Ayudantía 10

31/05/22

Breve Revisión prueba 2.

1. Pruebe que

$$\int_{-\pi}^{\pi} \underbrace{x^2}_{u} \underbrace{\cos(nx)dx}_{dv} = \frac{(-1)^n 4\pi}{n^2}. \qquad \textbf{(1 punto)}$$

$v = \dfrac{\text{sen}(nx)}{n}$

→ Integración por partes $\quad \int u\,dv = uv - \int v\,du$

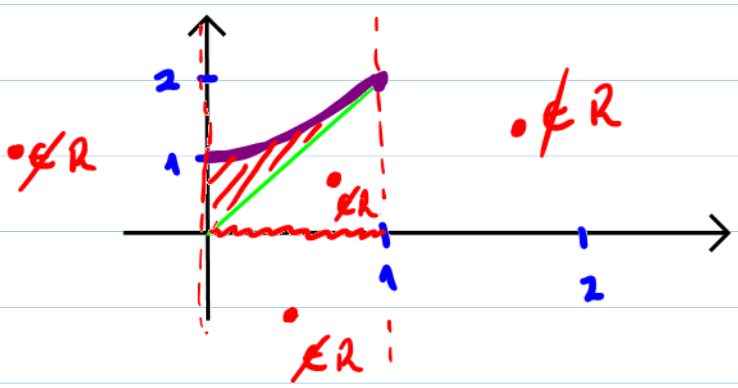
$$= x^2 \cdot \text{sen}(nx) \cdot \frac{1}{n} \Big|_{-\pi}^{\pi} - \underbrace{\int_{-\pi}^{\pi} 2x \cdot \frac{1}{n} \cdot \text{sen}(nx)\,dx}_{\text{Volver a I×P.}}$$

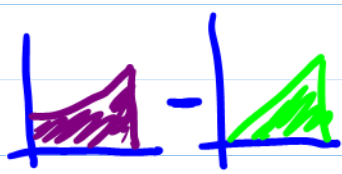
2. Considere la región plana dada por $R = \{(x,y) \in \mathbb{R}^2 : \underbrace{2x}_{f} \le y \le \underbrace{x^2+1}_{g},\ y \ge 0,\ 0 \le x \le 1\}$

   (a) Calcular el área de R. **(1 punto)**

   (b) Hallar el volumen del sólido generado al hacer girar R en torno al eje Y. **(1 punto)**

   (c) Encontrar la longitud de la curva frontera de R. **(1 punto)**





$= A(R)$

$$A(R) = \int_0^1 g(x)\,dx - \int_0^1 f(x)\,dx$$

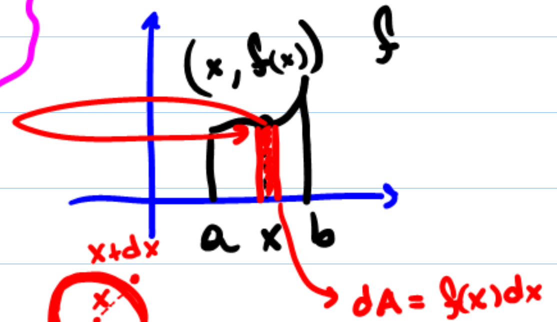
$$= \int_0^1 x^2+1\,dx - \int_0^1 2x\,dx$$

$$= \frac{x^3}{3} + x \Big|_0^1 - x^2 \Big|_0^1$$

$$= 1/3 + \cancel{1} - \cancel{1}$$

$$A(R) = 1/3$$

b) $f = 2x$; $g = x^2 + 1$



→ $V_f = 2\pi \int_0^1 f(x)\,x\,dx$

→ $V_g = 2\pi \int_0^1 g(x)\,x\,dx$

$$\boxed{V = V_g - V_f}$$

$$dv = \pi (x+dx)^2 f(x) - \pi x^2 \cdot f(x)$$

$$= \pi f(x) \left[(x+dx)^2 - x^2\right]$$

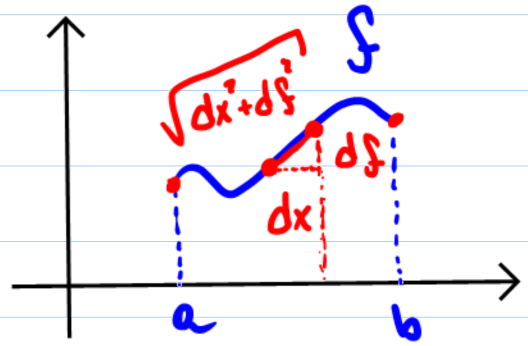
$$= \pi f(x) \cdot \left[2x\,dx + \cancel{dx^2}_{0}\right]$$

$$\sim 2\pi f(x)\, x\, dx$$

$$\boxed{V = \int dv = \int_a^b 2\pi f(x) \cdot x\, dx}$$

$$l = \int_a^b \sqrt{1 + (f')^2}\, dx$$

c) Long. de Arco.



$$l = \sum \sqrt{dx^2 + df^2} \quad / \; dx \to 0$$

$$l = \int_a^b \sqrt{dx^2 + df^2}$$

$$l = \int_a^b \left(dx^2 \left[1 + \left(\cancel{\frac{df}{dx}}_{f'}\right)^2\right]\right)^{1/2}$$

$$l = \int_a^b \left(1 + (f')^2\right)^{1/2} dx$$

3. Estudie la convergencia de las siguientes integrales

(a) $\displaystyle\int_0^1 \frac{1}{1-\cos(x)}\,dx$ **(1 punto)** (b) $\displaystyle\int_0^\infty \frac{1}{(2+x)\sqrt{x}}\,dx$ **(1 punto)**

a) $x = 2\alpha$ $\quad 1-\cos(2\alpha) = 2\,\text{sen}^2(\alpha)$

$dx = 2d\alpha$;

$$\int \frac{2d\alpha}{2\,\text{sen}^2(\alpha)}$$

$$= \int \text{cosec}^2(\alpha)\,d\alpha \quad ; \quad \cot(\alpha) = \frac{\cos(\alpha)}{\text{sen}(\alpha)} \Big)\,(\,)'$$

$$= -\cot(\alpha)\Big|_a^b \qquad \frac{-\text{sen}^2(\alpha)-\cos^2(\alpha)}{\text{sen}^2} = \frac{-1}{\text{sen}^2}$$

$$\int_0^1 \sim dx = \int_0^{1/2} \sim d\alpha = \underbrace{-\cot(\alpha)\Big|_0^1}_{\infty}$$

no converge

b) $$\int_0^\infty \frac{dx}{(2+x)\sqrt{x}} \quad ; \quad x = u^2$$

$dx = 2u\,du$

$$\int_0^\infty \frac{2\cancel{u}}{(2+u^2)\,\cancel{u}}\,du \qquad \sqrt{u^2} = |u| = u$$

$$= \int_0^\infty \frac{2du}{2+u^2} \quad ; \quad u = \sqrt{2}\tan(\alpha)$$

$u = \sqrt{2}\,\alpha \rightarrow du = \sqrt{2}\,d\alpha$

$$= \int_0^\infty \frac{2\sqrt{2}\,d\alpha}{2+2\alpha^2}$$

$$= \int_0^\infty \sqrt{2} \cdot \frac{d\alpha}{1+\alpha^2}$$

$$= \sqrt{2} \cdot \arctan(\alpha) \Big|_0^\infty = \frac{\sqrt{2}\cdot\pi}{4} = \frac{\pi}{\sqrt{8}}$$

~ 60:

Converge ✓ ; $\lim\limits_{\alpha\to\infty} \arctan(\alpha) = \frac{\pi}{4}$