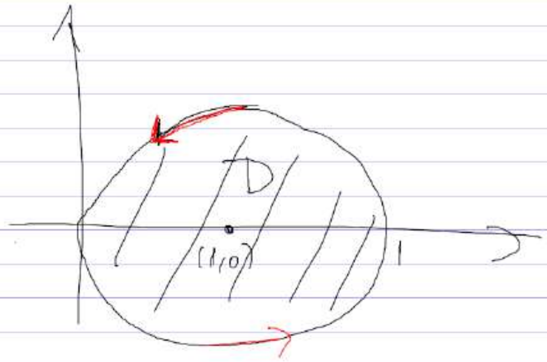
4. Calcule las siguientes integrales de línea

$$\int_\sigma F\cdot d\sigma \quad \int_\sigma G\cdot d\sigma$$

donde $F(x,y) = (y, e^{\sin(y^2)})$; y $G(x,y) = (-\frac{\partial h}{\partial y}, \frac{\partial h}{\partial x})$, donde $h:\mathbb{R}^2 \longrightarrow \mathbb{R}$ es de clase C^2 y $\Delta h = 1$. Aquí σ es la parametrización (en sentido antihorario) de la circunferencia de centro $(1,0)$ y radio 1.

a) $\displaystyle\int_\sigma F\cdot d\sigma$ $\qquad F(x,y) = (\underbrace{y}_{P}, \underbrace{e^{\sin y^2}}_{Q})$

σ parametriza al círculo de radio 1 y centro $(1,0)$.



$$\int_{\gamma} F \cdot d\sigma = \iint_{D} \left[\partial_x Q - \partial_y P \right] dA$$

$$= \iint_{D} (0 - 1)\, dA$$

$$= -\iint dA$$

$Q = e^{\sin y^2}$

$P = y$

$\partial_x Q = 0$

$$= \iint_D (0 - 1)\, dA$$

$$= -\iint_D dA$$

$$= -A(D)$$

$$= -\pi$$

$Q = e^{\sin y^2}$

$P = y$

$\partial_x Q = 0$

$\partial_y P = 1$

b) $\displaystyle\int_\sigma G \cdot d\sigma \quad , \quad G = \Big(\underbrace{-\frac{\partial h}{\partial y}}_{P} , \underbrace{\frac{\partial h}{\partial x}}_{Q}\Big)$

$$= -1$$

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b) $\displaystyle\int_\sigma G \cdot d\sigma$, $\quad G = \Big(\underset{\displaystyle P}{\overset{\|}{-\frac{\partial h}{\partial y}}} , \underset{\displaystyle Q}{\overset{\|}{\frac{\partial h}{\partial x}}} \Big)$

donde $h : \mathbb{R}^2 \to \mathbb{R}$ de clase C^1

y $\Delta h = 1$.

$$\int_\sigma G \cdot d\sigma \underset{\text{T. G}}{\overset{\downarrow}{=}} \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$= \iint_D \frac{\partial^2 h}{\partial x^2} - \left[-\frac{\partial^2 h}{\partial y^2} \right] dA$$

$$\int_{\sigma} G \cdot d\sigma \underset{\text{T.G}}{=} \iint_{D} \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

$$= \iint_{D} \frac{\partial^2 h}{\partial x^2} - \left[- \frac{\partial^2 h}{\partial y^2} \right] dA$$

$$= \iint_{D} \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) dA$$

$$= \iint_{D} \Delta h \, dA$$

$$= \iint_D \Delta h \, dA$$

$$= \iint_D 1 \, dA$$

$$= A(D)$$

$$= \pi$$