Ayudantía 9 — 24/08/22

a) $\int_0^\infty e^{-x}\,dx$; $\int_a^\infty f = \lim_{b\to\infty} \int_a^b f$

$= \lim_{b\to\infty} \int_0^b e^{-x}\,dx$

$= \lim_{b\to\infty} -e^{-x}\Big|_0^b$

$= \lim_{b\to\infty} \underbrace{\cancel{-e^{-b}}}_{0} - \left(-\underbrace{e^{-0}}_{1}\right)$ $\qquad e^{-0} = e^{0} = 1$

$= 1$

→ Criterio de convergencia:

Acotar: $f(x) \leq g(x) \leq h(x)$; $\forall\, x \in (a, \infty)$

Si $\int_a^\infty f$ y $\int_a^\infty h$ convergen

entonces $\int_a^\infty g$ converge.

b) $\int_0^\infty \sin(x) e^{-x}\,dx$

$-1 \leq \text{sen}(x) \leq 1 \quad / \cdot e^{-x}$

$-e^{-x} \leq e^{-x}\cdot \text{sen}(x) \leq e^{-x} \quad / \int_0^\infty$

$\int_0^\infty -e^{-x}\,dx$ y $\int_0^\infty e^{-x}$ convergen.

Luego $e^{-x}\,\text{sen}(x)$.

→ Otro criterio: $a \to b$

$\sim b \to \sim a$

Si $\int_a^{\infty} f$ converge, $\lim_{x\to\infty} f(x) = 0$.

$x = 2 \quad y = 3 \to x + y = 5$

$x + y \neq 5 \to x \neq 2$ ó $y \neq 3$

c) $\int_0^{\infty} \sin(x) e^{x} dx$

$\lim_{x\to\infty} \operatorname{sen}(x) e^x$ no existe.

$\int \frac{1}{1+x^2} = \arctan(x)$

sí

~~d) $\int_0^{\infty} \frac{1}{1+x^2} dx$~~

$\int_0^{\infty} \frac{dx}{1+x+\frac{x^2}{2}+\frac{x^3}{6}}$ $\frac{1}{p}$ $e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

Ojo: $f \leq \underbrace{1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}}_{p} \leq e^x$ $\quad \frac{1}{p} \leq f$

$e^{-x} \leq \frac{1}{p}$

Si $1 + x^2 \leq 1 + x + \frac{x^2}{2} + \frac{x^3}{6} = p$

$\int 0 \leq \frac{1}{p} \leq \int \frac{1}{1+x^2}$

↳ converge.

$\frac{x^3}{6} \leq p$

$0 < \frac{1}{p} \leq \int \frac{6}{x^3} dx$

Hay que probar: $1 + x^2 \leq 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$

$0 \leq x - \frac{x^2}{2} + \frac{x^3}{6}$

$0 \leq f(x)$?

Una idea. $f(0) \geq 0$

$f'(x) \geq 0 \quad \forall x \geq 0$

$\leq 1 - 1 + x - \frac{x^2}{2} + \frac{x^3}{6}$

$< 1 - \left(1 - x + \frac{x^2}{2} - \frac{x^3}{6}\right)$ e^{-x}

$< 1 - \left(P_{e^{-x}}(x)\right)$

$f(x) = x - \frac{x^2}{2} + \frac{x^3}{6}$

$f'(x) = 1 - x + \frac{x^2}{2} > 0$

$= \frac{1}{2}\left(2 - 2x + x^2\right)$

$= \frac{1}{2}\left(1 + 1 - 2x + x^2\right)$

$= \frac{1}{2}\left(1 + (1-x)^2\right)$

Finalmente: $1+x^2 \leq 1+x+\frac{x^2}{2}+\frac{x^3}{6} \leq e^x$

$$\int_0^\infty e^{-x} \leq \int_0^\infty \frac{1}{1+x+\frac{x^2}{2}+\frac{x^3}{6}} \leq \int_0^\infty \frac{1}{x^2+1}$$

$\hookrightarrow$ Converge.

$P(x) > 1$

$0 > 1 - P(x)$ queríamos

f converge

g ''

no sé.

f · g converge

g) $\displaystyle\int_0^\infty \frac{e^{-x}}{1+x}dx$

$x \geq 0$

$1+x \geq 1$

$\frac{1}{x+1} \leq 1 \quad \forall x \geq 0$

$$\int_0^\infty 0 \leq \frac{e^{-x}}{x+1} \leq \int_0^\infty e^{-x}$$

$\hookrightarrow$ converge.

$\int_0^\infty \frac{1}{x+1}dx$ no converge

$$\lim_{b\to\infty} \int_0^b \frac{1}{x+1}dx$$

$$= \lim_{b\to\infty} \ln(x+1)\Big|_0^b$$

$$= \lim_{b\to\infty} \ln(b+1)$$

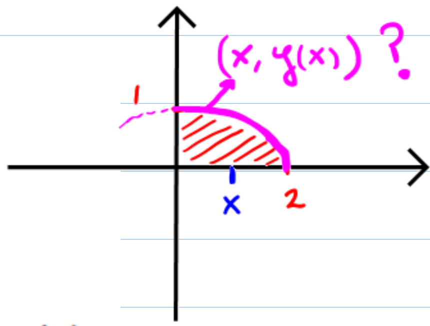
Cálculo de Áreas y perímetro.

2. Dada la elipse

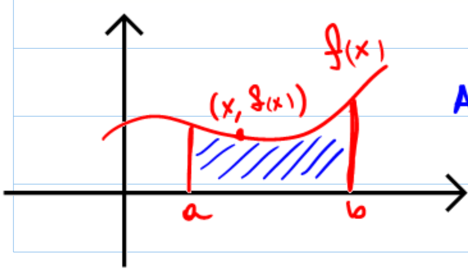
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

considere la sección que está en el primer cuadrante. Ahora:

$a=2 \quad b=1$



a) Calcule su área
b) Calcule su perímetro
c) Calcule el volumen del cuerpo generado al rotar la sección en torno al eje Y
d) Calcule el volumen del cuerpo generado al rotar la sección en torno al eje X



$$A = \int_a^b f(x)\,dx$$

$$\rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2\left(1 - \frac{x^2}{a^2}\right) \quad \sqrt{\ }$$

$$|y| = b\sqrt{1 - \frac{x^2}{a^2}}$$

$x < a$

$\frac{x}{a} < 1$

$$y = b\sqrt{1 - \frac{x^2}{a^2}}$$

$\cos^2 = 1 - \text{sen}^2$

$\text{sen}^2(x) = 1 - \cos^2(x)$

a) Area: $A = \int_0^a b\sqrt{1 - \frac{x^2}{a^2}}\, dx$

$x = a \cdot \text{sen}(\alpha)$ $\quad x = 0,\ \alpha = 0$

$dx = a\cos(\alpha)\, d\alpha$ $\quad x = a,\ \alpha = \frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}} b \underbrace{\sqrt{1 - \text{sen}^2(\alpha)}}_{\cos(\alpha)}\ a\cos(\alpha)\, d\alpha$$

$$= \int_0^{\frac{\pi}{2}} a \cdot b\ \cos^2(\alpha)\, d\alpha$$

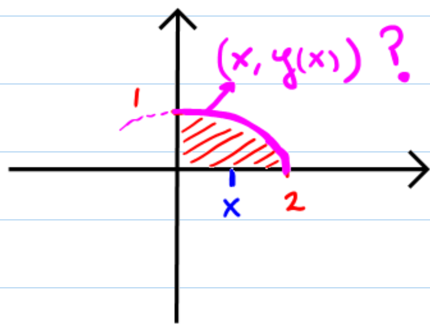
$\cos(2\alpha) = 2\cos^2(\alpha) - 1$

$\cos^2(\alpha) = \frac{1 + \cos(2\alpha)}{2}$

$$= ab\left[\frac{\alpha}{2} + \frac{\text{sen}(2\alpha)}{4}\right]_0^{\frac{\pi}{2}}$$

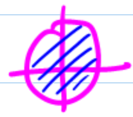
$$= ab\left[\frac{\pi}{4} + \cancel{\frac{\text{sen}\,\pi}{4}} - \cancel{\frac{\text{sen}(0)}{4}}\right]$$

$$= a \cdot b \cdot \frac{\pi}{4}$$



Área completa: πab

$a = b = r$; πr^2



Long. de Arco: $\int_a^b \sqrt{1 + (y')^2}\, dx$