General Nonlinear impurity in a photonic array: Green function approach

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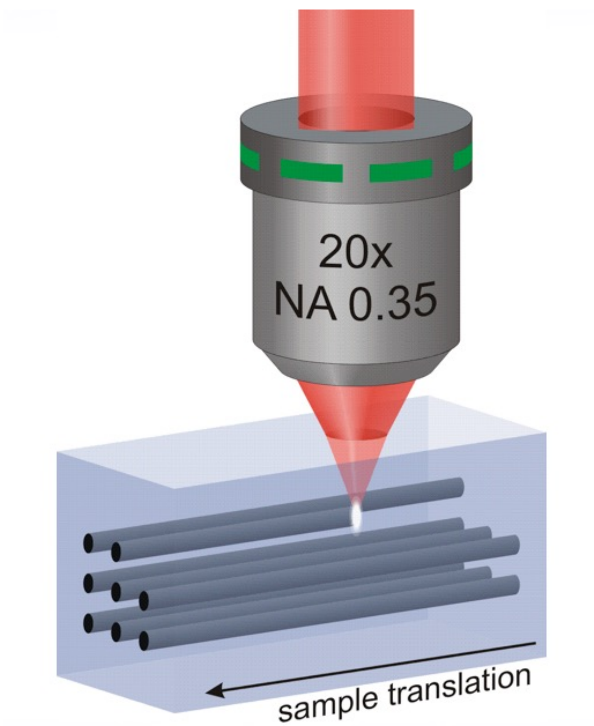
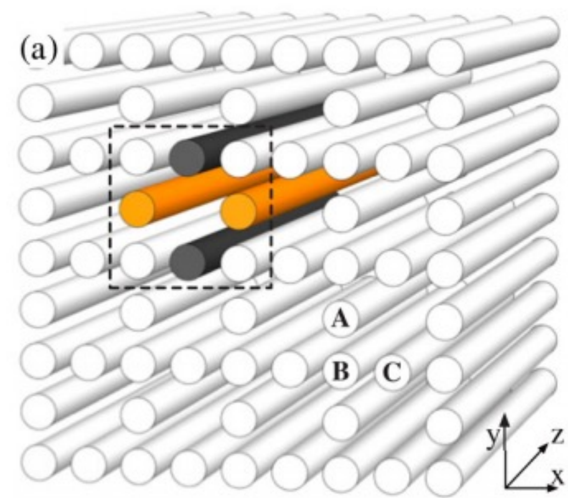
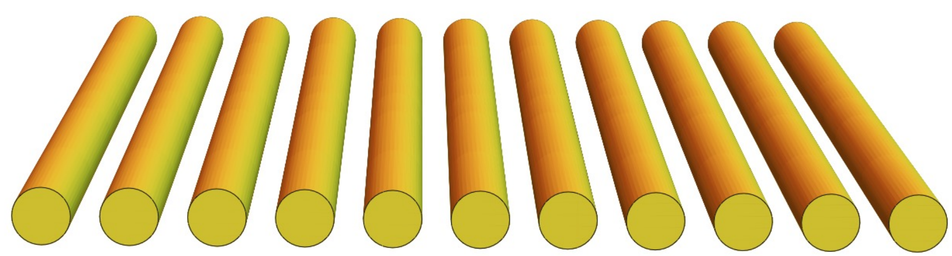
Phys. Rev. B, 74, 045412 (2006).
Phys. Rev. B 73, 014204 (2006).
Phys. Rev. B 71, 035404 (2005).

Optical waveguide arrays



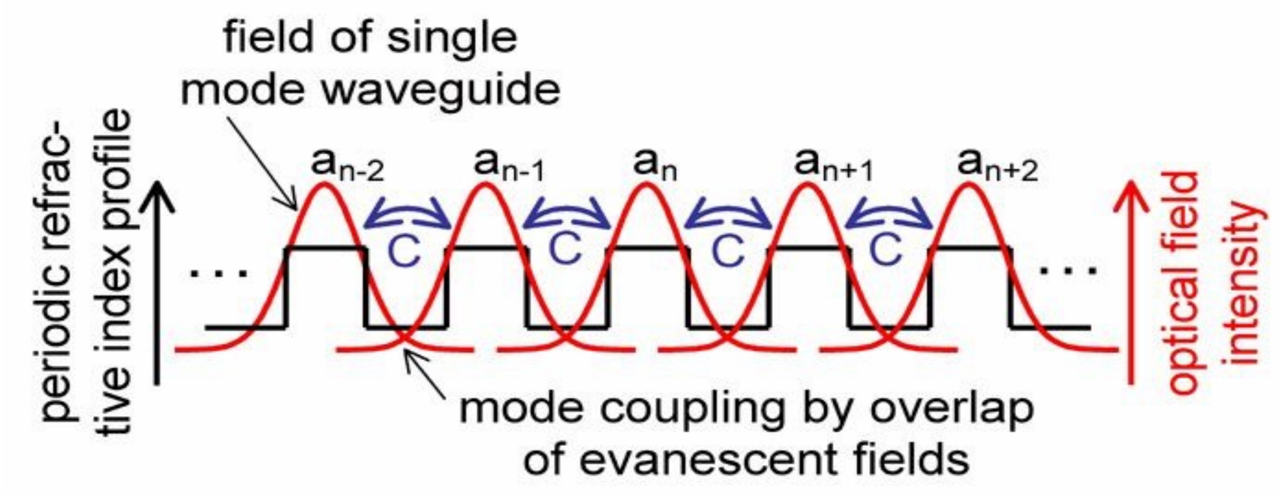


2D



1D

Fe:$LiNbO_3$

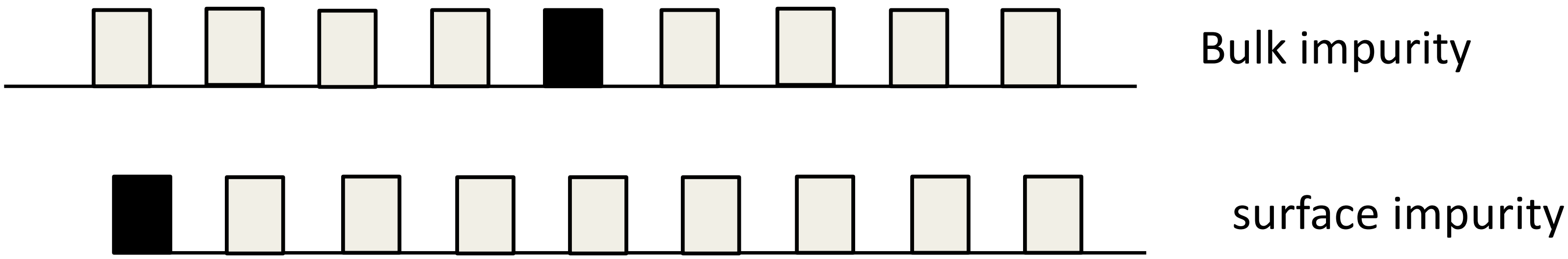


$$E(x,z) = \sum_n E_n(z)\phi(x-n)$$

Coupled-modes

$$i\left(\frac{dE_n}{dz}\right) + V(E_{n+1} + E_{n-1}) + \chi\ \delta_{n,d} f(|E_n|^2)E_n = 0$$

Intensity-dependent
index of refraction



What do we want to compute?

Form of the localized mode at the impurity site (bulk and surface) and transmission of plane waves across the impurity in **closed form**

The Hamiltonian !!

$$\tilde{H} = \tilde{H}_0 + \tilde{H}_1$$

$$\tilde{H}_0 = V \sum_{nn} (|n\rangle\langle m| + h.c.)$$

$$\tilde{H}_1 = \chi\ f(|E_d|^2)|d\rangle\langle d|$$

GREEN function $G(z) = 1/(z - \tilde{H})$

Poles of Green function → energies of bound states
Residues at poles → bound state amplitudes

NO GUESSWORK HERE!

Perturbative Expansion

$$G = G^{(0)} + G^{(0)} H_1 G^{(0)} + G^{(0)} H_1 G^{(0)} H_1 G^{(0)} + \cdots$$

$$G^{(0)} = 1/(z - H_0)$$

$$G_{mn} = G^{(0)}_{mn} + \frac{\varepsilon}{1 - \varepsilon\, G^{(0)}_{dd}}\, G^{(0)}_{md} G^{(0)}_{dn}$$

$$G_{mn} = \langle m|G|n\rangle \quad \text{and} \quad \epsilon = \chi\ f(|E_d|^2)$$

Bound state equation

$$1 = \epsilon\, G^{(0)}_{dd}(z_b) = \chi\ f(|E_d|^2)\ G^{(0)}_{dd}(z_b)$$

$$1 = f\left(-\frac{G^{(0)\,2}_{dd}(z_b)}{G'^{(0)}_{dd}(z_b)}\right) G^{(0)}_{dd}(z_b).$$

now,
$$G^{(0)}_{nd}(z) = \left(\frac{sgn(z)}{\sqrt{z^2-1}}\right)\left\{z - sgn(z)\sqrt{z^2-1}\right\}^{|n-d|}$$

$$1 = f\left(-\frac{\sqrt{z_b^2-1}}{|z_b|}\right)\frac{\mathrm{sgn}(z_b)}{\sqrt{z_b^2-1}}$$

Bound state energy equation

$$|E_n^{(b)}|^2 = Res\{G_{nd}\}_{z=z_b} = -\frac{G_{nd}^{(0)2}(z_b)}{G'^{(0)}_{dd}(z_b)}$$

Bound state amplitudes

$$\Rightarrow \quad |E_n|^2 = \frac{\sqrt{z_b^2-1}}{|z_b|}\left[z_b - \operatorname{sgn}(z_b)\sqrt{z_b^2-1}\right]^{2|n|}$$

Exponentially-decreasing profile $\sim e^{-\lambda|n|}$

Inverse localization length

$$\lambda = -\frac{1}{2}\log\left|z_b - \operatorname{sgn}(z_b)\sqrt{z_b^2-1}\right|$$

Special case: the saturable impurity

$$f(E_n) = \gamma\ \delta_{n,d}\ \left(\frac{1}{1+|E_n|^2} \right)$$

$$\frac{1}{\gamma} = \frac{z}{z^2 - 1 + |z|\sqrt{z^2-1}}$$

Energy equation

Analytic solution

$$z_b = -\left(\frac{1-\gamma^2}{6\gamma}\right) + \frac{1+10\gamma^2+\gamma^4}{6\gamma D(\gamma)} + \frac{D(\gamma)}{6\gamma}$$

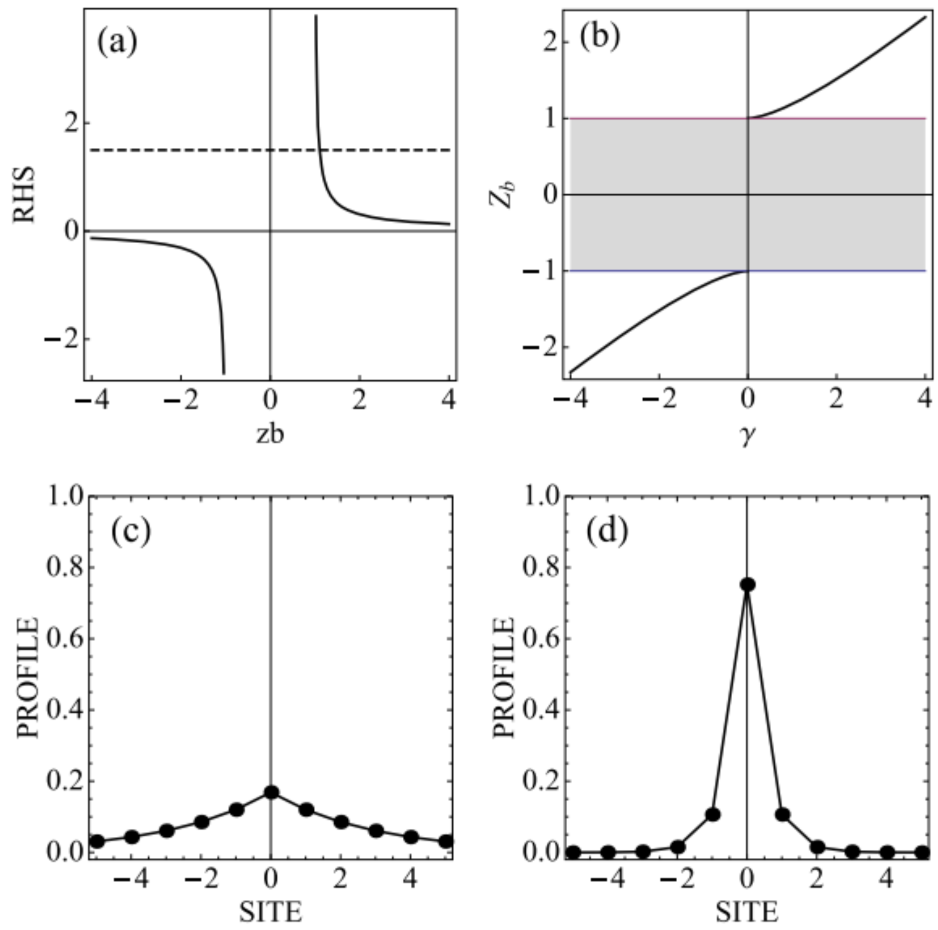
Bound state energy !

where

$$D(\gamma) = -1 + 39\gamma^2 + 15\gamma^4 + \gamma^6 + 6\sqrt{3}\gamma\sqrt{-1+11\gamma^2+\gamma^4}.$$

$$\left|E_n^{(b)}\right|^2 = \frac{\sqrt{z_b^2 - 1}}{|z_b|}\left(z_b - \mathrm{sgn}(z_b)\sqrt{z_b^2 - 1}\right)^{2|n-d|}$$

Bound state profile !



Bulk impurity

Surface impurity (d = 0)

Have to take into account the presence of boundary at d=0

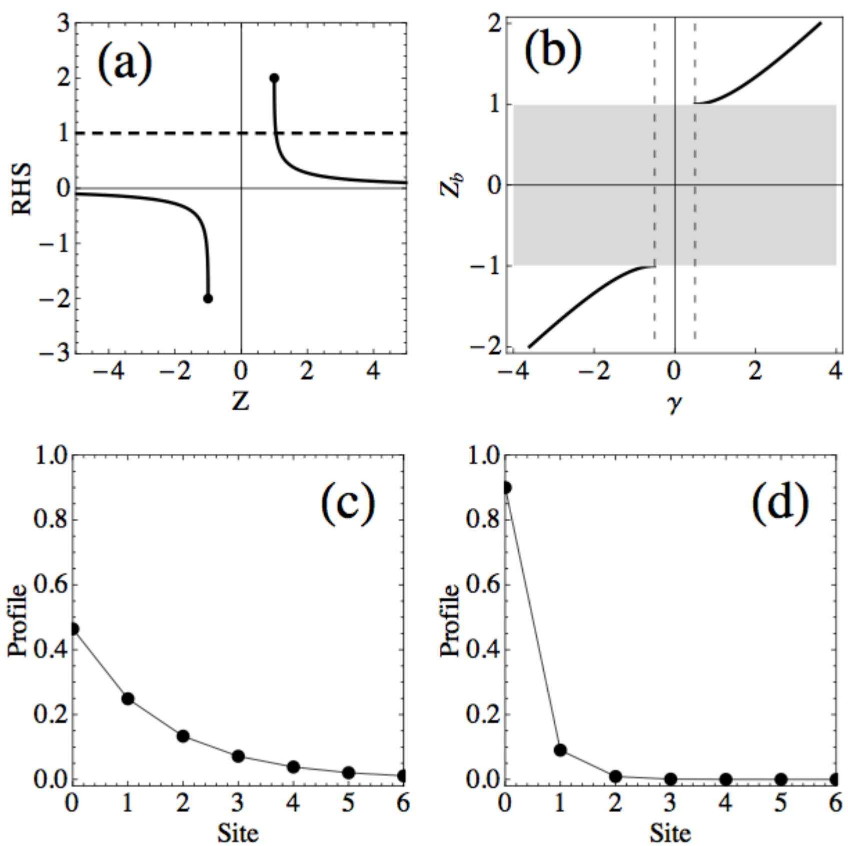
$$G_{mn}^{(0)} = G_{mn}^{\infty} - G_{m,-n-2}^{\infty}$$

$$G_{mn}^{(0)} = \frac{\mathrm{sgn(z)}}{\sqrt{z^2-1}} \left[z - \mathrm{sgn(z)}\sqrt{z^2-1} \right]^{|n-m|} - \frac{\mathrm{sgn(z)}}{\sqrt{z^2-1}} \left[z - \mathrm{sgn(z)}\sqrt{z^2-1} \right]^{|n+2+m|}$$

$$\frac{1}{\gamma} = \frac{2}{z + 3\ \mathrm{sgn(z)}\sqrt{z^2-1}}$$

$$z_b = (1/4)(-\gamma + 3\ \mathrm{sgn}(\gamma)\sqrt{2+\gamma^2})$$

$$|E_n^{(b)}|^2 = \alpha(z_b)(\ q(z_b)^{|n|} - q(z_b)^{|n+2|}\ )$$



Minimum nonlinearity strength needed

Surface impurity

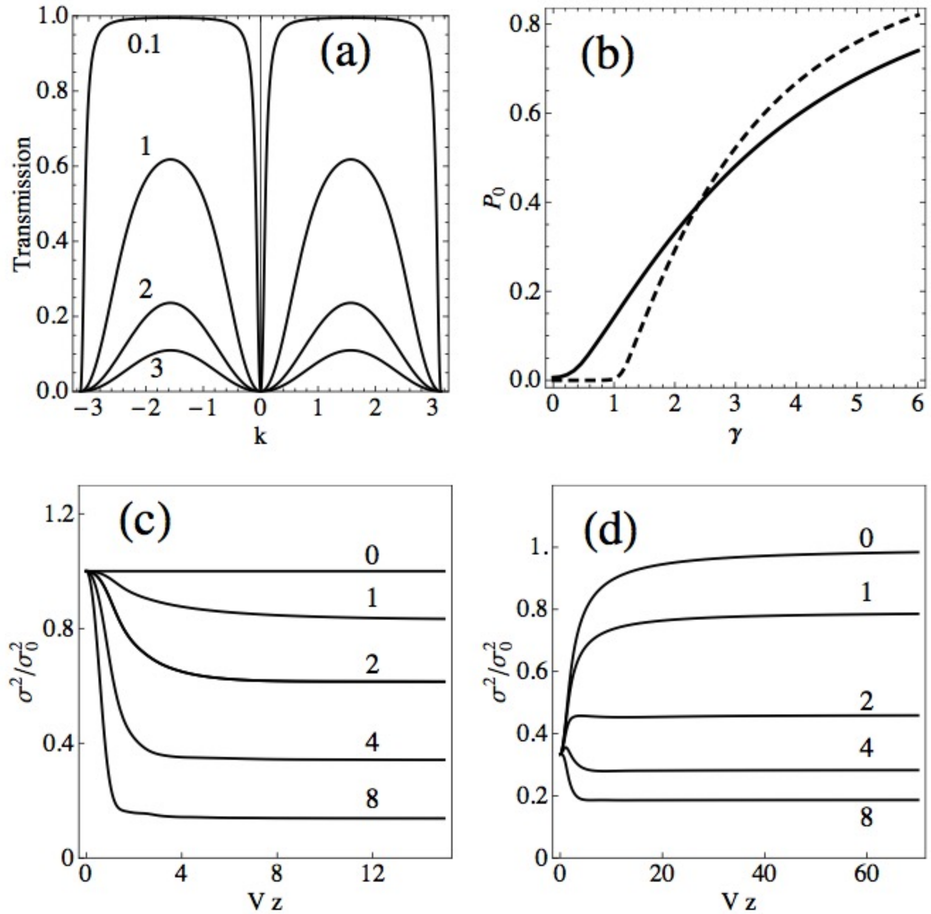
Dynamical properties

$$P_d = \lim_{L\to\infty} (1/L) \int_0^L |E_d(z)|^2 dz$$

$$\langle n^2 \rangle = \frac{\sum_n (n-d)^2 |E_n(z)|^2}{\sum_n |E_n(z)|^2}$$

ballistic propagation

$$\sigma(z) \to \sqrt{2}(Vz)$$



bulk surface

CONCLUSIONS

- Obtained Green function in closed form for 1D lattice with single general nonlinear impurity.
- Used Green function to obtain energy and bound state profile in closed form.
- Specialize to a saturable optical impurity
- In bulk case an impurity state is always possible.
- For surface case a minimum nonlinearity is needed.
- Bulk case shows no selftrapping transition.
- Surface case shows selftrapping transition.
- Asymptotic propagation of optical power shows ballistic character
- Method can be extended to higher dimensions