\frac{T_1}{T_2}}$

$V_2 = \sqrt{\frac{3KT_2}{m}}$

$\Rightarrow V_2 = \sqrt{\frac{T_2}{T_1}}\, V_1 = \sqrt{\frac{600}{300}} \times 1{,}270 = \boxed{1{,}796 \left[\frac{m}{s}\right]}$ ✓

[b) $V_{rms} = \sqrt{\frac{3KT}{m}}$

$\frac{1}{2}\cancel{m} V_e^2 = \frac{GM\cancel{m}}{R} \Rightarrow V_e^2 = \frac{2GM}{R} = 2R \underbrace{\left(\frac{GM}{R^2}\right)}_{g} = 2gR$

$V_{rms} = V_e \Rightarrow \left(\frac{3KT}{m}\right)^{1/2} = (2gR)^{1/2}$

$\Rightarrow \frac{3KT}{m} = 2gR \Rightarrow T = \frac{2mgR}{3K}$

$N_0 \rightarrow 14\,gr$
$1 \rightarrow x$
$\Rightarrow m = \frac{14}{6.02 \times 10^{23}} = 2.33 \times 10^{-23}\ gr.$

T=1.4 x 10^5 K

[2] $E_n = (n+\frac{1}{2})\hbar\omega \qquad n = 0, 1, 2, \dots$

$$P_n = A e^{-\frac{(n+\frac{1}{2})\hbar\omega}{kT}} = A e^{-\frac{\hbar\omega}{2kT}} \cdot e^{-\frac{n\hbar\omega}{kT}}$$

$$1 = \sum_{n=0}^{\infty} A e^{-\frac{\hbar\omega}{2kT}} \cdot e^{-\frac{n\hbar\omega}{kT}} = A e^{-\frac{\hbar\omega}{2kT}} \cdot \sum_{0}^{\infty} e^{-\frac{n\hbar\omega}{kT}}$$

$$S = 1 + e^{-\frac{\hbar\omega}{kT}} + e^{-\frac{2\hbar\omega}{kT}} + \dots$$

$$e^{-\frac{\hbar\omega}{kT}} S = e^{-\frac{\hbar\omega}{kT}} + e^{-\frac{2\hbar\omega}{\hbar\omega}} + \dots$$

$$S(1 - e^{-\frac{\hbar\omega}{kT}}) = 1 \Rightarrow \boxed{S = \frac{1}{1 - e^{-\frac{\hbar\omega}{\cancel{2}kT}}}}$$

$$1 = \frac{A e^{-\frac{\hbar\omega}{2kT}}}{1 - e^{-\frac{\hbar\omega}{kT}}} = \frac{A e^{-\frac{\hbar\omega}{2kT}}}{e^{-\frac{\hbar\omega}{kT}}(e^{\frac{\hbar\omega}{kT}} - 1)} = \frac{A e^{\frac{\hbar\omega}{2kT}}}{e^{\frac{\hbar\omega}{kT}} - 1}$$

$$\Rightarrow \boxed{A = (e^{\frac{\hbar\omega}{kT}} - 1) e^{-\frac{\hbar\omega}{2kT}}}$$

$$P_n = (e^{\frac{\hbar\omega}{kT}} - 1) \boxed{e^{-\frac{\hbar\omega}{2kT}} \times e^{-\frac{\hbar\omega}{2kT}}} e^{-n\frac{\hbar\omega}{kT}}$$

$$(e^{\frac{\hbar\omega}{kT}} - 1) e^{-\frac{\hbar\omega}{kT}} e^{-\frac{n\hbar\omega}{kT}} = (1 - e^{-\frac{\hbar\omega}{kT}}) e^{-\frac{n\hbar\omega}{kT}}$$

$\frac{\hbar\omega}{kT} = \frac{1.0}{300} \qquad P_{10} = (1 - e^{-0.003}) e^{-\frac{10}{300}} = 0.00321$

$$P_n = \left(1 - e^{-\frac{\hbar\omega}{KT}}\right) e^{-\frac{n\hbar\omega}{KT}}$$

$T \to 0 \quad e^{-\frac{\hbar\omega}{KT}} \to 0 \quad y \quad e^{-\frac{n\hbar\omega}{KT}} \to 0$ (excepto at $n=0$)

$$\boxed{P_{10}(T \to 0) = 0}$$

$T \to \infty, \quad e^{-\frac{\hbar\omega}{KT}} \to 1, \quad e^{-\frac{n\hbar\omega}{KT}} \to 1 \quad \forall n$

$$\Rightarrow \boxed{P_{10}(T \to \infty) = 0}$$

$$P_n = (1 - e^{-\beta})\, e^{-n\beta} \qquad \beta = \frac{\hbar\omega}{kT}$$

$$P_{10} = (1 - e^{-\beta})\, e^{-10\beta}$$

$$0 = \frac{dP_{10}}{dT} = \frac{d\beta}{dT} \cdot \frac{d}{d\beta} P_{10} = -\frac{\hbar\omega}{kT^2} \left\{ e^{-\beta} e^{-10\beta} + (1 - e^{-\beta})(-10)\, e^{-10\beta} \right\}$$

$$= -\frac{\hbar\omega}{kT^2}\, e^{-10\beta} \left[e^{-\beta} + (1 - e^{-\beta})(-10) \right]$$

$$= -\frac{\hbar\omega}{kT^2}\, e^{-10\beta} \left[e^{-\beta} - 10 + 10 e^{-\beta} \right]$$

$$\left[11 e^{-\beta} - 10 \right]$$

$$\Rightarrow 11 e^{-\beta} = 10 \Rightarrow e^{-\beta} = \frac{10}{11} \Rightarrow \qquad -\beta = \ln(10/11)$$

$$\Rightarrow \boxed{\beta = \ln(11/10)}$$

conocido

$$\therefore \frac{\hbar\omega}{kT} = \ln(11/10) \Rightarrow T = \frac{\hbar\omega}{k \ln(11/10)} = \frac{1}{\ln(11/10)} = \boxed{10.5}$$

$$P_{10} = (1 - e^{-\beta})\, e^{-10\beta} = (1 - e^{-\ln(11/10)})\, e^{-10\ln(11/10)} = \boxed{0.035}$$

$$\Rightarrow 3.5\%$$