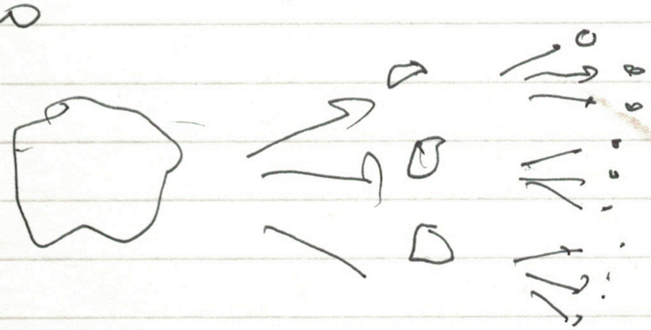
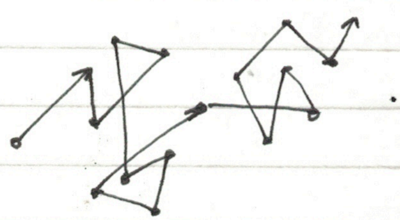
Naturaleza atómica de la Materia & Electricidad

Feynman → "Si ocurriera un cataclismo donde todo el conocimiento científico fuera destruido, y sólo se pudiera transmitir una frase a la siguiente generación de criaturas ¿qué frase contendría la mayor cant. de info. en el menor # de palabras? yo creo q' s la hipótesis atómica, o sea TODAS las cosas están hechas de átomos que se mueven alrededor en movimiento perpetuo, atrayéndose entre sí cuando están cerca, pero repeliéndose al apretarlos uno contra otro"



Brown (1860): mov. azaroso de partículas de polen suspendidas en líquido

→ "movimiento Browniano"

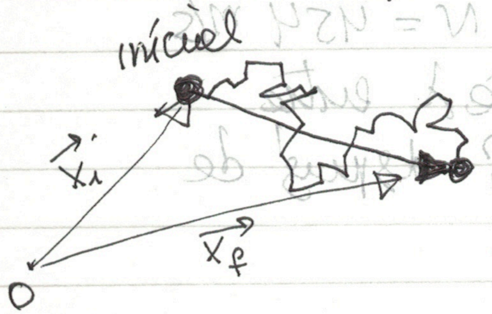


RANDOM WALK

tiempo total de observación $= t$

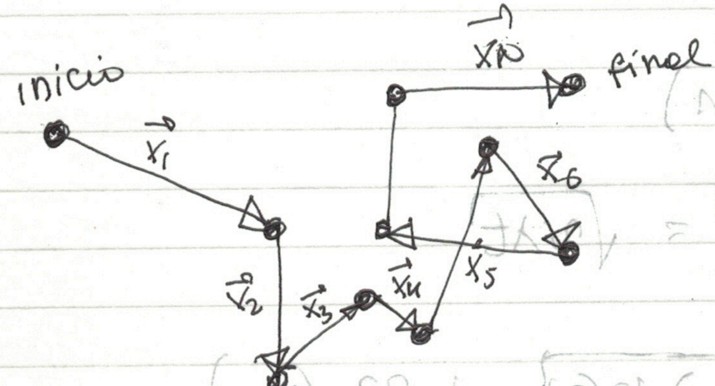
Dividí t en N intervalos $\Delta t = \dfrac{t}{N}$

Durante Δt se producen muchos choques: $\vec{X}_f = \vec{X}_i + \vec{L}$ ← random



supongo $\langle \vec{L}^2 \rangle = \lambda^2$ camino libre medio

fase aleatoria



$$\vec{X}_N = \vec{X}_{N-1} + \vec{L}$$

$$\vec{X}_N^2 = \vec{X}_{N-1}^2 + \vec{L}^2 + 2\vec{X}_{N-1}\cdot\vec{L}$$

Promedios sobre direcciones de $\vec{L}$:

$$\langle X_N^2 \rangle = \langle X_{N-1}^2 \rangle + \langle L^2 \rangle + 2|L||X_{N-1}|\underbrace{\langle \cos\theta \rangle}_{0}$$

$$\langle X_N^2 \rangle = \langle X_{N-1}^2 \rangle + \lambda^2 = \langle X_{N-2}^2 \rangle + 2\lambda^2$$

$$= \underbrace{\langle X_0^2 \rangle}_{0} + N\lambda^2 \quad \therefore \boxed{\langle X_N^2 \rangle = N\lambda^2} \quad \rightarrow$$

pero $N \propto t$

$$\langle \vec{X}_N^{\,2} \rangle = \alpha t \Rightarrow \boxed{\sqrt{\langle X_N^2 \rangle} = \beta t^{1/2}}$$

difusión

EJ: sea una molécula con $v = 454$ m/s que se mueve una distancia λ entre colisiones ¿cuál es $\sqrt{\langle x^2 \rangle}$ después de $t = 10$ seg?

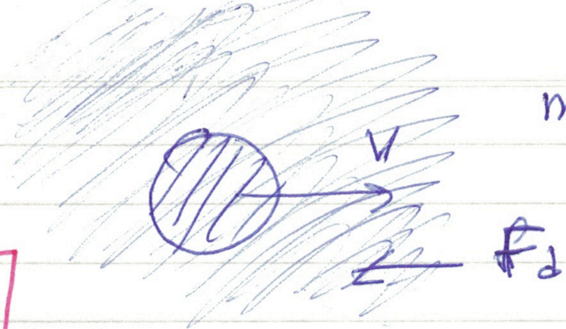
$$\lambda \simeq \cancel{\quad} 7.40 \times 10^{-8} \text{ (m)}$$

$$\sqrt{\langle x^2 \rangle} = \lambda \sqrt{N} = \lambda \sqrt{\frac{vt}{\lambda}} = \sqrt{v \lambda t}$$

$$= \sqrt{454 \left(\tfrac{m}{s}\right) (7.40 \times 10^{-8}\,m)\, 10 (s)} = 1.83 \text{ (cm)}$$

el cual es mucho menor que

$$v \times t = 454 \left(\tfrac{m}{s}\right) \times 10 (s) = 4543 \text{ (m)}.$$



medio viscoso

$$\vec{F_d} = -\gamma \vec{v}$$

ESCALA MACROSCOPICA

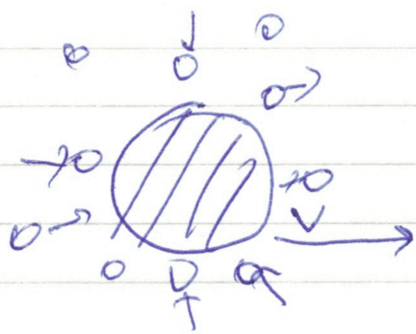
En presencia de $\vec{F}$ externa, la partícula alcanza una $\vec{v}$ <u>terminal</u>:

$$F_{ext} = F_d \Rightarrow \gamma v = F_{ext}$$

$$\boxed{v = \frac{F_{ext}}{\gamma}} \qquad (1)$$

Ley de Stokes
$\gamma = 6\pi a \eta$

ESCALA MICROSCOPICA:



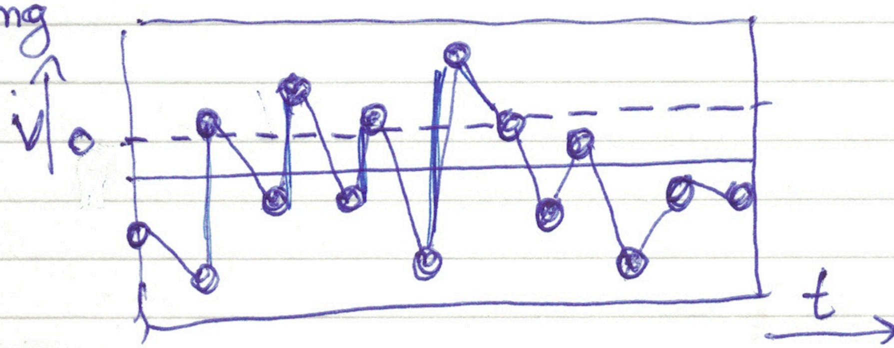
bombardeo moleculas

<u>Modelo simple</u>: una colisión cada Δt. Como resultado de c/ colisión, el objeto adquiere $\vec{v}_0$ random

<u>Entre colisiones</u>, sólo actúa F_{ext}.

$$\Rightarrow \quad v = \overset{\text{random}}{v_0} + \left(\frac{F_{ext}}{m}\right)\Delta t$$

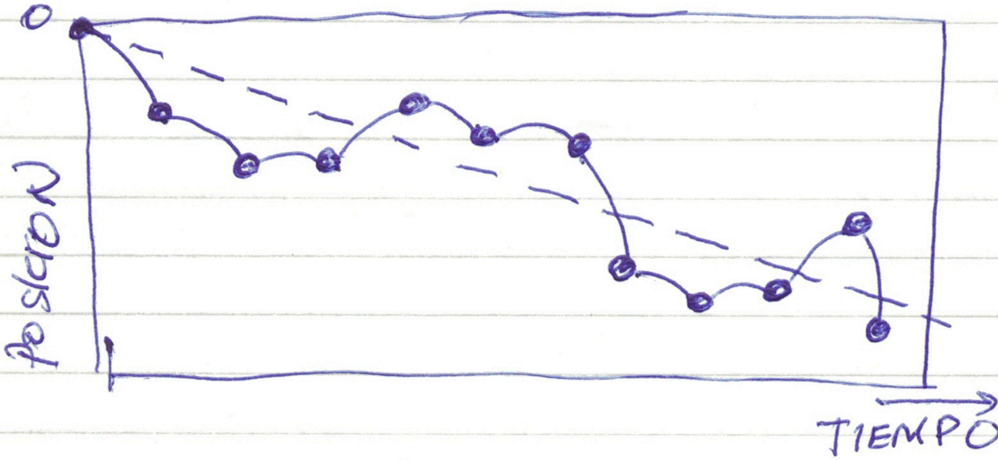
ej: $F = -mg$



promedio de $\vec{v}$ es negativo.

Entre 2 colisiones tenemos:

$$X = \underset{\substack{\uparrow \\ \text{random}}}{X_0} + \underset{\substack{\uparrow \\ \text{random}}}{V_0}\,\Delta t + \frac{1}{2}\left(\frac{F_{ext}}{m}\right)(\Delta t)^2$$



$$\langle x \rangle = \langle v \rangle t$$
$$= -cte \cdot t$$

$$\langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{X - X_0}{\Delta t} = \frac{1}{2}\left(\frac{F_{ext}}{m}\right)\Delta t$$

y como $F_{ext} = \gamma \langle v \rangle \Rightarrow \langle v \rangle = \frac{1}{2}\frac{\gamma \langle v \rangle}{m}\Delta t$

$$\Rightarrow \boxed{\underset{\text{macro}}{\gamma} = \frac{2m}{\underset{\text{micro}}{\Delta t}}\,.} \qquad (2)$$

Toma $F_{ext} = 0$.

De la caminata al azar, después de N colisiones

$$\langle x^2 \rangle = N(\Delta x)^2 = \frac{(\Delta x)^2 t}{\Delta t} = 2Dt$$

$$\Rightarrow D = \frac{(\Delta x)^2}{2\Delta t} \quad \text{coeficiente de difusión} \qquad (3)$$

$$\Rightarrow \langle x^2 \rangle = 2D t \qquad (4)$$

Ahora, equipartición $\Rightarrow \langle v^2 \rangle = \frac{KT}{m} \qquad (5)$

pero $\langle v^2 \rangle = \left(\frac{\Delta x}{\Delta t}\right)^2 = \frac{2(\Delta x)^2}{2\Delta t\, \Delta t} = \frac{2D}{\Delta t} \overset{(5)}{=} \frac{KT}{m}$

$$\Rightarrow \Delta t = \frac{2Dm}{KT} \qquad (6)$$

volviendo a (2):

$$\gamma = \frac{\cancel{2m}}{\cancel{2Dm}} KT = \frac{KT}{D}$$

Relac. de Einstein - Smoluchowski

$$\Rightarrow \boxed{\gamma D = K_B T} \qquad (7)$$

macroscópicos medibles

microscópico

Ejercicio: Como se miden γ y D?

$(7) \Rightarrow \gamma D = \left(\frac{R}{N_A}\right) T \Rightarrow \boxed{N_A = \frac{RT}{\gamma D}} \qquad (8)$

se puede medir N_A usando MOV. Browniano

Resultado: excelente acuerdo con estimaciones previas no-moleculares $\Rightarrow$ fuerte evidencia para la existencia de átomos y moléculas.

El problema del "Random walk"

Sup. una partícula sumergida en un medio. Supondremos que las fuerzas que actúan sobre ella son de 2 tipos:

(a) Fuerzas debidas a la carencia de balance en el ~~borde~~ bombardeo molecular sobre la partícula.

(b) Fuerza viscosa proporcional a la velocidad de la partícula (Ley de Stokes)

Por simplicidad, sup. mov. 1-dimensional.

Sea X = resultante, en cualq. instante, de las fuerzas debidas al bombardeo

$F = -6\pi a \eta (dx/dt) = -\mu (dx/dt)$, fuerza viscosa.

Ec. de mov:
$$m \frac{d^2x}{dt^2} = -\mu \frac{dx}{dt} + X \qquad (1) \qquad / \times 2x$$

$$m\, 2x \frac{d^2x}{dt^2} = -\mu \cdot 2x \frac{dx}{dt} + 2Xx \qquad (2)$$

pero $2x \dfrac{dx}{dt} = \dfrac{d}{dt}(x^2)$ (3), y diferenciando c/r a t

$$2\left(\frac{dx}{dt}\right)^2 + 2x\frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(x^2)$$

$$\Rightarrow 2x\frac{d^2x}{dt^2} = \frac{d^2}{dt^2}(x^2) - 2\left(\frac{dx}{dt}\right)^2 \qquad (4)$$

sustituyendo (3) y (4) en (2), queda:

$$m\frac{d^2}{dt^2}(x^2) - 2m\left(\frac{dx}{dt}\right)^2 = -\mu\frac{d}{dt}(x^2) + 2Xx \qquad (5)$$

" "

$$\langle F\rangle = \frac{1}{t}\int_0^t f(t')\,dt'$$

$$\left\langle \frac{dF}{dt}\right\rangle - \frac{d\langle F\rangle}{dt} = \frac{1}{t}\langle F\rangle$$

Y $\frac{1}{2}m\overline{v^2} = \frac{3}{2}KT = \frac{3}{2}\frac{R}{N}T$

luego: $m\overline{\left(\frac{dx}{dt}\right)^2} = \frac{RT}{N}$

la ec. que nos interesa queda:

$$m\frac{d^2}{dt^2}(\overline{x^2}) - \frac{2RT}{N} = -\mu\frac{d}{dt}(\overline{x^2})$$, poniendolo $w = \frac{d}{dt}(\overline{x^2})$

$$m\frac{dw}{dt} = \frac{2RT}{N} - \mu w \quad \text{ó} \quad \frac{dw}{dt} + \frac{\mu}{m}w = \frac{2RT}{Nm}$$

con solución: $w(t) = \frac{2RT}{N\mu} + A\,e^{-\left(\frac{\mu}{m}\right)t}$ (6)

$\tau = \frac{m}{\mu}$: tiempo de relajación $= \frac{\frac{4}{3}\pi a^3\rho}{6\pi a\eta} = \frac{2a^2\rho}{9\eta}$

En un caso típico: $a = 10^{-4}$ cm , $\rho \sim 1$, $\eta \approx 10^{-2}$ cgs $\Rightarrow \tau \sim 2\cdot 10^{-7}$ segs.

luego el término exponencial es despreciable para cualq. tiempo razonable de observación.

Luego (6) puede escribirse como:

$$w = \frac{d}{dt}(\overline{x^2}) = \frac{2RT}{N\mu} \Rightarrow \overline{x^2} = \frac{2RT}{N\mu}t = \frac{RT}{3\pi a\eta N}t$$

$$\therefore \quad \overline{x^2} = \frac{KT}{3\pi a\eta}t \qquad (7)$$ $\left\{\text{ecuación de Einstein - Smoluchowski}\right\}$ (1905)

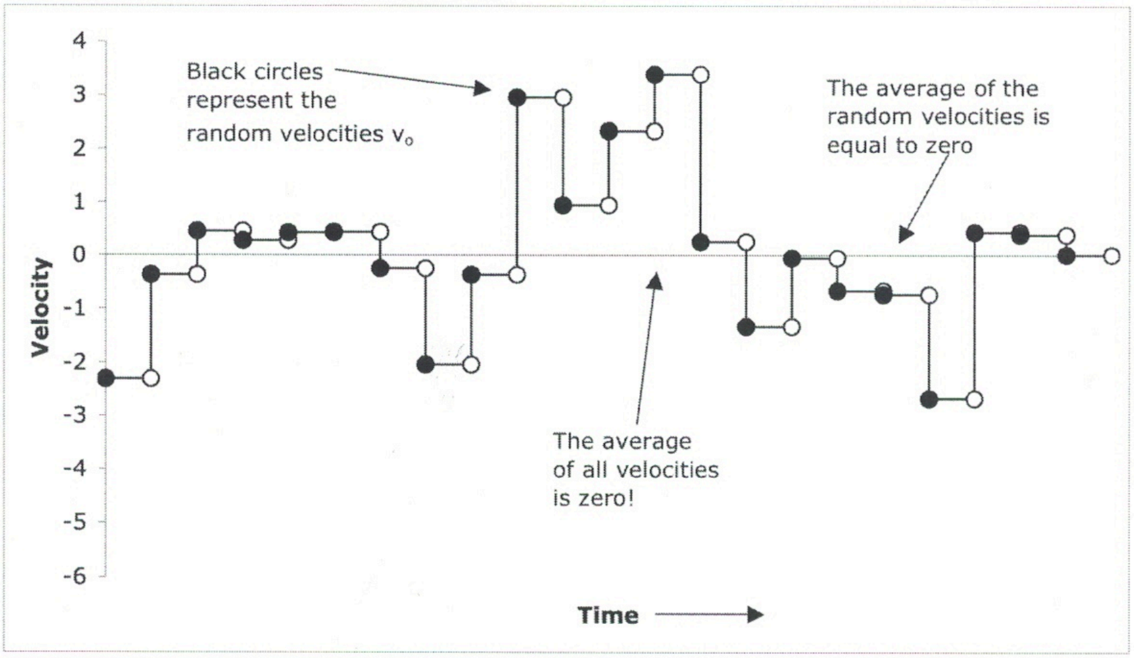
veamos si estos desplazamientos pueden ser observados:

sea $t = 60$ segs. , $a = 10^{-4}$ cms , $\eta \approx 10^{-2}$ (agua)

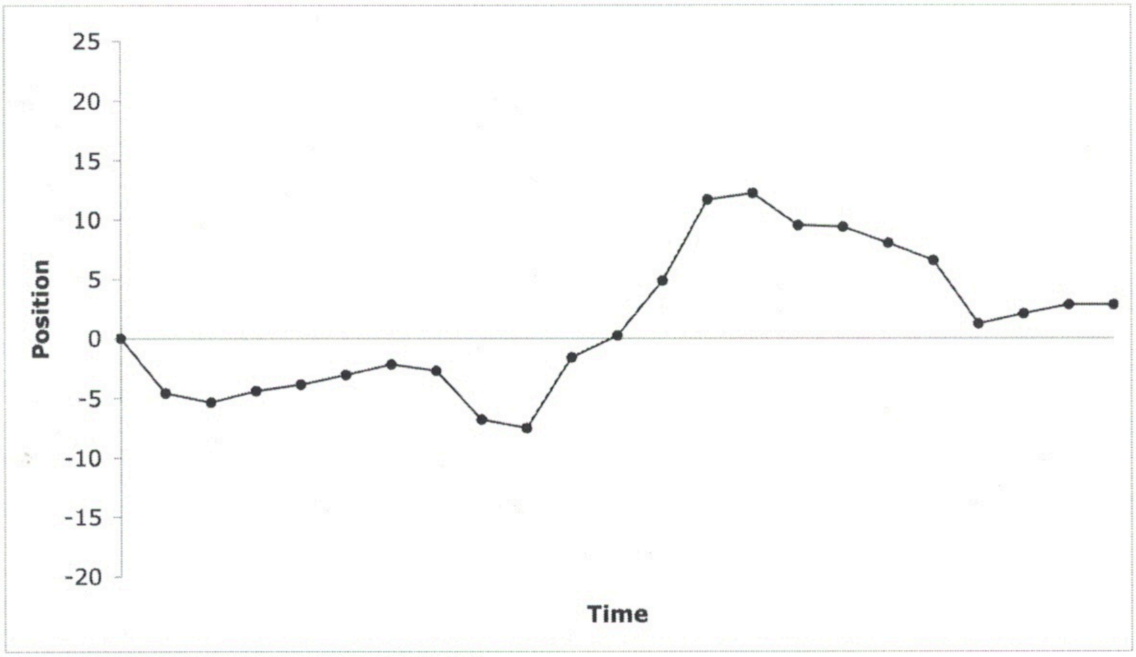
$$\boxed{f = \frac{2m}{\Delta t}}$$

According to this equation, the *macroscopic* viscous drag coefficient of an object with mass m depends only on the frequency of *microscopic* collisions with that object! This relationship should make sense: if the collisions are very frequent (so Δt is small), then the viscous drag coefficient will be larger. Next time you pull a spoon out of a jar of honey, think about how the extremely rapid collisions between the "honey molecules" and the spoon give rise to the viscous drag you feel on the spoon!

We can immediately see the connection to Brownian motion if we *remove the constant external force* from our above discussion. Then the velocity as a function of time would look like this: the object moves at a *constant random velocity* over each time interval:



Now, if we integrate the velocities to find the position, we see that the *average* position of the object is zero, but the object still "wanders" up and down over time:



Compare this graph of positon with the previous one: in this case, the object moves with a constant velocity between each collision. This is an example of what physicists and mathematicians refer to as a *random walk*: the object moves randomly, taking a series of small "steps." Each step can be in either direction (up or down, in this example).

Because a random walk is, by definition, random, we can only inquire about the *average* behavior of an object undergoing a random walk. For instance, we note that the average *displacement* of the object is always zero, since each step is equally likely to be up or down. Thus, in some sense, the object, on average, doesn't "go anywhere."

This observation is misleading, however, because the average *distance from the origin* will *not* be zero. We can better characterize the average distance by considering the *mean square displacement*, or $<x^2>$. (We use this definition because we want the "distance" to always be positive, so we square the displacement to obtain a positive measure of distance.)

Why will the mean square displacement not be zero? Consider a random walk of four steps, where the steps are Δx_1, Δx_2, Δx_3, and Δx_4. If the object starts at $x = 0$, then the final displacement of the object is given by $(\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4)$. What happens if we *square* that? We get a hideous mess that looks something like this:

$$(\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4)^2 = (\Delta x_1)^2 + (\Delta x_2)^2 + (\Delta x_3)^2 + (\Delta x_4)^2 + (\Delta x_1)(\Delta x_2) + (\Delta x_1)(\Delta x_3) + \ldots$$

where we have omitted a number of the "cross terms" $(\Delta x_i)(\Delta x_j)$ in which $i \neq j$. Now what can we say about the *sign* of all of these terms? We know that the squared terms such as $(\Delta x_1)^2$ will *always* be positive, regardless of the sign of Δx_1. However, the *cross* terms will, *on average*, be equal to **zero**, because each cross term could have either a positive or negative sign, depending on the signs of the individual Δx_i. Thus, the mean square displacement is given by dropping all of the cross terms in the above expression:

$$\langle x^2 \rangle = \langle (\Delta x_1 + \Delta x_2 + \Delta x_3 + \Delta x_4)^2 \rangle = \langle (\Delta x_1)^2 \rangle + \langle (\Delta x_2)^2 \rangle + \langle (\Delta x_3)^2 \rangle + \langle (\Delta x_4)^2 \rangle$$

The bottom line is that *random displacements add in quadrature*: the square of the overall displacement is equal to the sum of the squares of the individual displacements. You may recall a similar formula for adding standard deviations of random variables: the square of the standard deviation of the sum of a number of random variables is given by the sum of the squares of the individual standard deviations:

$$\sigma^2 = (\sigma_1)^2 + (\sigma_2)^2 + (\sigma_3)^2$$