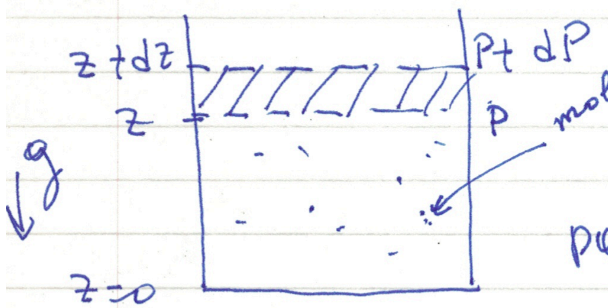
Fórmula barométrica



$$dP = -\rho g\, dz = -\frac{Nmg}{V}\, dz$$

pero gas ideal $PV = N k_B T$

$$\Rightarrow \frac{N}{V} = P/k_B T$$

$$\Rightarrow dP = -\frac{mgP}{k_B T}\, dz \Rightarrow \frac{dP}{P} = -\frac{mg\, dz}{k_B T}$$

$$\Rightarrow \ln P = \text{cte} - \frac{mgz}{k_B T}$$

$$\Rightarrow \boxed{P(z) = P(0)\, e^{-\frac{mgz}{k_B T}}}$$

$$\boxed{N(z) = N(0)\, e^{-\frac{mgz}{k_B T}}}$$

La prob. de hallar la molécula a altura z es...

$$f(z) \propto e^{-\frac{mgz}{k_B T}} = e^{-U/k_B T}$$

Integrals

$$I_n = \int_0^\infty v^n e^{-\alpha v^2} dv \qquad (1)$$

<u>$n = 2p+1$</u>

$$I_n = \int_0^\infty v^{2p} v\, e^{-\alpha v^2} dv = \frac{1}{2}\int_0^\infty (v^2)^p e^{-\alpha (v^2)} d(v^2) = \frac{1}{2}\int_0^\infty x^p e^{-\alpha x} dx$$

$$= \frac{1}{2\alpha^{p+1}} \cdot \underbrace{\int_0^\infty s^p e^{-s} ds}_{p!} = \boxed{\frac{p!}{2\alpha^{p+1}}} \qquad (2)$$

<u>$n = 2p$</u>

$$I_n = \int_0^\infty v^{2p} e^{-\alpha v^2} dv = (-1)^p \frac{d^p}{d\alpha^p} \underbrace{\int_0^\infty e^{-\alpha v^2} dv}_{\frac{1}{\sqrt{\alpha}} \int_0^\infty e^{-x^2} dx}$$

$$I \equiv \int_0^\infty e^{-x^2} dx$$

$$I^2 = \int_0^\infty e^{-x^2} dx \int_0^\infty e^{-y^2} dy = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$$

$$r^2 = x^2 + y^2$$

$$r\,dr\,d\phi = dx\,dy$$

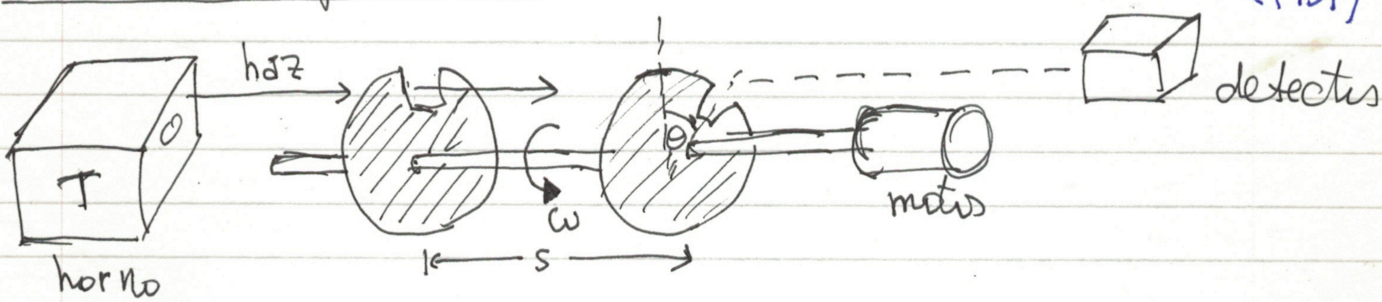
$$I^2 = \int_0^{\pi/2} d\phi \int_0^{\infty} dr\, r\, e^{-r^2} = \frac{\pi}{2}\int_0^{\infty} r\, e^{-r^2}\, dr =$$

$$= \frac{\pi}{4}\int_0^{\infty} e^{-r^2}\, dr^2 = \frac{\pi}{4}\underbrace{\int_0^{\infty} e^{-s}\, ds}_{1} = \frac{\pi}{4}$$

$$\boxed{I = \frac{\sqrt{\pi}}{2}}$$

$$\therefore \boxed{I_n = (-1)^p \frac{\sqrt{\pi}}{2}\frac{d^p}{d\alpha^p}\left(\alpha^{-1/2}\right)} \qquad (n = 2p).$$

comprobac. experimental (J.F. Zartman, 1931) PR 37, 383-39 (1931)



para ω dado, sólo pasan al detector aquellas moléculas que cumplan: $t = \frac{s}{v} = \frac{\theta}{\omega}$

$\Rightarrow v = \frac{s\theta}{\omega}$. Variando ω ó θ, se puede medir directa/. el # de moléculas en un rango de v dado.

Los resultados concuerdan con M-B

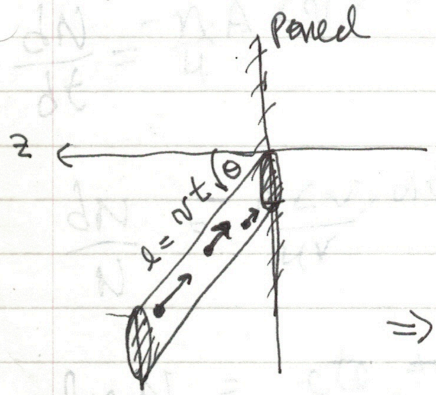
Ejercicio: usando **M-B** calcular como fn de T:

(a) $\bar{v}$ (rapidez promedio) $= \sqrt{\dfrac{8k_BT}{\pi m}}$

(b) rapidez más probable v_{mp}

(c) veloc. cuadrática media: $\sqrt{\overline{v^2}}$

Calcular la tasa de escape de moléculas desde una pequeña abertura de área A, en un contenedor.



$$\Delta N = nAvt\cos\theta\, f(\vec{v})$$

$$\frac{\Delta N}{A\Delta t} = n\, v\cos\theta\, f(\vec{v})$$

$$\Rightarrow \text{flujo} = \int n v \cos\theta\, f(\vec{v})\, d^3v$$

$$= n\left(\frac{m}{2\pi kT}\right)^{3/2} \iiint e^{-\frac{mv^2}{2kT}} v^3 \cos\theta \sin\theta \, dv\, d\theta\, d\phi$$

$$= 2\pi n\left(\frac{m}{2\pi kT}\right)^{3/2} \underbrace{\int_0^{\pi/2} \sin\theta\cos\theta\, d\theta}_{1/2} \underbrace{\int_0^{\infty} v^3 e^{-\frac{mv^2}{2kT}} dv}_{\underbrace{\cancel{1/(2m/2kT)^2}}_{2\left(\frac{kT}{m}\right)^2}} = n\sqrt{\frac{kT}{2\pi m}}$$

$$\frac{n}{4}\bar{v}$$ (12

$$\frac{dN}{dt} = -\frac{n}{4} A \langle v \rangle = -\frac{NA}{4V} \langle v \rangle$$

$$\frac{dN}{N} = -\frac{A\langle v \rangle}{4V} dt$$

$$\ln N = cte - \frac{A\langle v \rangle t}{4V}$$

$$\boxed{N = N(0)\, e^{-\frac{A\langle v \rangle t}{4V}}}$$

$$P = P(0)\, e^{-\frac{A\langle v \rangle t}{4V}}$$

$$\boxed{P = P(0)\, e^{-\frac{A}{V}\sqrt{\frac{kT}{2\pi m}}\, t}}$$

Ej: calc. tiempo de vaciado.