3. Sea $F:\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ dada por $F(x,y) = (2018x^{2017}y^{2017} + y, 2017x^{2018}y^{2016} + x)$. Calcule

$$\int_\sigma F(x,y)\cdot d\sigma$$

donde σ es una parametrización (en el sentido contrario a las agujas del reloj) de la intersección entre la parábola $y = x^2$ y el circulo unitario.

• Veamos si el campo es conservativos, es decir, si existe $f:\mathbb{R}^2 \to \mathbb{R}$ tq $\nabla f = \overline{F}$.

Calculamos:

$$\nabla \times F = \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \quad (\text{rotor en } \mathbb{R}^2)$$

• $\frac{\partial Q}{\partial x} = 2017\, y^{2016}\; 2018\, x^{2017} + 1$

- $\dfrac{\partial Q}{\partial x} = 2017\, y^{2016}\, 2018\, x^{2017} + 1$

- $\dfrac{\partial P}{\partial y} = 2018\, x^{2017} \cdot 2017\, y^{2016} + 1$

$\therefore \quad \nabla \times F = 0.$

$\therefore \quad \exists f : \quad \nabla f = F = (\partial_x f, \partial_y f)$

Igualando coordenadas:

$$\partial_x f = 2018\, x^{2017} y^{2017} + y \qquad (1)$$

Igualando coordenadas:

$$\partial_x f = 2018\, x^{2017} y^{2017} + y \qquad (1)$$

$$\partial_y f = 2017\, x^{2018} y^{2016} + x \qquad (2)$$

Integrando respecto a x (1):

$$f(x,y) = \cancel{2018}\, y^{2017} \cdot \frac{x^{2018}}{\cancel{2018}} + yx + C(y)$$

Derivo respecto a y:

$$\partial_y f = 2017\, y^{2016} x^{2018} + x + C'(y)$$

Derivo respecto a y:

$$\partial_y f = 2017\, y^{2016} \cdot x^{2018} + x + c'(y)$$

Comparando con (2):

$$c'(y) = 0 \Rightarrow c(y) = C \in \mathbb{R}.$$

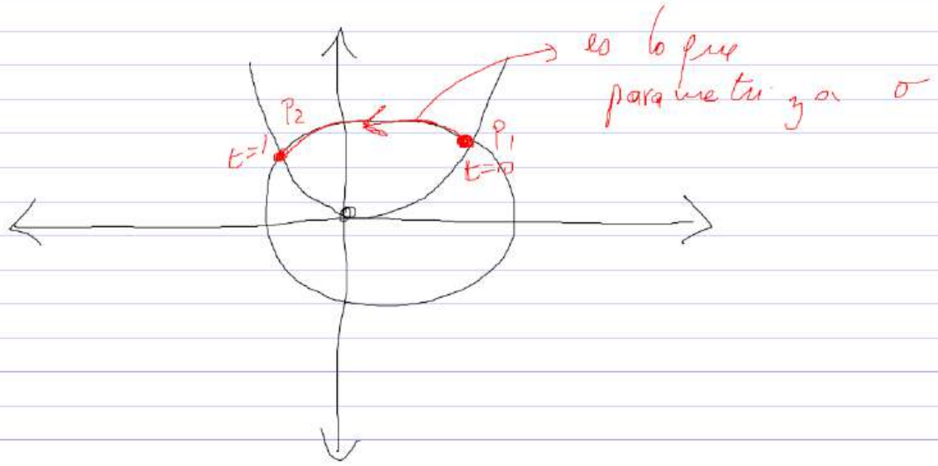
$$\therefore \boxed{f(x,y) = y^{2017}\, x^{2018} + yx + C}$$

Así,

$$\int F \cdot d\sigma = \int \nabla f \cdot d\sigma$$

$$\int_{\sigma} F \cdot d\sigma = \int_{\sigma} \nabla f \cdot d\sigma$$

$$= f(\sigma(1)) - f(\sigma(0))$$



$$\boxed{\begin{aligned} \sigma(0) &= P_1 \\ \sigma(1) &= P_2 \end{aligned}}$$

Intersectando $x^2 + y^2 = 1$

$y = x^2$

$$\Rightarrow P_1 = \left(\sqrt{\frac{-1+\sqrt{5}}{2}} \, , \, \frac{-1+\sqrt{5}}{2} \right)$$

$$P_2 = \left(-\sqrt{\frac{-1+\sqrt{5}}{2}} \, , \, \frac{-1+\sqrt{5}}{2} \right)$$

$$\int F = f(P_2) - f(P_1)$$

$$\boxed{\sigma(1) = P_2}$$

$$y = x^2$$

$$\Longrightarrow P_1 = \left(\sqrt{\frac{-1+\sqrt{5}}{2}} \, , \, \frac{-1+\sqrt{5}}{2} \right)$$

$$P_2 = \left(-\sqrt{\frac{-1+\sqrt{5}}{2}} \, , \, \frac{-1+\sqrt{5}}{2} \right)$$

$$\therefore \int_\sigma F = f(P_2) - f(P_1)$$

//

4. Calcule las siguientes integrales de línea