CLASE 2

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$$F_{\mathrm{resistiva}} = -\,\gamma\ \mathbf{v}\ \text{siempre?}$$

NO

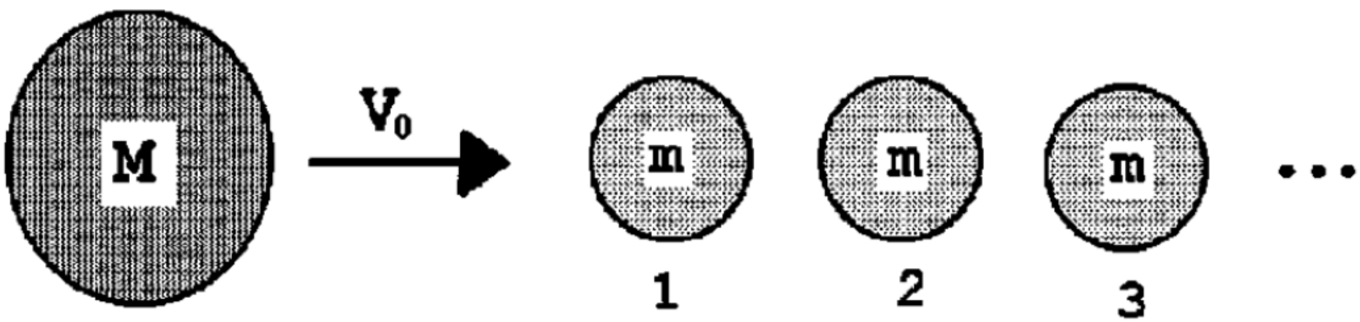


Fig. 1. A body of mass M, speed V_0, undergoes a one-dimensional collision with an array of identical molecules of mass m at rest, with $m<M$. The momentum lost by the body as it moves forward is exchanged among the molecules away from the body in an orderly manner, without backscattering.

$$V_1 = V_0 - \left(\frac{2m}{M+m}\right)(V_0 - v_0),$$

$$v_1 = V_0 + \left(\frac{M-m}{M+m}\right)(V_0 - v_0).$$

Despues de la primera colision

Suponer v0=0

$$V_1 = \left(\frac{M-m}{M+m}\right) V_0 \qquad v_1 = \frac{2M}{M+m} V_0 > V_1 .$$

it is easy to calculate the speed of the body after an arbitrary number, N, of collision events (all of them with molecule #1):

$$V_N = \left(\frac{M-m}{M+m}\right)^N V_0 . \tag{5}$$

Assuming an average density of medium molecules ρ, the number of collisions after traversing a distance x will be ρx, and Eq. (5) can be cast as

$$V(x) = \left(\frac{1-r}{1+r} \right)^{\rho x} V_0 , \tag{6}$$

where we have defined $r = m/M$ as the mass ratio. In going over to the continuum, we have to assume that an infinitesimal interval dx contains very many molecules, as in Hydrodynamics. Equation (6) means an exponential decay of speed with distance traversed inside the medium, since it can be rewritten as $V(x) = V_0 \exp(-\alpha x)$ with $\alpha = \rho \log[(1+r)/(1-r)]$.

We can define a characteristic distance, the *half-range* R, as the distance traveled inside the medium necessary to reduce the kinetic energy of the body to a half. This implies $V = V_0/\sqrt{2}$. After equating Eq. (6) (with $x = R$) to $V_0/\sqrt{2}$ and solving for R, we obtain:

$$R = \frac{1}{2\rho}\log(2) \Bigg/ \log\left(\frac{1+r}{1-r}\right). \tag{7}$$

Let us now calculate the force acting on the body. According to Newton's second law: $F = Ma$ and $a = dV/dt = (\partial V/\partial x)(\partial x/\partial t) = (\partial V/\partial x)V(x)$. Using $dA^x/dx = A^x \log(A)$, we obtain

$$a = -\rho \log\left(\frac{1+r}{1-r}\right) V^2 \tag{8}$$

which implies,

$$F = Ma = -\gamma V^2, \tag{9}$$

where

$$\gamma \equiv m\rho \log\left(\frac{1+r}{1-r}\right) \Big/ r \tag{10}$$

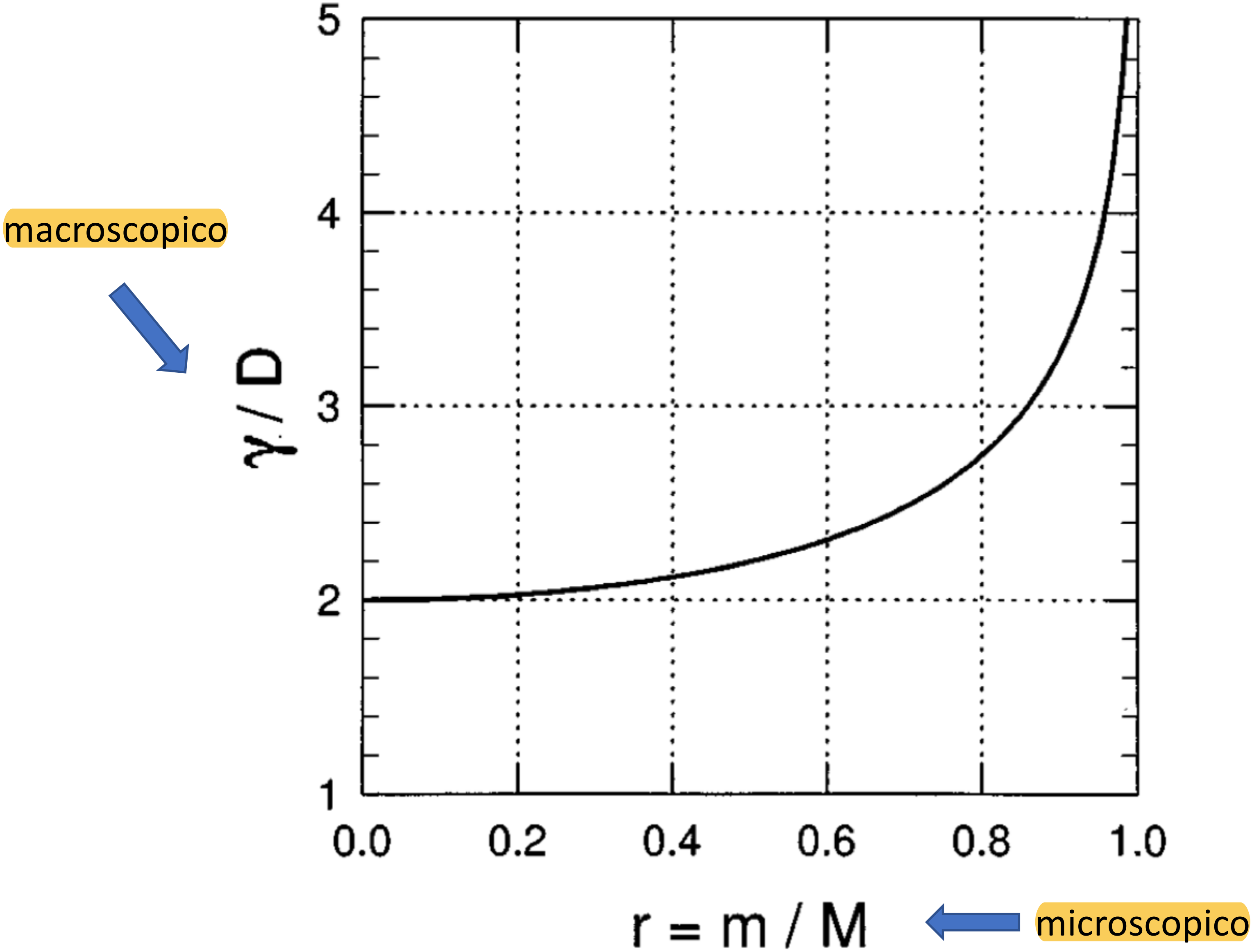


Fig. 3. Resistive coefficient γ as a function of molecule/body mass ratio.

EL problema del "Random walk"

Sup. una partícula sumergida en un medio. Supondremos que las fuerzas que actúan sobre ella son de 2 tipos:

(a) Fuerzas debidas a la carencia de balance en el bombardeo molecular sobre la partícula.

(b) Fuerza viscosa proporcional a la velocidad de la partícula (Ley de Stokes)

Por simplicidad, sup. mov. 1-dimensional.

Sea X = resultante, en cualq. instante, de las fuerzas debidas al bombardeo

$$F = -6\pi a \eta (dx/dt) = -\mu (dx/dt) \quad \text{, fuerza viscosa.}$$

Ec. de MOV:

$$m \frac{d^2x}{dt^2} = -\mu \frac{dx}{dt} + X \qquad (1) \qquad / \times 2x$$

$$m\, 2x \frac{d^2x}{dt^2} = -\mu \cdot 2x \frac{dx}{dt} + 2 X x \qquad (2)$$

pero $2x \dfrac{dx}{dt} = \dfrac{d}{dt}(x^2)$ (3), y diferenciando c/r a t

$$2\left(\frac{dx}{dt}\right)^2 + 2x \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(x^2)$$

$$\Rightarrow 2x \frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(x^2) - 2\left(\frac{dx}{dt}\right)^2 \qquad (4)$$

, sustituyendo (3) y (4) en (2), queda:

$$m \frac{d^2}{dt^2}(x^2) - 2m\left(\frac{dx}{dt}\right)^2 = -\mu \frac{d}{dt}(x^2) + 2 X x \qquad (5)$$

" "

Y $\frac{1}{2} m \overline{v^2} = \frac{3}{2} KT = \frac{3}{2} \frac{R}{N} T$

luego: $m \overline{\left(\frac{dx}{dt}\right)^2} = \frac{RT}{N}$.

la ec. que nos interesa queda:

$$m \frac{d^2}{dt^2}(\overline{x^2}) - \frac{2RT}{N} = -\mu \frac{d}{dt}(\overline{x^2})$$, poniendo $\omega = \frac{d}{dt}(\overline{x^2})$

$$m \frac{d\omega}{dt} = \frac{2RT}{N} - \mu\omega \qquad ó \qquad \frac{d\omega}{dt} + \frac{\mu}{m}\omega = \frac{2RT}{Nm}$$

con solución: $\omega(t) = \frac{2RT}{N\mu} + A e^{-\left(\frac{\mu}{m}\right)t}$ (6)

$\tau = \frac{m}{\mu}$: tiempo de relajación $= \frac{\frac{4}{3}\pi a^3 \rho}{6\pi a \eta} = \frac{2a^2\rho}{9\eta}$

En un caso típico: $a = 10^{-4}$ cm , $\rho \sim 1$, $\eta \approx 10^{-2}$ cgs $\Rightarrow \tau \sim 2\cdot 10^{-7}$ seg.

Luego el término exponencial es despreciable para cualq. tiempo razonable de observación.

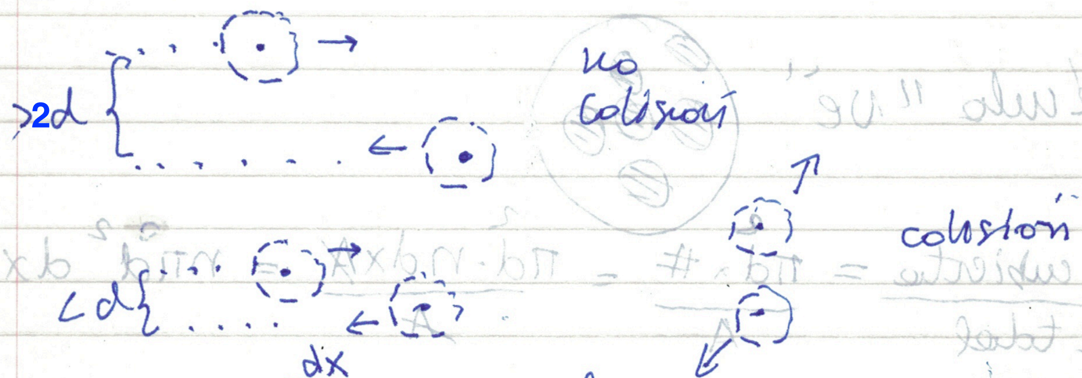
Luego (6) puede escribirse como:

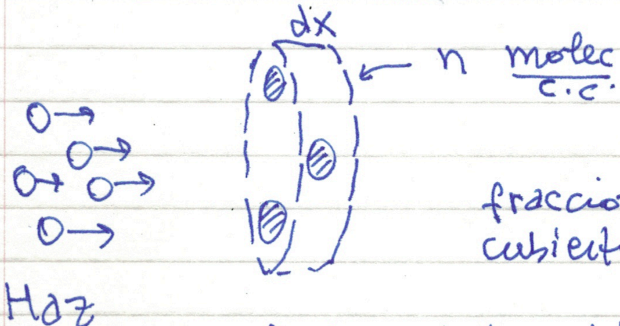
$$\omega = \frac{d}{dt}(\overline{x^2}) = \frac{2RT}{N\mu} \Rightarrow \overline{x^2} = \frac{2RT}{N\mu} t = \frac{RT}{3\pi a \eta N} t$$

$$\therefore \quad \overline{x^2} = \frac{KT}{3\pi a \eta} t \qquad (7)$$ {ecuación de Einstein-Smoluchowski (1905)}

Camino libre medio λ: distancia media recorrida por una molécula entre colisiones con otras moléculas.

sea d = "radio" de una molécula





fracción área cubierta $= \pi d^2 \cdot n\,dx = n\pi d^2\,dx \equiv \alpha\,dx$

$\Rightarrow$ fracción moléculas removidas del haz $= \alpha\,dx$

si $N = N_0$ en $x=0$

en dx: $dN = -N\alpha\,dx \Rightarrow \boxed{N(x) = N(0)\,e^{-\alpha x}}$

P. de recorrer x sin chocar $\propto e^{-\alpha x}$

Cálc. de λ : P. de llegar a x sin chocar, pero no más allá = p. de llegar a x, multip. por la p. de chocar en dX $= e^{-\alpha x}\alpha\,dx$

$\lambda = \int_0^\infty x\,Q(x)\,dx$

$$\lambda = \frac{\int_0^\infty x\, e^{-\alpha x} \alpha\, dx}{\int_0^\infty e^{-\alpha x} \alpha\, dx} = \frac{1}{\alpha} = \frac{1}{n\pi d^2}$$

$$\therefore \boxed{\lambda = \frac{1}{n\pi d^2}}$$

EX: λ para aire a STP.

$N_0 = 6.02 \times 10^{23} \longleftrightarrow$ mole 22.4 lts

$\Rightarrow n = 2.7 \times 10^{19}$ (1/cc) ✓

$d_{aire} = ?$ suponer $d_{aire} \simeq d_{H_2O}$

1 gr. de H_2O ocupa 1 cc y contiene $\sim 6 \times 10^{23}$ moléculas

$\Rightarrow v = 3 \times 10^{-23}$ cc $\sim \frac{4\pi}{3} d^3 \Rightarrow d \simeq (3 \times 10^{-23})^{1/3} = 3 \times 10^{-8}$ cm

$$\Rightarrow \lambda \simeq \frac{1}{2.7 \times 10^{19} \times \pi \times (3 \times 10^{-8})^2} = 10^{-5} \text{ (cm)}.$$

$$\Rightarrow P(x) = e^{-x/\lambda}$$

$$P(1\,\text{cm}) = e^{-10^5} = e^{-100000} = 10^{-43.429}$$

Espacio profundo: 0.5 átomos/cc

$\Rightarrow \lambda \simeq 7 \times 10^{14}$ cm $= 74$ años-Luz