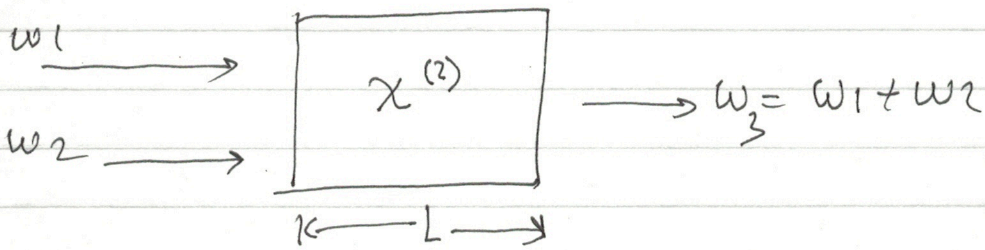
Ec. de onda acoplados para la generación de suma de frecuencias



En ausencia de nolinealidad (bajas intensidades)

$$\vec{E}_3(z,t) = A_3 e^{i(k_3 z - \omega_3 t)} + c.c. \qquad (1)$$

$$k_3 = \frac{n_3 \omega_3}{c}, \qquad n_3 = \sqrt{\epsilon^{(1)}(\omega_3)} \qquad (2)$$

Cuando se aumenta la intensidad, $A_3 \to A_3(z)$ lentamente variable

$$P_{NL}(z,t) = P_3 e^{-i\omega_3 t} + c.c. \qquad (3)$$

donde $P_3 = 4d E_1 E_2 \quad \left(= 2\chi^{(2)} E_1 E_2\right)$ (4)

$$E_i(z,t) = E_i e^{-i\omega_i t} + c.c. \quad (i = 1,2)$$

con $E_i = A_i e^{i k_i z}$ (5)

$$\Rightarrow P_3 = 4d A_1 A_2 e^{i(k_1 + k_2) z} \qquad (6)$$

se sust. todo esto en la ec. de onda:

$$\left(\frac{\partial^2 A_3}{\partial z^2} + 2ik_3\frac{\partial A_3}{\partial z} \overbrace{- \cancel{k_3^2 A_3} + \cancel{\frac{\epsilon^{(1)}(\omega_3)\,\omega_3^2 A_3}{c^2}}}^{\text{se cancelan mutua'/.}}\right) e^{i(k_3 z-\omega_3 t)} + c.c$$

$$= -\frac{16\pi d\omega_3^2}{c^2}A_1A_2\, e^{i((k_1+k_2)z-\omega_3 t)} + c.c.$$

$$\Rightarrow \frac{d^2A_3}{dz^2} + 2ik_3\frac{dA_3}{dz} = -\frac{16\pi d\omega_3^2}{c^2}A_1A_2\, e^{i(k_1+k_2-k_3)z}$$

Ahora suponemos $\left|\frac{d^2A_3}{dz^2}\right| \ll \left|k_3\frac{dA_3}{dz}\right|$

(Aprox. de amplitud lenta'/. variable)

$$\boxed{\frac{dA_3}{dz} = \frac{8\pi i d\omega_3^2}{k_3c^2}A_1A_2\, e^{i(\Delta k)z}} \qquad (7)$$

donde $\Delta k \equiv k_1+k_2-k_3$ (parámetro de mismatch) (8)

Dentro del medio no-lineal, también varían A_1 y A_2.
Repitiendo el procedimiento anterior para ω_1, ω_2:

$$\boxed{\frac{dA_1}{dz} = \frac{8\pi i d\omega_1^2}{k_1c^2}A_3A_2^*\, e^{-i\Delta k z}} \qquad (9)$$

$$\boxed{\frac{dA_2}{dz} = \frac{8\pi i d\omega_2^2}{k_2c^2}A_3A_1^*\, e^{-i\Delta k z}} \qquad (10)$$

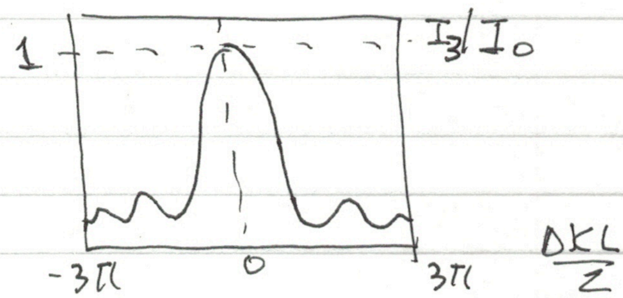
Tarea: obtener el set completo (7), (9), (10) a partir de las ec. de onda y la aprox. de amplitud lenta'/. variable.

Caso I: $A_1(z)\,,\ A_2(z) \simeq$ cte.

$$A_3(L) = \frac{8\pi i \omega_3^2 d A_1 A_2}{k_3 c^2}\int_0^L e^{i\Delta k z}\, dz = \frac{8\pi i d \omega_3^2 A_1 A_2}{k_3 c^2}\left(\frac{e^{i\Delta k L}-1}{i\Delta k}\right)$$

Intensidad: (a la salida) $\quad I_3 = \frac{n_3 c}{2\pi}|A_3|^2 = \frac{32\pi d^2 \omega_3^4 |A_1|^2 |A_2|^2 n_3}{k_3^2 c^3}\left|\frac{e^{i\Delta k L}-1}{\Delta k}\right|^2$

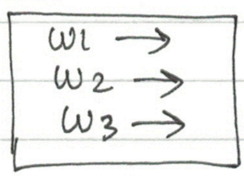
$$\downarrow \quad I_3 = I_0\left[\frac{\sin\left(\frac{\Delta k L}{2}\right)}{\left(\frac{\Delta k L}{2}\right)}\right]^2$$



$\frac{I_3(L)}{I_0}$ es máximo para $\Delta k = 0$ ("phase-matched")

Relaciones de Manley-Rowe

$$I_i = \frac{n_i c}{2\pi} A_i A_i^*$$



$$\frac{dI_i}{dz} = \frac{n_i c}{2\pi}\left[A_i^* \frac{dA_i}{dz} + A_i \frac{dA_i^*}{dz}\right]$$

$$\Rightarrow \frac{dI_1}{dz} = 4 d\omega_1 \left(i A_3 A_1^* A_2^* e^{-i\Delta k z} + c.c.\right) = -8 d\omega_1 \,\mathrm{Im}\left[A_3 A_1^* A_2^* e^{-i\Delta k z}\right.$$

$$\frac{dI_2}{dz} = -8 d\omega_2 \,\mathrm{Im}\left[A_3 A_1^* A_2^* e^{-i\Delta k z}\right]$$

y $\quad \frac{dI_3}{dz} = -8 d\omega_3 \, \mathrm{Im}\left[A_3^* A_1 A_2 e^{i\Delta k z}\right] = 8 d\omega_3 \, \mathrm{Im}\left[A_3 A_1^* A_2^* \times e^{-i\Delta k z}\right]$

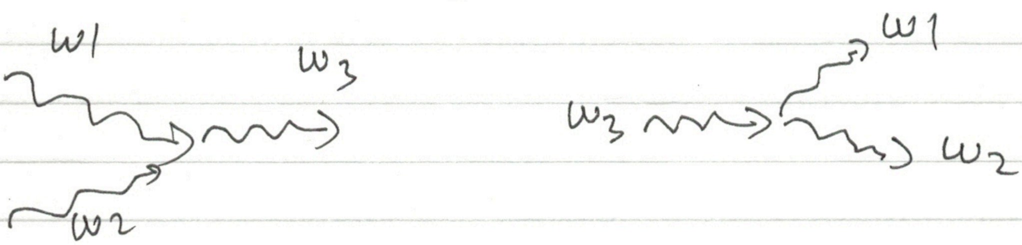
$$\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right) = \frac{d}{dz}\left(\frac{I_2}{\omega_2}\right) = -\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)$$

MC: $\frac{I_i}{\omega_i} = \frac{\#\ \text{fotones}}{\text{u. de área} \cdot \text{u. det.}}$

$\Rightarrow$ tasa de creación (destrucción) de fotones a frec. ω_1

= tiene " " " " " " ω_2

= tiene destrucción (creación) " " " ω_3



Tarea 2, Demostrar que la intensidad total $I_1 + I_2 + I_3$ es conservada.

$$\frac{dI_1}{dz} + \frac{dI_2}{dz} + \frac{dI_3}{dz} = -8d \, \mathrm{Im}\left\{A_3 A_1^* A_2^* e^{-i\Delta k z}\right\}\left\{\underbrace{\omega_1 + \omega_2 - \omega_3}_{0}\right\} = 0$$

Case II :

campo E_2 fuerte, $\boxed{A_2(z) \simeq cte}$ (PUMP) (1)
E_1 débil

$$\Rightarrow \frac{dA_1}{dz} = K_1 A_3 e^{-i\Delta k z} \qquad (2)$$

$$\frac{dA_3}{dz} = K_3 A_1 e^{i\Delta k z} \qquad (3)$$

donde $K_1 \equiv \dfrac{8\pi i \omega_1^2 d A_2^*}{k_1 c^2}$; $K_3 = \dfrac{8\pi i \omega_3^2 d A_2}{k_3 c^2}$ (4)

$\Delta k \equiv k_1 + k_2 - k_3$ parámetro de mismatch. (5)

(a) $\Delta k = 0$: $\dfrac{dA_1}{dz} = K_1 A_3$, $\dfrac{dA_3}{dz} = K_3 A_1$

$$\Rightarrow \frac{d^2A_1}{dz^2} = K_1 \frac{dA_3}{dz} = K_1 K_3 A_1 = -K^2 A_1 \qquad (6)$$

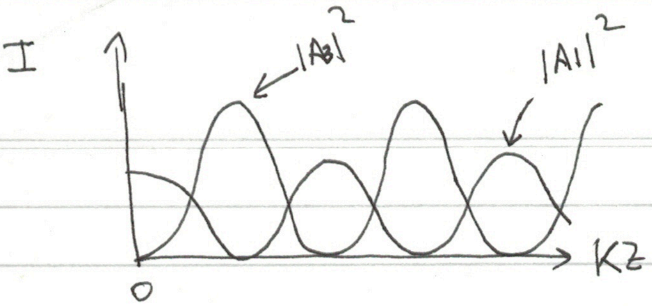
donde $K^2 = \dfrac{64\pi^2 \omega_1^2 \omega_3^2 d^2 |A_2|^2}{k_1 k_3 c^4}$ (6')

luego $A_1(z) = B\cos(Kz) + C\sin(Kz)$

y $A_3(z) = \dfrac{1}{K_1}\dfrac{dA_1}{dz} = -\dfrac{BK}{K_1}\sin(Kz) + \dfrac{CK}{K_1}\cos(Kz)$

si $A_3(0) = 0 \wedge A_1(0) \neq 0$,

$$\Rightarrow \boxed{\begin{aligned} A_1(z) &= A_1(0)\cos(Kz) \\ A_3(z) &= -A_1(0)\frac{K}{K_1}\sin(Kz) \end{aligned}} \qquad (7)$$



$\left(\begin{array}{l} A_2 \simeq cte, \\ \Delta K = 0 \end{array} \right)$

(b) $\underline{\Delta K \neq 0}$

$$\left.\begin{array}{l} \dot{A}_1 = K_1 A_3 e^{-i\Delta K z} \\ \dot{A}_3 = K_3 A_1 e^{+i\Delta K z} \end{array}\right\} (8)$$

$$\Rightarrow \ddot{A}_1 = K_1 \dot{A}_3 e^{-i\Delta K z} + K_1 A_3 (-i\Delta K) e^{-i\Delta K z}$$

$$= K_1 \cancel{e^{-i\Delta K z}} (K_3 A_1) \cancel{e^{i\Delta K z}} - i K_1 \Delta K A_3 e^{-i\Delta K z}$$

$$= K_1 K_3 A_1 - i \frac{\cancel{K_1} \Delta K \dot{A}_1 \cancel{e^{-i\Delta K z}}}{\cancel{K_1} \cancel{e^{-i\Delta K z}}} = K_1 K_3 A_1 - i \Delta K \dot{A}_1$$

$$\therefore \frac{d^2 A_1}{dz^2} = K_1 K_3 A_1 - i \Delta K \frac{dA_1}{dz} \qquad (9)$$

$A_1 \sim e^{\lambda z}$: $\lambda^2 = K_1 K_3 - i \Delta K \lambda$

$$\lambda^2 + i\Delta K \lambda - K_1 K_3 = 0 \Rightarrow \lambda = \frac{-i\Delta K \pm \sqrt{-\Delta K^2 + 4 K_1 K_3}}{2}$$

ou seja, $\lambda = -i \frac{\Delta K}{2} \pm \frac{i}{2} \sqrt{\Delta K^2 + 4K^2}$

$$\therefore A_1(z) = e^{-i\frac{\Delta K}{2} z} \left\{ \alpha e^{i\sqrt{K^2 + (\Delta K/2)^2} z} + \beta e^{-i\sqrt{K^2 + (\Delta K/2)^2} z} \right\}$$

ou

$$A_1(z) = e^{-i\frac{\Delta K}{2} z} \left[\alpha \cos(gz) + \beta \sin(gz) \right] \qquad (10)$$

donde $g(z) \equiv \sqrt{K^2 + (\Delta K/2)^2}$

Por su parte, $A_3(z) = K_1^{-1} e^{i\Delta k z} \dot{A}_1$

$$= K_1^{-1} e^{i\Delta k z}\left\{-i\frac{\Delta k}{2} e^{-i\frac{\Delta k}{2}z}\left[\alpha\cos(gz)+\beta\sin(gz)\right] + \right.$$

$$\left. + e^{-i\frac{\Delta k}{2}z}\left[-\alpha g\sin(gz)+\beta g\cos(gz)\right]\right\}$$

$$= K_1^{-1} e^{i\frac{\Delta k}{2}z}\left\{\left(-i\frac{\Delta k}{2}\alpha+\beta g\right)\cos(gz) - \left(\alpha g - i\frac{\Delta k}{2}\beta\right)\sin(gz)\right\} \qquad (11)$$

$$A_1(0) = \alpha,$$

$$A_3(0) = K_1^{-1}\left(-i\frac{\Delta k}{2}\alpha+\beta g\right) = K_1^{-1}\left(-i\frac{\Delta k}{2}A_1(0)+\beta g\right)$$

$$A_3(0) = -\frac{i\Delta k}{2K_1}A_1(0) + \beta\frac{g}{K_1} \Rightarrow \boxed{\beta = \frac{K_1}{g}A_3(0) + i\frac{\Delta k}{2g}A_1(0)} \qquad (12)$$

$$\therefore\ A_1(z) = e^{-i\frac{\Delta k}{2}z}\left[A_1(0)\cos(gz) + \left(+i\frac{\Delta k}{2g}A_1(0) + \frac{K_1}{g}A_3(0)\right)\times\sin(gz)\right]$$

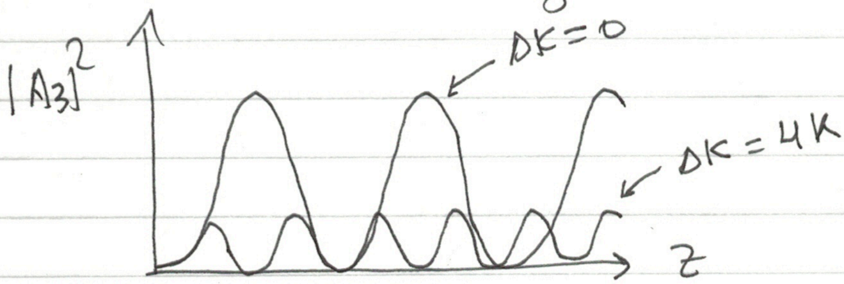
$$A_3(z) = e^{+i\frac{\Delta k}{2}z}\left[A_3(0)\cos(gz) + \left(-i\frac{\Delta k}{2g}A_3(0) + \frac{K_3}{g}A_1(0)\right)\sin(gz)\right] \qquad (13)$$

<u>Caso especial</u>: $A_3(0)=0$

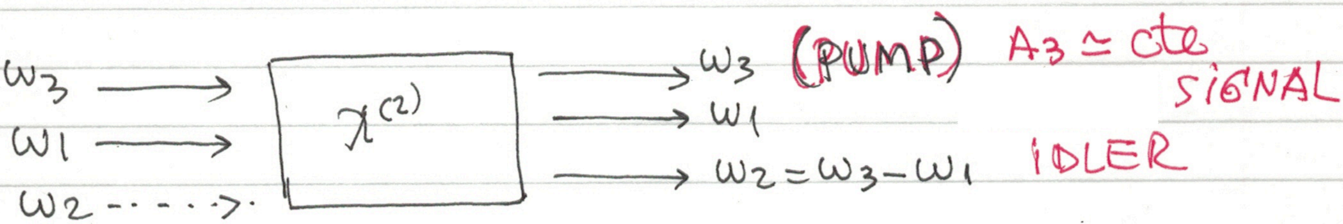
$$\Rightarrow A_1(z) = e^{-i\frac{\Delta k}{2}z}\left[A_1(0)\cos(gz) + i\frac{\Delta k}{2g}A_1(0)\sin(gz)\right]$$

$$A_3(z) = e^{+i\frac{\Delta k}{2}z}\frac{K_3}{g}A_1(0)\sin(gz) \qquad (14)$$

$$\Rightarrow \quad I_3 = |A_3(z)|^2 = \frac{|A_1(0)|^2 |K_3|^2}{g^2} \sin^2(gz) \qquad \left(g = \sqrt{K^2 + (\Delta K/2)^2}\right)$$



Diferencia de frecuencia, amplificación paramétrica



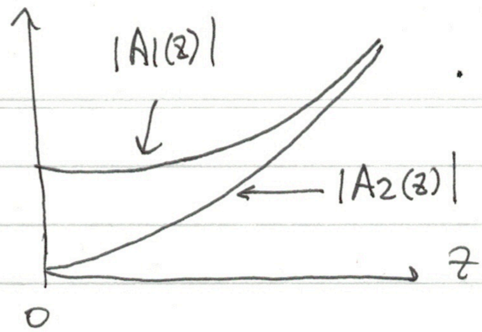
$$\left.\begin{aligned} \frac{dA_1}{dz} &= \frac{8\pi i \omega_1^2 d}{k_1 c^2} A_3 A_2^* e^{i\Delta k z} \\ \frac{dA_2}{dz} &= \frac{8\pi i \omega_2^2 d}{k_2 c^2} A_3 A_1^* e^{i\Delta k z} \end{aligned}\right\} \quad (1)$$

(a) $\underline{\Delta K = 0}$:
$$\frac{d^2 A_2}{dz^2} = \frac{64\pi^2 \omega_1^2 \omega_2^2 d^2}{k_1 k_2 c^4} A_3 A_3^* A_2 \equiv K^2 A_2 \qquad (2)$$

$$\Rightarrow \quad A_2(z) = C\sinh(Kz) + D\cosh(Kz) \qquad (3)$$

Si $A_2(0) = 0$, $A_1(0)$ = arbitrario

$$\Rightarrow \quad \left.\begin{aligned} A_1(z) &= A_1(0)\cosh(Kz) \\ A_2(z) &= i\,\frac{8\pi \omega_2^2 d |A_3|^2}{A_3} A_1(0) \sinh(Kz) \end{aligned}\right\} \quad (4)$$



(b) $\Delta k \neq 0$ (Demostrar como Tarea)

$$A_1(z) = \left[A_1(0)\left(\cosh(gz) - \frac{i\Delta k}{2g}\sinh(gz)\right) + \frac{\kappa_1}{g} A_2^*(0)\sinh(gz) \right] e^{i\frac{\Delta k}{2}z}$$

$$A_2(z) = \left[A_2(0)\left(\cosh(gz) - \frac{i\Delta k}{2g}\sinh(gz)\right) + \frac{\kappa_2}{g} A_1^*(0)\sinh(gz) \right] e^{i\frac{\Delta k}{2}z}$$

donde $g \equiv \sqrt{\kappa_1 \kappa_2^* - (\Delta k/2)^2}$, $\kappa_j \equiv \dfrac{8\pi i \omega_j^2 d A_3}{k_j c^2}$

Generación de $2^{\underline{da}}$ armónica

$$\omega_1 \longrightarrow \boxed{\chi^{(2)}} \longrightarrow \omega_2 = 2\omega_1$$

$$E(z,t) = E_1(z,t) + E_2(z,t) \tag{1}$$

$$E_j(z,t) = E_j(z)\, e^{-i\omega_j t} + c.c. \tag{1'}$$

$$E_j(z) = A_j(z)\, e^{i k_j z} \tag{1''}$$

donde $k_j = n_j \omega_j / c$, $n_j = \sqrt{\epsilon^{(1)}(\omega_j)}$

$$\frac{\partial^2 E_j}{\partial z^2} - \frac{\epsilon^{(1)}(\omega_j)}{c^2}\frac{\partial^2 E_j}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P_j}{\partial t^2} \tag{2}$$

$$P_{NL}(z,t) = P_1(z,t) + P_2(z,t) , \tag{3}$$

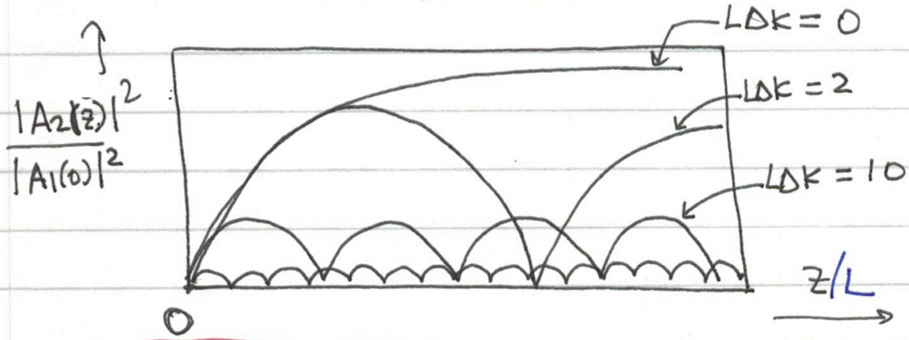
con $P_j(z,t) = P_j(z)\, e^{-i\omega_j t} + c.c. \quad (j=1,2)$ (3')

donde
$$\left.\begin{aligned} P_1(z) &= 2\chi^{(2)} E_2 E_1^* = \overbrace{2\chi^{(2)}}^{4d} A_2 A_1^* e^{i(k_2-k_1)z} \\ P_2(z) &= \chi^{(2)} E_1^2 = \underbrace{\chi^{(2)}}_{2d} A_1^2 e^{i2k_1 z} \end{aligned}\right\} (4)$$

$$\Rightarrow \left.\begin{aligned} \frac{dA_1}{dz} &= \frac{8\pi i \omega_1^2 d}{k_1 c^2} A_2 A_1^* e^{-i\Delta k z} \\ \frac{dA_2}{dz} &= \frac{4\pi i \omega_2^2 d}{k_2 c^2} A_1^2 e^{i\Delta k z} \end{aligned}\right\} (5)$$

donde $\Delta k \equiv 2k_1 - k_2$

sistema (5) puede ser resuelto en forma exacta, por medio de fns. elípticas de Jacobi.



$A_2(0) = 0$
$A_1(0) \neq 0$

Tarea: Integrar (5) numéricamente y graficar

$\frac{I_2(z)}{I}$ v/s $\frac{z}{L}$ para $A_1(0) = \sqrt{I}$, $A_2(0) = 0$

para varios valores de $\Delta s \equiv L\Delta k$

Para $\Delta k \neq 0$, la eficiencia de conversión a 2da armónica decrece dramáticamente.