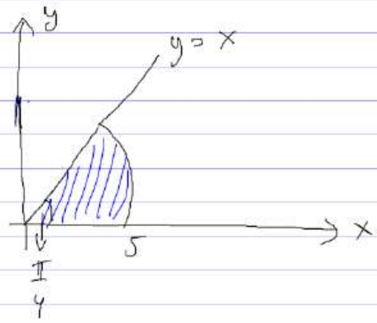
2. Calcule la integral doble

$$\int_R \underbrace{x e^{\sqrt{x^2+y^2}}}_{f(x,y)} \, dA$$

$$\sqrt{x^2+y^2} \leq 5$$

donde $R = \{(x,y) \in \mathbb{R}^2 : y \geq 0, \boxed{x \geq y}, \underline{x^2+y^2 \leq 25}\}$.

$$\boxed{\begin{aligned} x &= r \cdot \cos\theta \\ y &= r \cdot \sin\theta \end{aligned}}, \quad r \in [0, 5], \quad \theta \in \left[0, \frac{\pi}{4}\right].$$



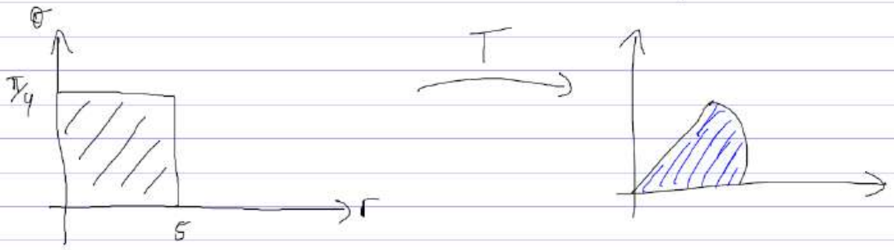
$$T(r,\theta) = (r \cdot \cos\theta, \, r \sin\theta)$$

$$r \in [0, 5], \quad \theta \in [0, \pi/4].$$

$$R^* = \left\{ (r,\theta) : 0 \leq r \leq 5, \quad 0 \leq \theta \leq \frac{\pi}{4} \right\}$$

θ

$$R = \{ (r,\theta) : \quad 0 \leq r \leq 5 \quad , \quad 0 \leq \theta \leq \frac{\pi}{4} \}$$



$$\int_{T(R^*) = R} f \underset{T.C.V}{=} \int_{R^*} f(T(r,\theta)) \cdot \left| \frac{\partial (x,y)}{\partial (r,\theta)} \right| dA .$$

$$= \int_{R^*} f\left(r.\cos\theta , r\sin\theta \right) \cdot \left| \frac{\partial (x,y)}{\partial (r,\theta)} \right| dA$$

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$$\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right| = \left|JT(r,\theta)\right| = \left|\begin{pmatrix}\cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta\end{pmatrix}\right|$$

$$= r$$

por Fubini,

$$\int_0^5 \int_0^{\pi/4} r\cos\theta \cdot e^{\sqrt{r^2\cos^2\theta + r^2\sin^2\theta}} \cdot r \, d\theta \, dr$$

$$= \int_0^5 \int_0^{\pi/4} r^2 \cos\theta \cdot e^{r} \, d\theta \, dr$$

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$$= \int_0^5 \int_0^{\pi/4} r^2 \cos\theta \cdot e^r \, d\theta \, dr$$

$$= \int_0^5 r^2 e^r \, dr \int_0^{\pi/4} \cos\theta \, d\theta$$

$$= \int_0^5 r^2 e^r \, dr \left(\sin\theta \Big|_0^{\pi/4} \right)$$

$$= \frac{\sqrt{2}}{2} \underbrace{\int_0^5 r^2 e^r \, dr}$$

$$* \int^5 r^2 e^r dr \qquad u = r^2 \implies du = 2r\,dr$$

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$$* \int_0^5 r^2 e^r\,dr \qquad u = r^2 \Rightarrow du = 2r\,dr$$

$$dv = e^r\,dr \Rightarrow v = e^r$$

$$\int_0^5 r^2 e^r\,dr = e^r r^2 \Big|_0^5 - 2\int_0^5 r e^r\,dr$$

$$= 25e^5 - 2\int_0^5 r e^r\,dr$$

$$= 17e^5 - 2$$

por lo tanto,

$$\iint_R x e^{\sqrt{x^2+y^2}}\,dA = \frac{\sqrt{2}}{2}\left(17e^5 - 2\right)$$

$$= 25e^5 - 2\int_0^5 re^r\,dr$$

$$= 17e^5 - 2$$

por lo tanto, $\displaystyle\iint_R x e^{\sqrt{x^2+y^2}}\,dA = \frac{\sqrt{2}}{2}\cdot\left(17e^5 - 2\right).$

3. Sea $F : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ dada por $F(x,y) = (\underbrace{2018x^{2017}y^{2017} + y}_{P}, \underbrace{2017x^{2018}y^{2016} + x}_{Q})$. Calcule

$$\int_\sigma F(x,y)\cdot d\sigma$$

donde σ es una parametrización (en el sentido contrario a las agujas del reloj) de la intersección entre la parábola $y = x^2$ y el circulo unitario.