Discrete Photonics in Waveguide Arrays

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http://fisica.ciencias.uchile.cl/nonopt/NLOG.html
http://www.cefop.cl//

Why study physics of discrete systems?

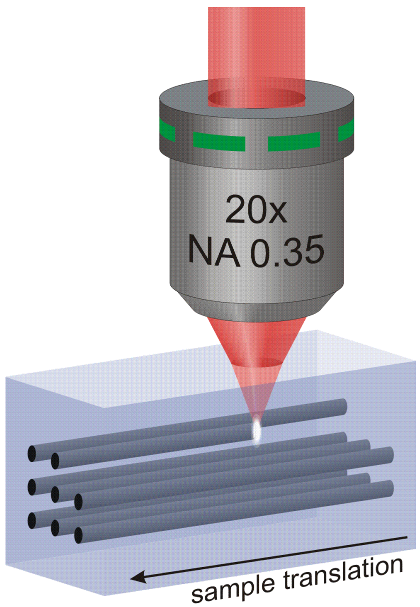
Testbed to test general phenomenology

Richer physics than continuous counterpart

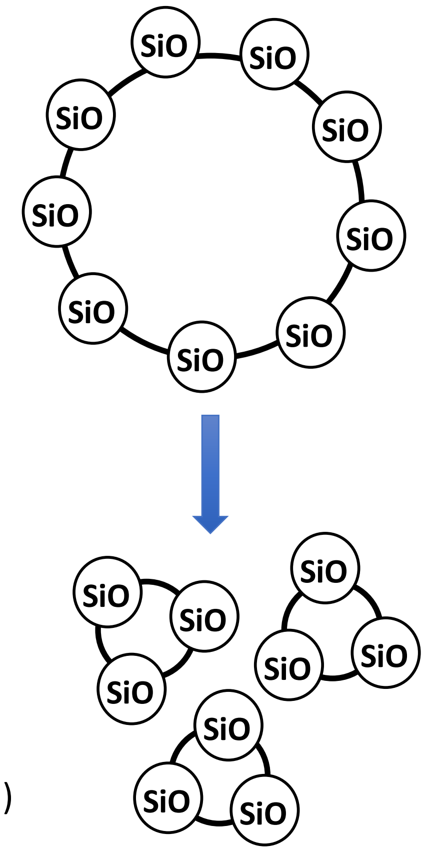
Greater potential for applications

Waveguides in fused silica

A. Szameit et al, Opt. Express 13,10552 (2005).

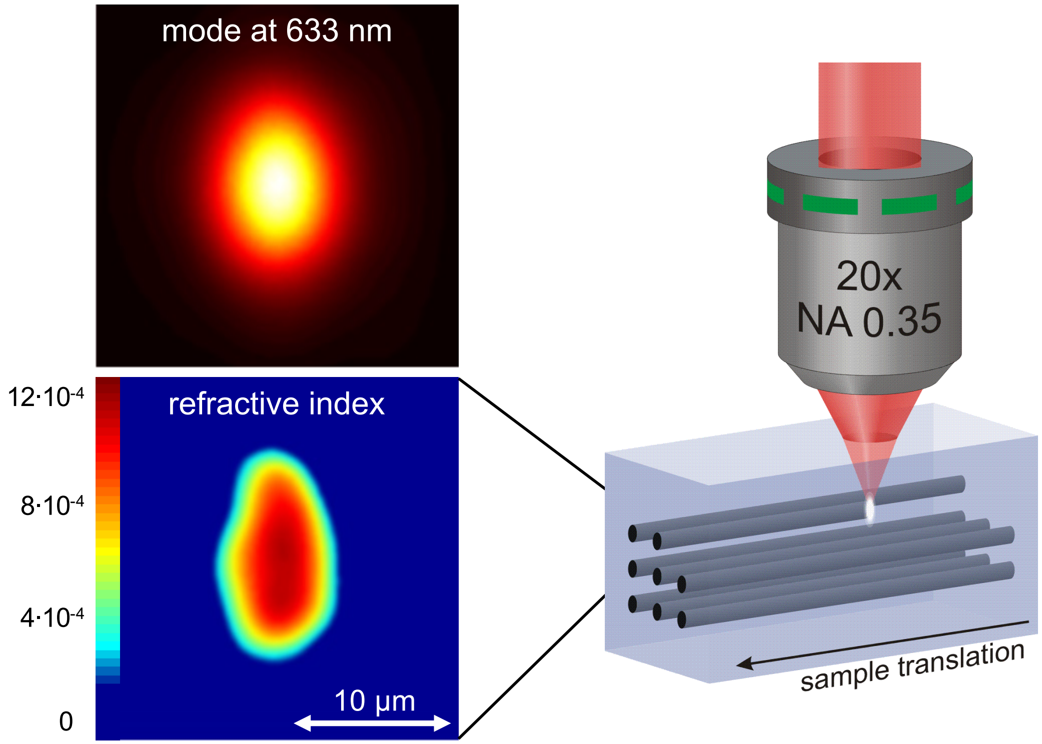


9-ring structure



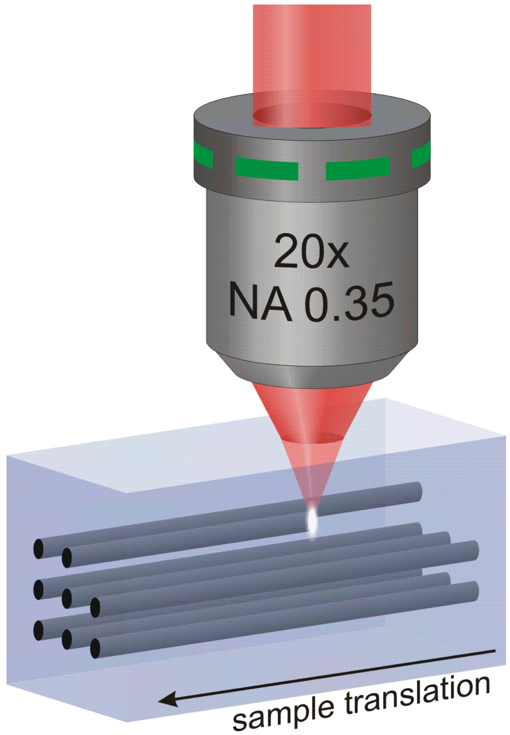
3-ring structures
(densification,
refractive index increase)

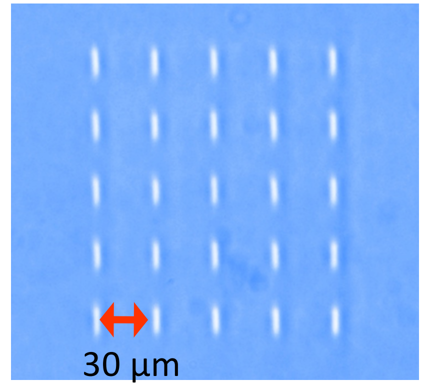
Waveguides in fused silica

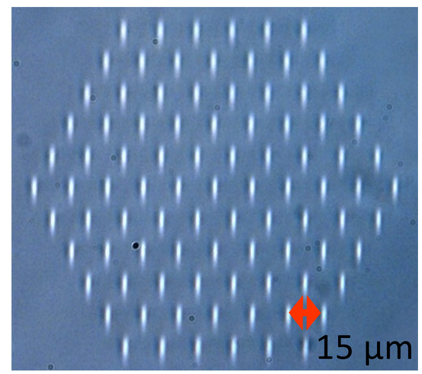


typical Δn = 10^{-3}

Waveguides in fused silica



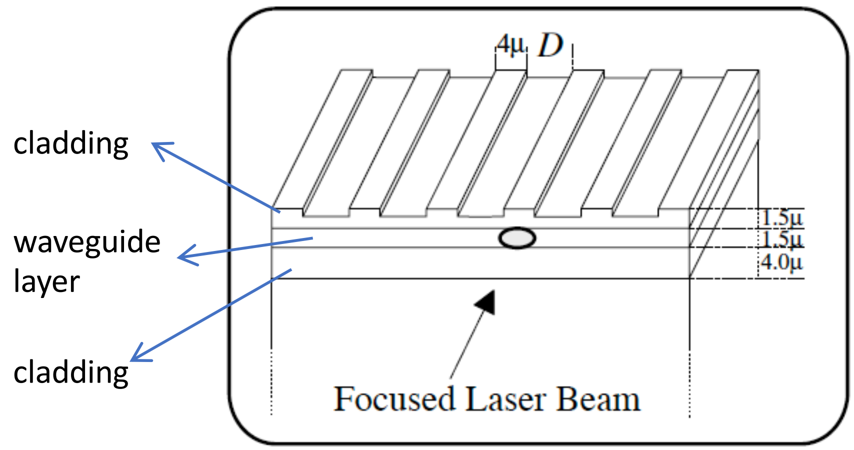




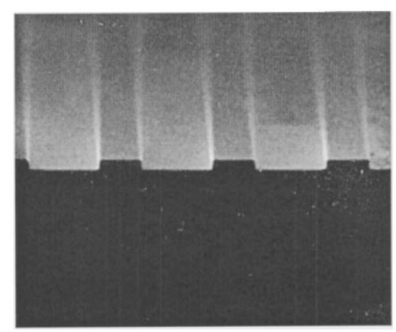
AS et al., Opt. Lett. **33**, 663 (2008).
AS et al., Appl. Phys. B **82**, 507 (2006).

Semiconductor Waveguides

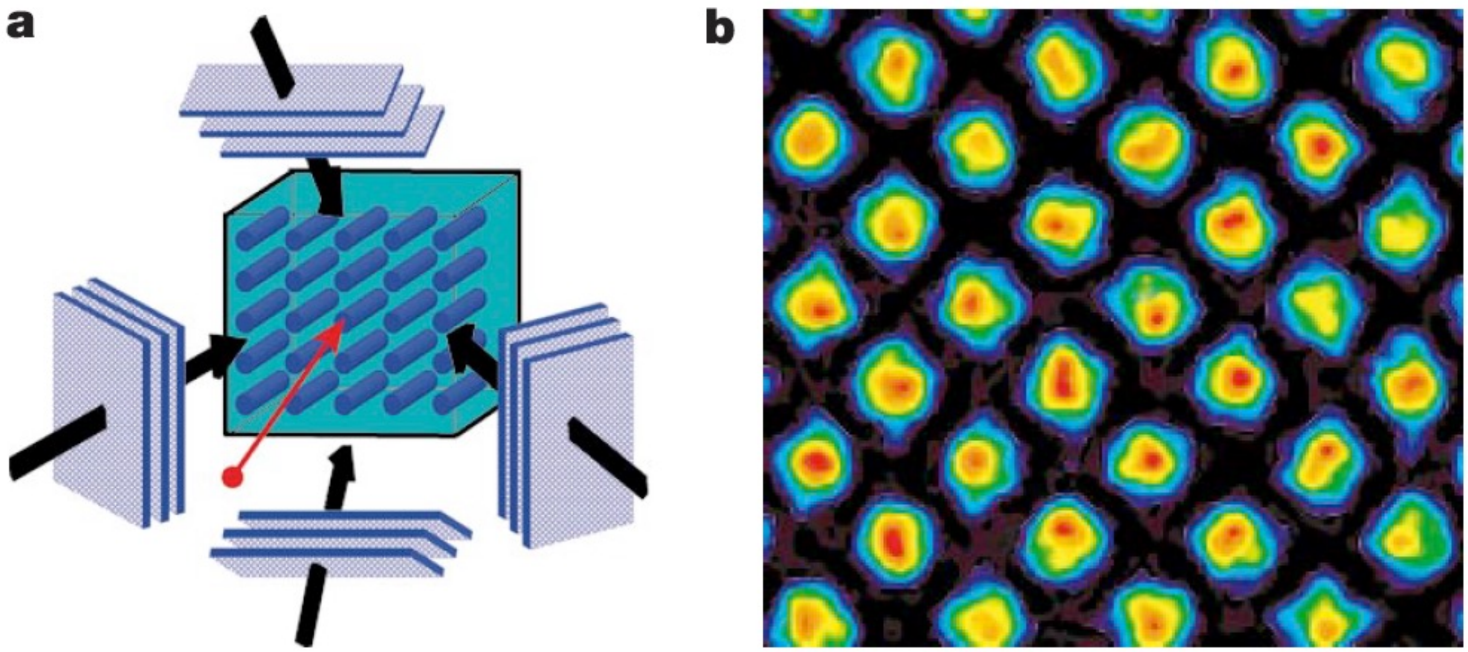
P. Millar, J.S. Aitchson,J.U. Kang, G.I. Stegeman, J. Opt. Soc. Am. B 14, 3224 (1997).



Substrate: Ga As
Cladding: $Al_{0.24}Ga_{0.76}As$
Waveguide layer:$Al_{0.18}Ga_{0.82}As$



Photorefractive Waveguides



Light→releases electrons→drift→local E fields→electro-optic effect→distribution of refractive indices

Coupled-modes theory

Maxwell:

$$\vec{\nabla} \cdot \vec{D} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

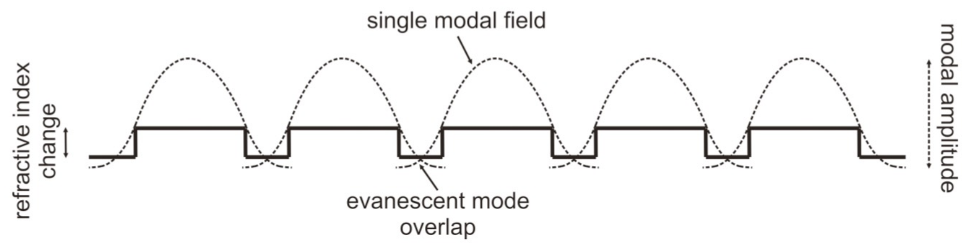
$$\vec{\nabla} \times \vec{H} = \frac{\partial}{\partial t}\vec{D} \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\vec{D} = \vec{E} + \vec{P} \qquad \vec{H} = \vec{B} + \vec{M}$$

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2 \vec{P}}{\partial t^2}$$

$$\vec{P} = \chi^{(1)}\vec{E} + \chi^{(3)}|\vec{E}|^2\vec{E}$$

Teoría de modos acoplados



$$E(x,z) = \sum_{n=-\infty}^{\infty} C_n(z)\phi(x - x_n)$$

$$\left|\frac{d^2 C_n}{dz^2}\right| \ll k_0 \left|\frac{dC_n}{dz}\right| \qquad n = n_0 + n_2|E|^2$$

Kerr

$$i\frac{dC_n}{dz} + V(C_{n+1} + C_{n-1}) + \gamma|C_n|^2 C_n = 0$$

Discrete nonlinear Schrodinger (DNLS) equation

Coupled-modes theory

$$P = \sum_n |C_n|^2$$

$$H = \sum_n \{V(C_n C^*_{n+1} + C^*_n C_{n+1}) + (\gamma/2)|C_n|^4\}$$

Conserved quantities

$$C_n = u_n \exp(i\beta z)$$

Stationary mode

$$-\beta u_n + (u_{n+1} + u_{n-1}) + \chi |u_n|^2 u_n = 0$$

Nonlinear eigenvalue equation

Coupled-modes theory

Finding the localized nonlinear mode

$$-EC_n + V(C_{n+1} + C_{n-1}) + \chi|C_n|^2 C_n = 0$$

$$\lambda \equiv E/V, \qquad \phi_n \equiv \sqrt{\chi/V}\, C_n$$

$$-\lambda\phi_n + (\phi_{n+1} + \phi_{n-1}) + |\phi_n|^2 \phi_n = 0$$

$$\vec{F}(\vec{\phi}) = 0 \quad \text{use Newton-Raphson}$$

Need good seed (anticontinuous limit)
Find many solution families (characterized by power
vs prop.const. curve

Coupled-modes theory

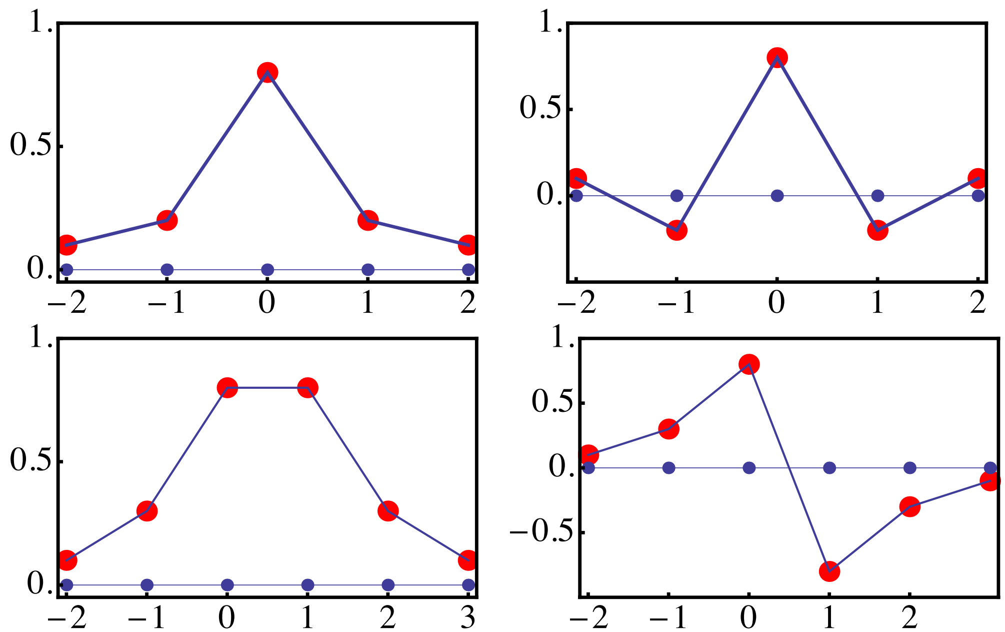
Pure 1D CASE

odd unstaggered

odd staggered

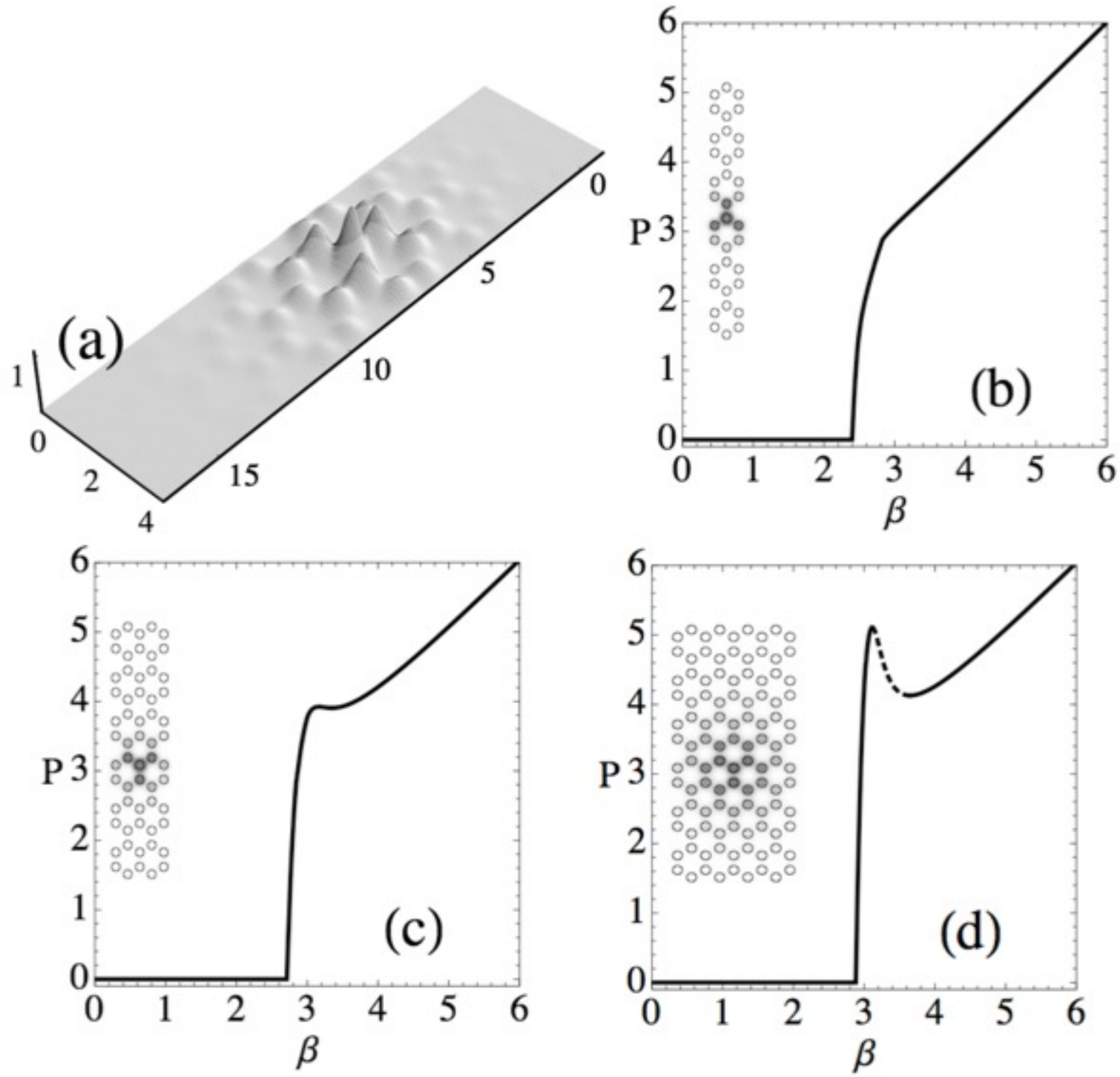
even

twisted



$$E(x,z) = \sum_{n=-\infty}^{\infty} \sqrt{V/\chi}\ \phi_n(z)\ \psi(x - x_n)$$

Coupled-modes theory



Example: Graphene ribbon

Coupled-modes theory

Linear stability

$$C_n(z) = \phi_n e^{-i\lambda z} \qquad \text{sol. of DNLS}$$

$$C_n(z) \rightarrow (\phi_n + \delta\phi_n) e^{-i\lambda z}, \qquad |\delta\phi_n / \phi_n| \ll 1$$

$$\Longrightarrow \quad \text{Equation for} \;\; \delta\phi_n = \delta u_n + i\delta v_n$$

$$\text{define } \delta\vec{u} = (\delta u_1, \delta u_2, ..., \delta u_N), \qquad \delta\vec{v} = (\delta v_1., \delta v_2, ..., \delta v_N)$$

$$\mathcal{A}_{nm} = \delta_{n,m+1} + \delta_{n,m-1} + (\lambda + \phi_n^2)\delta_{n,m}$$

$$\mathcal{B}_{nm} = \delta_{n,m+1} + \delta_{n,m-1} + (\lambda + 3\phi_n^2)\delta_{n,m}$$

$$\boxed{\delta\ddot{\vec{U}} + \mathcal{B}\mathcal{A}\; \delta\vec{U} = 0 \;\; \text{and} \;\; \delta\ddot{\vec{V}} + \mathcal{A}\mathcal{B}\; \delta\vec{V} = 0}$$

Coupled-modes theory

$$\{\mathrm{m}\}=\text{eigenvalues of } \mathcal{A}\mathcal{B} = \text{eigenvalues of } \mathcal{B}\mathcal{A}$$

instability gain

$$G^* = \mathrm{Max}\left\{\sqrt{(1/2)(-\mathrm{Re}[\mathrm{m}] + \sqrt{\mathrm{Re}[\mathrm{m}]^2 + \mathrm{Im}[\mathrm{m}]^2})}\right\}$$

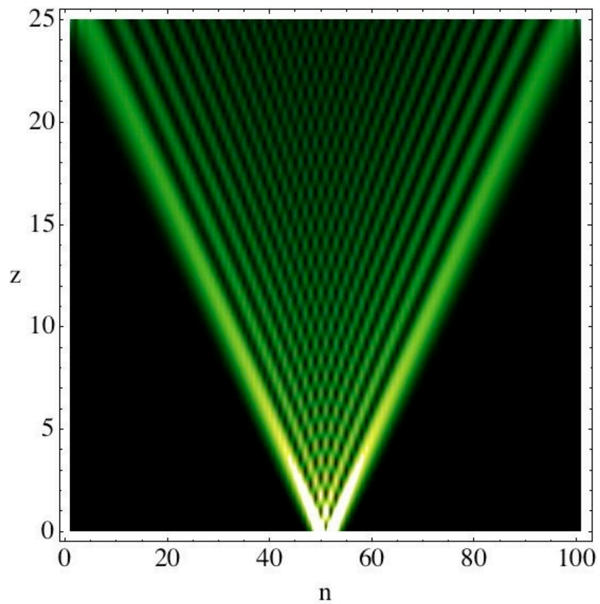
$$G^* = 0 \quad \text{stable}$$

$$G^* > 0 \quad \text{unstable}$$

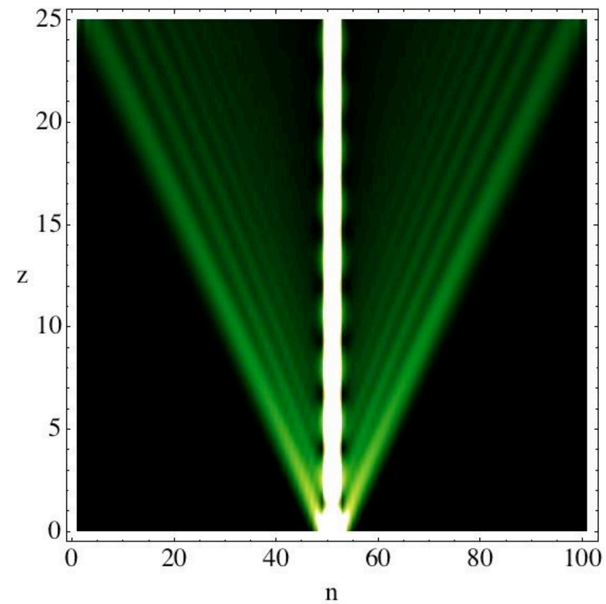
Coupled-modes theory

Numerical propagation

Discrete diffraction

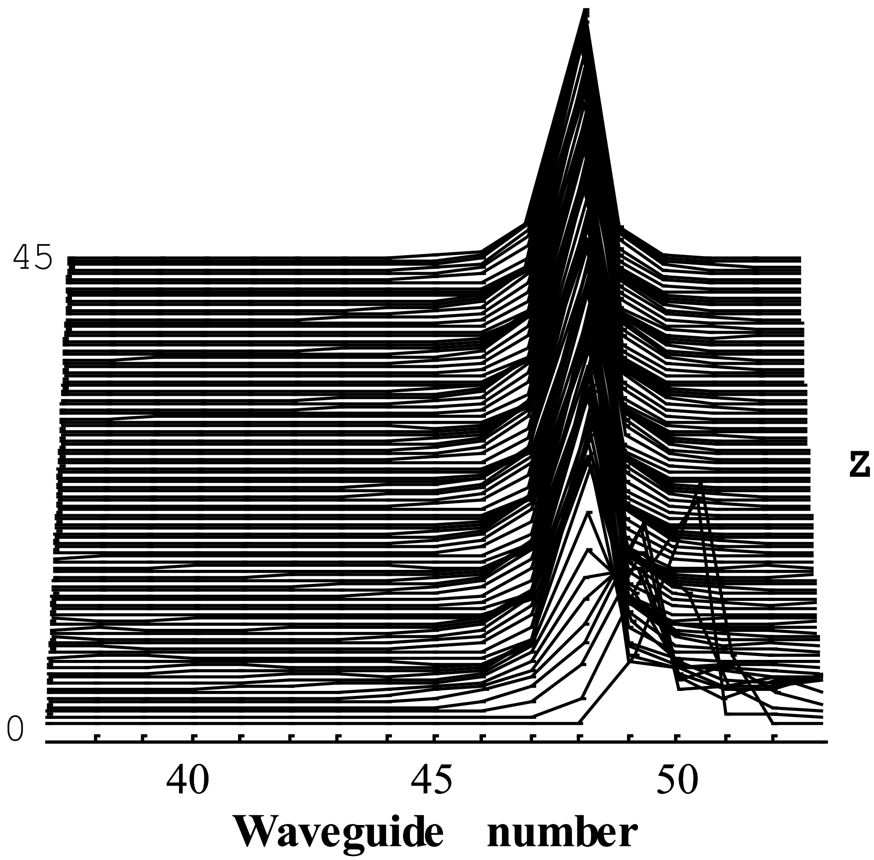


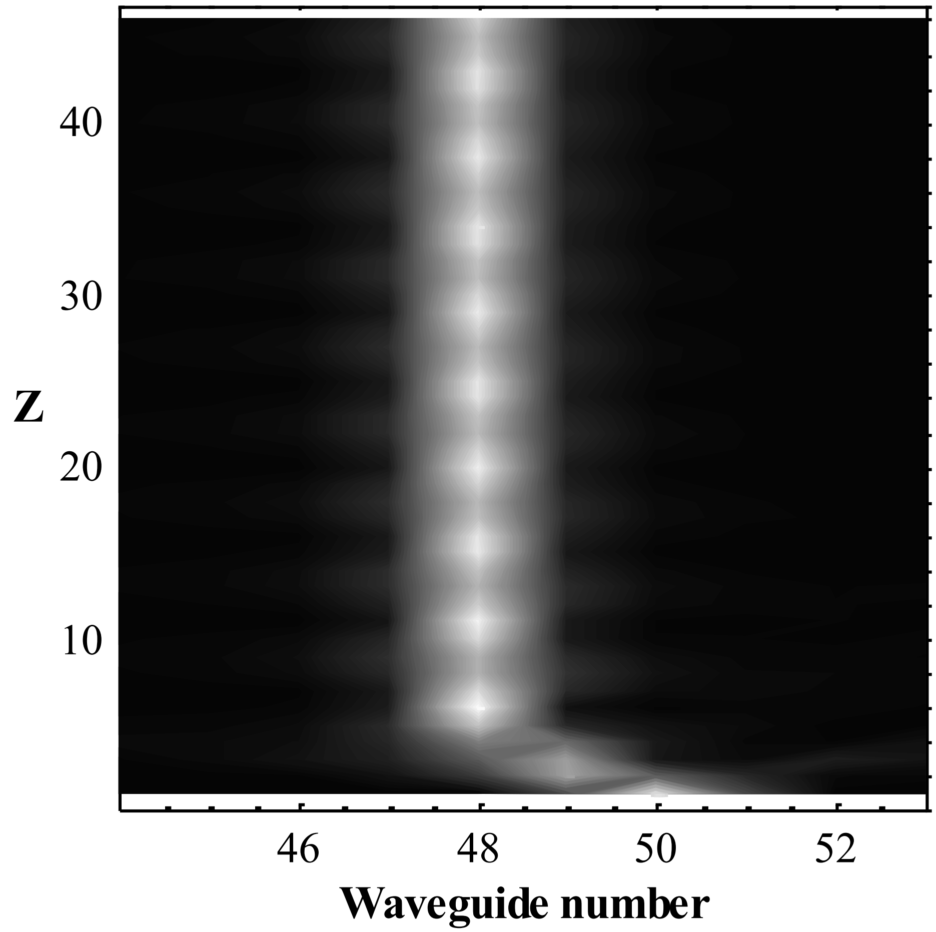
Discrete soliton formation



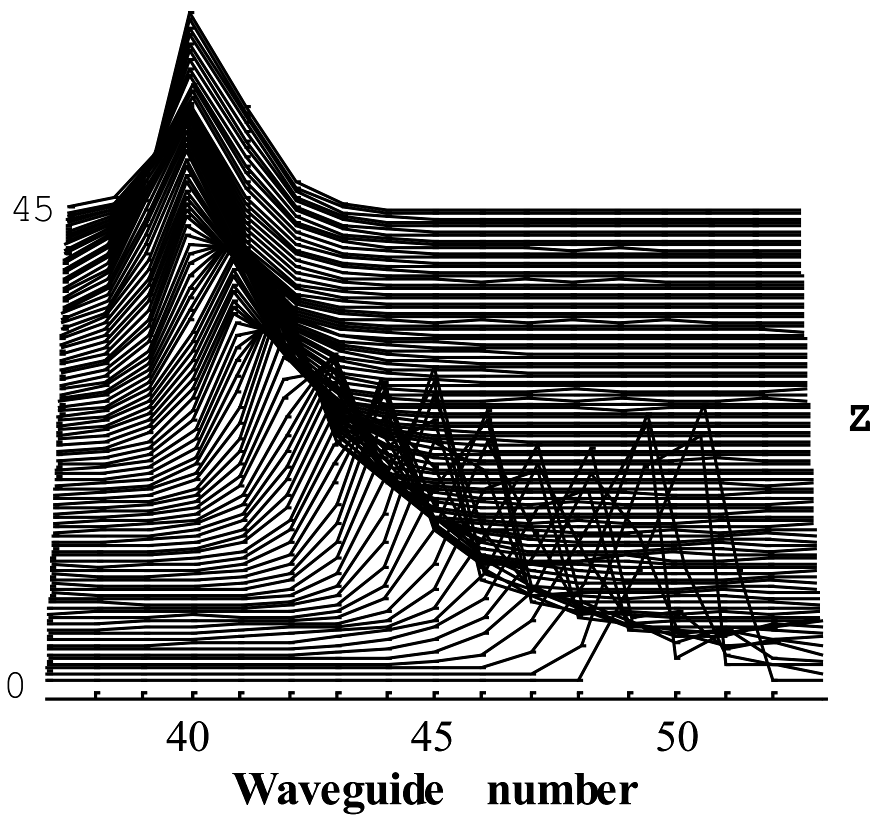
Arreglos cúbicos: Switching a 2 guías

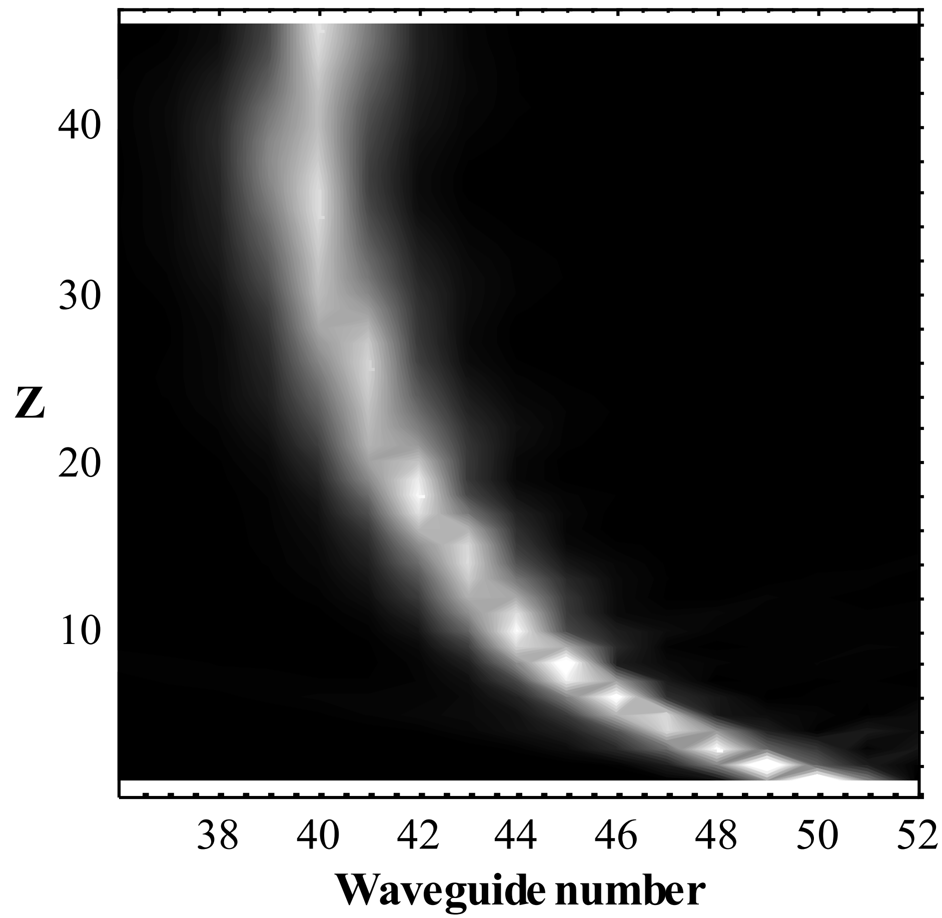
$$u_n(0) = \Phi_n e^{-ik(n-n_c)}$$



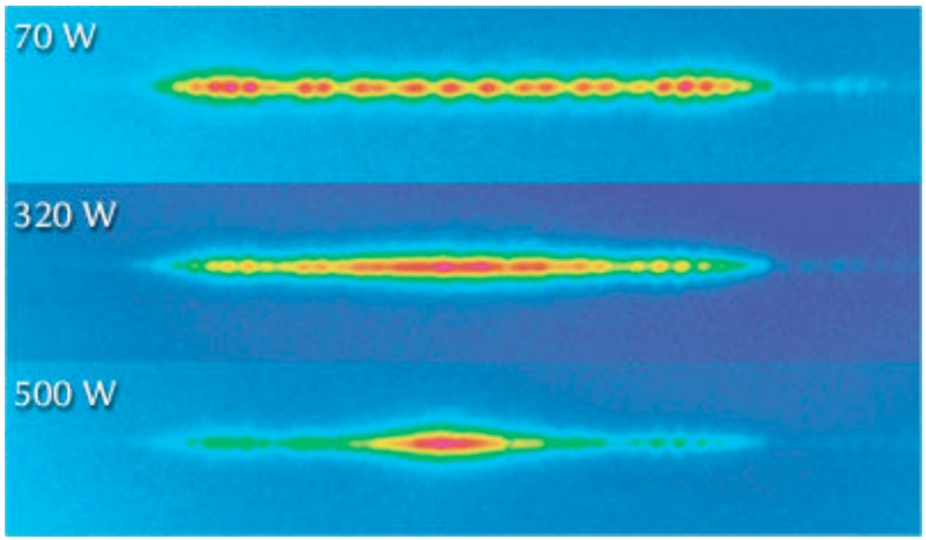


Arreglos cúbicos: Switching a 10 guías





First experimental observation of discrete soliton

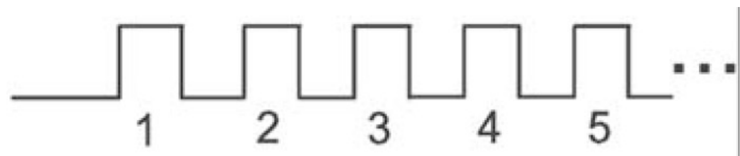


Difraccion discreta

Soliton discreto

H. Eisenberg at al, PRL 81, 3383 (1998).

Surface Solitons



$$i\frac{dE_1}{dz} + \alpha E_1 + E_2 + \gamma |E_1|^2 E_1 = 0,$$

$$i\frac{dE_n}{dz} + \alpha E_n + (E_{n+1} + E_{n-1}) + \gamma |E_n|^2 E_n = 0$$

Stationary mode: $E_n(z) = \exp(i\beta z) E_n$

γ=0 : $E_n \sim \sin(kn) \quad \beta = \alpha + 2\cos(k) \quad k = \frac{m\pi}{N+1}$

NO SURFACE MODE

Surface Solitons

$\gamma \neq 0$

Use Newton-Raphson + judicious initial condition (antiadiabatic limit

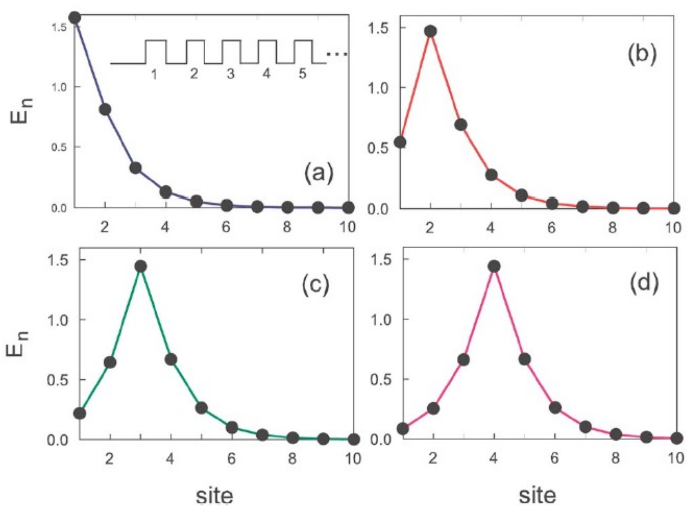


Fig. 1. (Color online) Examples of surface localized modes at $\beta=3$ in an array of focusing waveguides ($\gamma=+1$) centered at distances d of (a) 0, (b) 1, (c) 2, (d) 3 from the array edge.

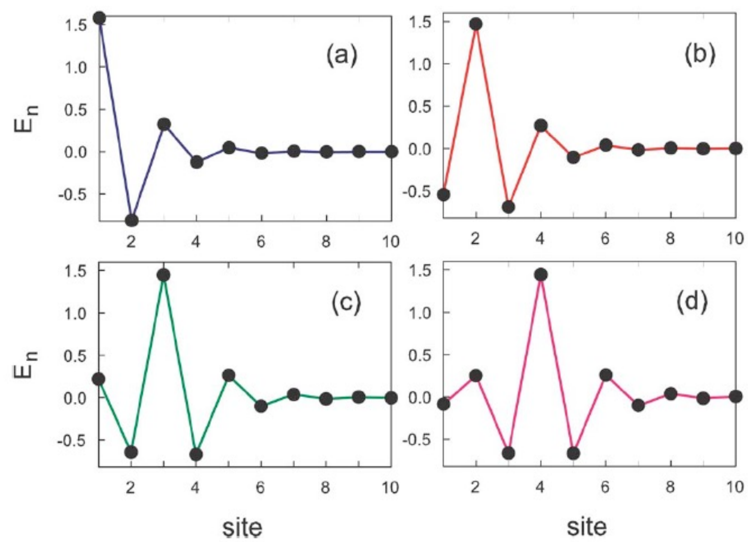


Fig. 2. (Color online) Examples of localized surface modes at $\beta=-3$ in an array of defocusing waveguides ($\gamma=-1$) located at distances d of (a) 0, (b) 1, (c) 2, (d) 3 from the array edge.

Surface Solitons

$$\text{Power} = \sum_n |E_n|^2$$

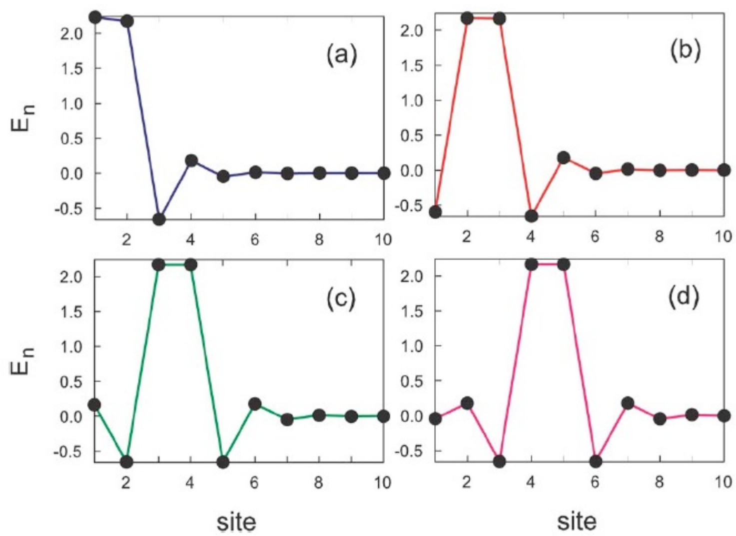


Fig. 5. (Color online) Examples of stable flat-topped localized surface modes at $\beta = -4$ in the array of defocusing waveguides ($\gamma = -1$) centered between various sites near the edge.

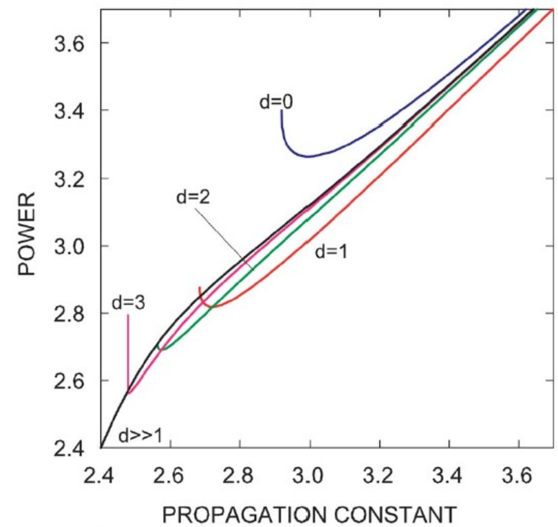


Fig. 3. (Color online) Normalized power versus propagation constant β for the surface modes shown in Fig. 1 located at distances $d = 0, 1, 2, 3$ from the surface. The darkest curve corresponds to the discrete soliton in an infinite array.

Surface Solitons

Existence and stability: The constraint method

$$H = -\sum_n (E_n E^*_{n+1} + E^*_n E_{n+1}) - (1/2)\sum_n |E_n|^4$$

$$X = \sum_n n|E_n|^2 / \sum_n |E_n|^2$$

$E_n \neq 2$

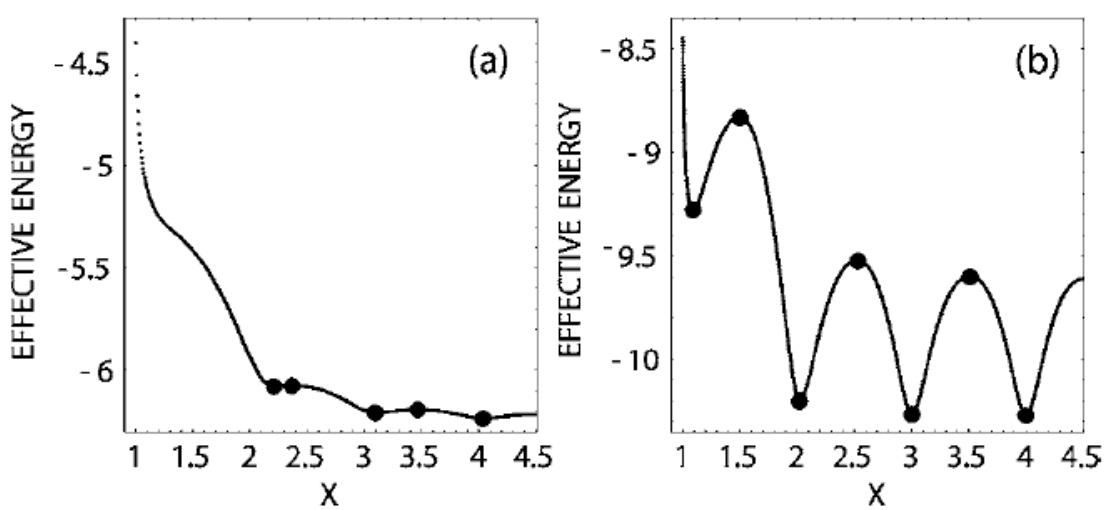
(1) Compute an odd mode centered at n. Obtain all $\{E_n\}$ and power P
(2) Fix amplitude at n+1 to be $E_{n+1} + \epsilon$
(3) Solve all NR equations for E_m $(m \neq n+1)$ keeping power fixed at P, arriving to intermediate state centered between n and $n+1$.
(4) Obtain X and H for intermediate state.
(5) increase ϵ and repeat procedure until amplitudes at sites n and $n+1$ coincide (even mode).

$$U_{\text{eff}} = H(X)$$

$$dH/dX = 0$$ Stationary solutions

Surface Solitons

$P < P_c$ $P > P_c$



$P < P_c$: surface is repulsive

even modes unstable

Surface solitons

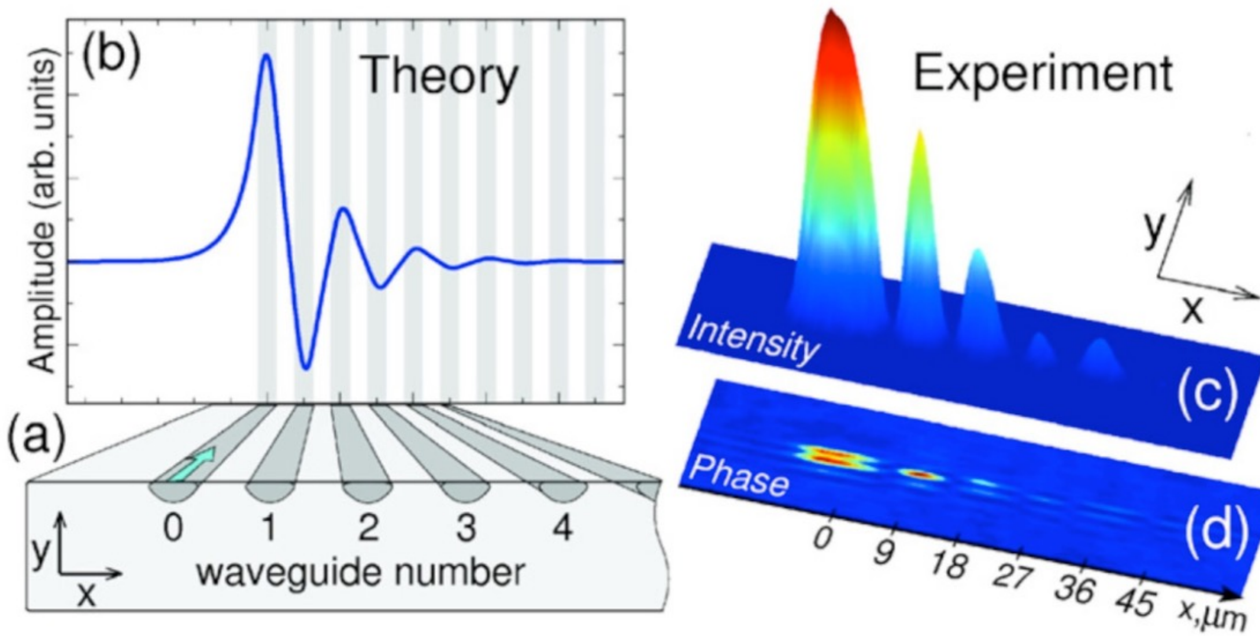


Fig.2: Theoretical prediction (a, b) and experimental observation (c, d) of nonlinear Tamm states in a truncated photonic lattice. (a) Schematic of the waveguide array geometry; (b) theoretical profile of a nonlinear Tamm statea surface gap soliton. (c) three-dimensional representation of the nonlinear surface state observed above the localization threshold. (x,y) are the horizontal and vertical sample coordinates, respectively. (d) Experimental plane–wave interferogram demonstrating the staggered phase structure of the nonlinear Tamm state (from M. I. Molina and Y. S. Kivshar, ref [14]).