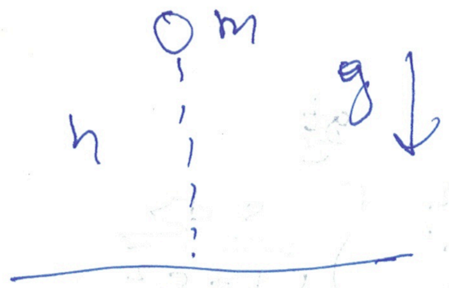
#2



calcular tiempo de caida

$$F = mg = \frac{d}{dt}(m\gamma u) \Rightarrow g = \frac{d}{dt}(\gamma u) = \gamma\frac{du}{dt} + u\frac{d\gamma}{dt}$$

$$= \gamma\frac{du}{dt} + u\cdot\frac{d\gamma}{du}\frac{du}{dt}$$

$$g = \left(\frac{du}{dt}\right)\left(\gamma + u\frac{d\gamma}{du}\right)$$

$$\gamma = \left(1-\left(\frac{u}{c}\right)^2\right)^{-1/2} \Rightarrow \frac{d\gamma}{du} = -\frac{1}{2}\left(1-\left(\frac{u}{c}\right)^2\right)^{-3/2}\cdot\frac{(-2u)}{c^2} = \frac{u}{c^2}\left(1-\left(\frac{u}{c}\right)^2\right)^{-3/2}$$

$$g = \left(\frac{du}{dt}\right)\left[\gamma + \frac{(u/c)^2}{(1-(u/c)^2)^{3/2}}\right] = \left(\frac{du}{dt}\right)\left(\frac{1-\cancel{(u/c)^2}+\cancel{(u/c)^2}}{(\quad)^{3/2}}\right) =$$

$$\boxed{g = \left(1-\left(\frac{u}{c}\right)^2\right)^{-3/2}\left(\frac{du}{dt}\right)}$$

$$\underbrace{\int_0^t g\,dt}_{gt} = \int_0^u du\left(1-\left(\frac{u}{c}\right)^2\right)^{-3/2} = \left.\frac{u}{\sqrt{1-(u/c)^2}}\right|_0^u = \frac{u}{\sqrt{1-(u/c)^2}}$$

$$\therefore (gt)^2 = \frac{u^2}{1-\left(\frac{u}{c}\right)^2} \Rightarrow (gt)^2 - (gt)^2\left(\frac{u}{c}\right)^2 = u^2$$

$$(gt)^2 = u^2\left(1+\left(\frac{gt}{c}\right)^2\right)$$

$$\therefore \boxed{u(t) = \frac{gt}{\sqrt{1+\left(\frac{gt}{c}\right)^2}}}$$

$$\lim_{t\to\infty} u(t) = c$$

límite no-relativista: $c\to\infty$, $u\to gt$.

$$\frac{dy}{dt} = \frac{gt}{\left(1+\left(\frac{gt}{c}\right)^2\right)^{1/2}}$$

$$y(t) = h + \int_0^t \frac{gt\,dt}{\left(1+\left(\frac{gt}{c}\right)^2\right)^{1/2}} = \frac{c^2}{g}\int_0^{\frac{gt}{c}} \underbrace{\frac{s\,ds}{\sqrt{1+s^2}}}_{\frac{d}{ds}\left(\sqrt{1+s^2}\right)}$$

$$= \frac{c^2}{g}\left(\sqrt{1+s^2}\right)\Big|_0^{\frac{gt}{c}} = \frac{c^2}{g}\left[\sqrt{1+\left(\frac{gt}{c}\right)^2}-1\right]$$

$$\boxed{y(t) = h - \frac{c^2}{g}\left(-1+\sqrt{1+\left(\frac{gt}{c}\right)^2}\right)}$$

$$y=0 \Rightarrow h = \frac{c^2}{g}(\cdots) \Rightarrow \left(\frac{gh}{c^2}\right) = 1+\sqrt{\quad}$$

$$\sqrt{\quad} = 1+\left(\frac{gh}{c^2}\right) \Rightarrow 1+\left(\frac{gt}{c}\right)^2 = \left(1+\left(\frac{gh}{c^2}\right)\right)^2$$

$$-1+\left(1+\left(\frac{gh}{c^2}\right)\right)^2 = \left(\frac{gt}{c}\right)^2$$

$$\Rightarrow \boxed{t = \frac{c}{g}\left[-1+\left(1+\frac{gh}{c^2}\right)^2\right]^{1/2}}$$

$$t = \sqrt{\frac{2g}{h}}\sqrt{1+\frac{gh}{2c^4}} \geq \sqrt{\frac{2g}{h}} = t_{NR}. \quad //$$

carga electrica en campo uniforme

$$F = eE = \frac{dp}{dt}$$

$$\frac{d}{dt}(m\gamma u) = eE$$

$$\frac{d}{dt}\left(\frac{u}{\left(1-\left(\frac{u}{c}\right)^2\right)^{1/2}}\right) = \frac{eE}{m}$$

$$\frac{\left(\frac{du}{dt}\right)}{\sqrt{1-(u/c)^2}} + u\cdot\left(-\frac{1}{\cancel{2}}\right)\left(\quad\right)^{-3/2}\cdot\frac{-\cancel{2}u}{c^2}\frac{du}{dt}$$

$$\left(\frac{u}{c}\right)^2\left(\quad\right)^{-3/2}$$

$$\left(\frac{du}{dt}\right)\left[\frac{1}{\left(1-(u/c)^2\right)^{1/2}} + \frac{(u/c)^2}{\left(1-\left(\frac{u}{c}\right)^2\right)^{3/2}}\right]$$

$$\frac{1-\cancel{\left(\frac{u}{c}\right)^2}+\cancel{\left(\frac{u}{c}\right)^2}}{\left(1-\left(\frac{u}{c}\right)^2\right)^{3/2}}$$

$$\left(\frac{du}{dt}\right)\cdot\frac{1}{\left(1-\left(\frac{u}{c}\right)^2\right)^{3/2}} = \frac{eE}{m}$$

$$\boxed{\frac{du}{dt} = \frac{qE}{m}\left(1-\left(\frac{u}{c}\right)^2\right)^{3/2}}$$

mas facil:

$$\frac{d}{dt}(m\gamma u) = qE$$

$$m\gamma u = qEt$$

$$\gamma u = \frac{qEt}{m}$$

$$\underbrace{\gamma^2 u^2}_{} = \left(\frac{qEt}{m}\right)^2$$

$$c^2(\gamma^2 - 1) = \left(\frac{qEt}{m}\right)^2$$

$$\gamma^2 = 1 + \left(\frac{qEt}{mc}\right)^2 = \frac{1}{1-\left(\frac{u}{c}\right)^2}$$

$$\Rightarrow 1 - \left(\frac{u}{c}\right)^2 = \frac{1}{1+\left(\frac{qEt}{mc}\right)^2}$$

$$\left(\frac{u}{c}\right)^2 = 1 - \frac{1}{1+(\,)^2} = \frac{\cancel{1}+(\,)^2-\cancel{1}}{1+(\,)^2} = \frac{(qEt/mc)^2}{1+(qEt/mc)^2}$$

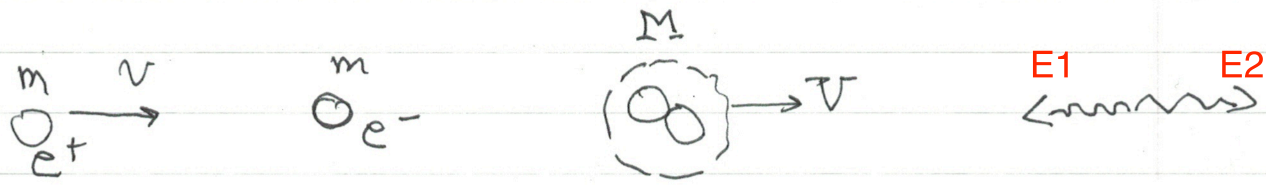
$$\boxed{\frac{u}{c} = \frac{(qEt/mc)}{\sqrt{1+(qEt/mc)^2}}} = \frac{dx}{dt}$$

$$u = \frac{(qEt/m)}{\sqrt{1+(qEt/mc)^2}} = \frac{dx}{dt}$$

$\frac{qE}{m}$

$$x = \left(1+\left(\frac{qEt}{mc}\right)^2\right)^{1/2}\frac{mc^2}{qE} - \frac{mc^2}{qE}$$

$$x(t) = \frac{mc^2}{qE}\left[\sqrt{1+\left(\frac{qEt}{mc}\right)^2} - 1\right]$$



$$m\gamma(v)v + 0 = M\gamma(V)V \qquad (1)$$

$$m\gamma(v)c^2 + mc^2 = M\gamma(V)c^2 \qquad (2)$$

$$\frac{\cancel{m}\gamma(v)v}{\cancel{m}\gamma(v)\cancel{c^2} + \cancel{m}\cancel{c^2}} = \frac{V}{\cancel{c^2}}$$

$$V = \frac{\gamma(v)v}{\gamma(v)+1} = \boxed{\left(\frac{\gamma}{\gamma+1}\right)v}$$

luego decae en 2 fotones:

$$M\gamma_2 c^2 = E_1 + E_2$$

$$M\gamma_2 V c = E_1 - E_2$$

$$\Rightarrow \boxed{E_1 = \frac{1}{2} M\gamma_2 c^2\left(1 + \frac{V}{c}\right)}$$

$$\boxed{E_2 = \frac{1}{2} M\gamma_2 c^2\left(1 - \frac{V}{c}\right)}$$

but $M\gamma_2 \overset{(2)}{=} m\gamma_1 + m$
$= m(\gamma_1 + 1)$

$$E_1 = \frac{1}{2} mc^2(1+\gamma_1)\left(1 + \frac{V}{c}\right)$$

$$E_2 = \frac{1}{2} mc^2(1+\gamma_1)\left(1 - \frac{V}{c}\right)$$

#2. Mov. de proyectil en Relatividad Especial.

Una partícula de masa m [redacted] lanzada con una veloc. inicial $v_0\hat{x}$, en presencia de una $\vec{F}$ externa constante $-F\hat{y}$. Encuentra $v_x(t)$ y $v_y(t)$ explícito(s) como funciones de F, m y v_0. Demuestre analíticamente que $|\vec{v}| = \sqrt{v_x^2+v_y^2} < c$ para cualquier instante.

$$\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \frac{d}{dt}(m\gamma v_x) = 0 \Rightarrow m\gamma v_x = \frac{m v_0}{\sqrt{1-(v_0/c)^2}} \equiv p_0$$

$$\frac{d}{dt}(m\gamma v_y) = -F \Rightarrow m\gamma v_y = -Ft$$

$$\therefore \quad \begin{aligned} m\gamma v_x &= p_0 \\ m\gamma v_y &= -Ft \end{aligned} \quad \Rightarrow \quad \frac{v_x}{v_y} = \frac{-p_0}{Ft} \Rightarrow \boxed{v_y = -\frac{Ft}{p_0} v_x}$$

$$\left.\begin{aligned} m^2\gamma^2 v_x^2 &= p_0^2 \\ m^2\gamma^2 v_y^2 &= (Ft)^2 \end{aligned}\right\} \quad m^2\gamma^2 v^2 = p_0^2 + (Ft)^2$$

but $$m\gamma^2 v^2 = \frac{m v^2}{1-(v/c)^2} = (mc)^2(\gamma^2-1)$$

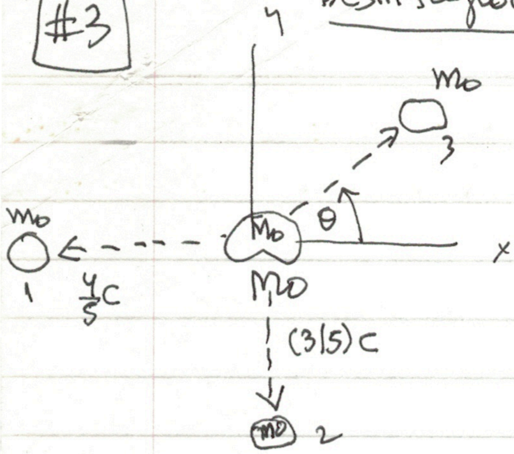
$$\Rightarrow (mc)^2(\gamma^2-1) = p_0^2 + (Ft)^2 \Rightarrow \boxed{\gamma^2 = 1 + \left(\frac{p_0}{mc}\right)^2 + \left(\frac{Ft}{mc}\right)^2}$$

then) $$v_x = \frac{p_0}{m\gamma} = \frac{p_0 c}{\sqrt{(mc)^2 + p_0^2 + (Ft)^2}}$$

$$v_y = \frac{-Fct}{\sqrt{(mc)^2 + p_0^2 + (Ft)^2}}$$

$$|\vec{v}| = c\left[\frac{p_0^2 + (Ft)^2}{(mc)^2 + p_0^2 + (Ft)^2}\right]^{1/2} < c$$, regardless of t. indep. del signo de F.

#3 Desintegración de una partícula



$$M_0 \cancel{c^2} = m_0\gamma_1 \cancel{c^2} + m_0\gamma_2 \cancel{c^2} + m_0\gamma_3 \cancel{c^2}$$

$$\frac{M_0}{m_0} = \gamma_1 + \gamma_2 + \gamma_3(\gamma_3)$$

$$\boxed{\begin{aligned} m_0\gamma_1 v_1 &= \cancel{m_0}\gamma_3 v_3 \cos\theta \\ \cancel{m_0}\gamma_2 v_2 &= \cancel{m_0}\gamma_3 v_3 \sin\theta \end{aligned}}$$

$$\Rightarrow \qquad \tan\theta = \frac{\gamma_2 v_2}{\gamma_1 v_1} = \frac{v_2}{v_1}\left(\frac{1-(v_1/c)^2}{1-(v_2/c)^2}\right)^{1/2}.$$

also, $\gamma_1^2 v_1^2 + \gamma_2^2 v_2^2 = \gamma_3^2 v_3^2 = \dfrac{v_3^2}{1-\left(\frac{v_3}{c}\right)^2} = -1 + \dfrac{1}{1-\left(\frac{v_3}{c}\right)^2}$

$$\Rightarrow 1-\left(\frac{v_3}{c}\right)^2 = \frac{1}{1+\left(\gamma_1\frac{v_1}{c}\right)^2 + \left(\gamma_2\frac{v_2}{c}\right)^2}$$

$$\Rightarrow \frac{v_3}{c} = \left[\frac{\left(\gamma_1\frac{v_1}{c}\right)^2 + \left(\gamma_2\frac{v_2}{c}\right)^2}{1+\left(\gamma_1\frac{v_1}{c}\right)^2 + \left(\gamma_2\frac{v_2}{c}\right)^2}\right]^{1/2} < 1 \text{ always.}$$

$$\Rightarrow \qquad \gamma_3 = \sqrt{1+\left(\gamma_1\frac{v_1}{c}\right)^2 + \left(\gamma_2\frac{v_2}{c}\right)^2}$$

$$\Rightarrow \frac{M}{m_0} = \gamma_1 + \gamma_2 + \left(1+\left(\gamma_1\frac{v_1}{c}\right)^2 + \left(\gamma_2\frac{v_2}{c}\right)^2\right)^{1/2}$$

using $v_1 = \frac{4}{5}c$; $v_2 = \frac{3}{5}c$ $\Rightarrow$ $\boxed{\begin{aligned} \frac{M}{m_0} &= 4.74 \\ \frac{v_3}{c} &= 0.837 \\ \theta &= 29.4^\circ \end{aligned}}$

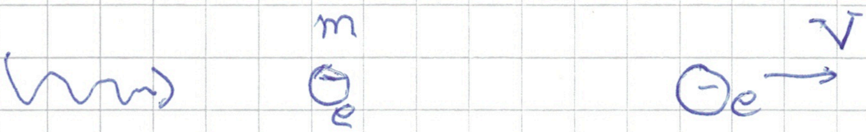
Fisica contemporánea I

1. Demuestre rigurosamente que los siguientes procesos son imposibles:
   (a) Un solo fotón choca contra un electrón estacionario y le transfiere toda su energía.
   (b) Un solo fotón en el vacío es transformado en un electrón mas un positrón.
   (c) Un positrón rápido y un electrón estacionario se aniquilan mutuamente, produciendo un solo fotón.

2. Considere el decaimiento de un núcleo radiactivo en reposo por emisión de partículas alfa

$$X \to Y + \alpha \tag{1}$$

   Definimos la energá de desintegración Q como $Q \equiv (M_X - M_Y - M_\alpha)c^2$. Esta energía debe ser compartida entre la partícula α y el núcleo Y con el objeto de conservar la energía y el momento del proceso de decaimiento. Encuentre una expresión para la energía cinética K_α en términos de Q y las masas en reposo de los núcleos.



$$\boxed{\begin{aligned} E_\gamma + mc^2 &= m\gamma c^2 \\ \frac{E_\gamma}{c} &= m\gamma v \end{aligned}}$$

$$\boxed{\begin{aligned} E_\gamma + mc^2 &= m\gamma c^2 \\ E_\gamma &= m\gamma c v \end{aligned}}$$

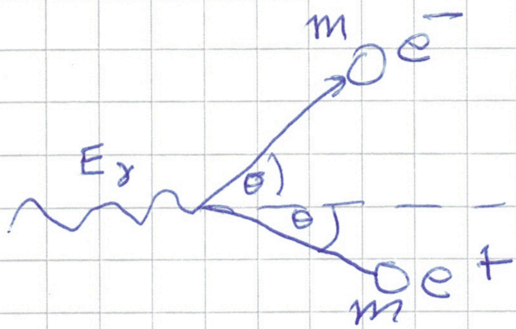
$$\cancel{m}c^2 = \cancel{m}\gamma c^2 - \cancel{m}\gamma c v$$

$$\cancel{c^2} = \gamma \cancel{c^2}\left(1 - \frac{v}{c}\right)$$

$$1 = \gamma\left(1 - \frac{v}{c}\right) = \frac{1}{\sqrt{1-(v/c)^2}} \cdot 1-(v/c).$$

$$= \frac{1}{\sqrt{(1+\frac{v}{c})\cancel{(1-\frac{v}{c})}}} \cancel{\sqrt{1-(v/c)}}\sqrt{1-(v/c)}$$

$$1 = \sqrt{\frac{1-(v/c)}{1+(v/c)}} < 1$$



$$E_\gamma = 2E_e$$

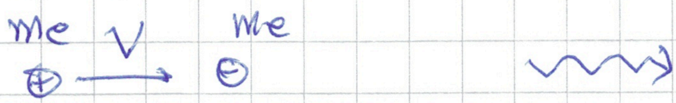
$$\frac{E_\gamma}{c} = P_e\cos\theta + P_e\cos\theta = 2P_e\cos\theta$$

$$0 = 2E_e - 2cP_e\cos\theta$$

$$\Rightarrow \quad E_e = cP_e\cos\theta < cP_e$$

$$\downarrow$$

$$\sqrt{(P_e c)^2 + (m_e c^2)^2} < cP_e \qquad \times$$



$$E_e + mc^2 = E_\gamma$$

$$m\gamma v + 0 = \frac{E_\gamma}{c}$$

$$= E_e + mc^2$$

$$cP_e = \sqrt{(cP_e)^2 + (mc^2)^2} + mc^2 > P_e c \qquad \times$$

#8

$$\boxed{\begin{aligned} E &= mc^2 + K \\ E^2 &= c^2p^2 + m^2c^4 \end{aligned}}$$

$$K_\alpha = E_\alpha - m_\alpha c^2 = \sqrt{P_\alpha^2 c^2 + m_\alpha^2 c^4} - m_\alpha c^2$$

$$K_y = \sqrt{P_\alpha^2 c^2 + m_y^2 c^4} - m_y c^2 \qquad (P_\alpha = P_y)$$

$$Q = K_\alpha + K_y = K_\alpha + \sqrt{P_\alpha^2 c^2 + m_y^2 c^4} - m_y c^2 \qquad (1)$$

$$E_\alpha^2 = (K_\alpha + m_\alpha c^2)^2 = P_\alpha^2 c^2 + m_\alpha^2 c^4$$

$$K_\alpha^2 + \cancel{m_\alpha^2 c^4} + 2m_\alpha c^2 K_\alpha = P_\alpha^2 c^2 + \cancel{m_\alpha^2 c^4} \qquad (2)$$

$$\Rightarrow \quad P_\alpha^2 c^2 = K_\alpha^2 + 2m_\alpha c^2 K_\alpha \qquad (3)$$

$$(1) \Rightarrow (Q + m_y c^2 - K_\alpha)^2 = P_\alpha^2 c^2 + m_y^2 c^4$$

$$\overset{(3)}{=} K_\alpha^2 + 2m_\alpha c^2 K_\alpha + m_y^2 c^4$$

$$(Q + m_y)^2 + \cancel{K_\alpha^2} - 2(Q + m_y)K_\alpha = \cancel{K_\alpha^2} + 2m_\alpha c^2 K_\alpha + m_y^2 c^4$$

$$(Q + m_y c^2)^2 - m_y^2 c^4 = [2m_\alpha c^2 + 2(Q + m_y)] K_\alpha$$

$$Q^2 + \cancel{m_y^2 c^4} + 2m_y c^2 Q - \cancel{m_y^2 c^4} = [\cdots]$$

$\rightarrow$

$$\Rightarrow \quad K_\alpha = \frac{Q^2 + 2 m_y c^2 Q}{2\left(m_\alpha c^2 + m_y c^2 + Q\right)}$$

Límite no-rel: $c \to \infty$

$$K_\alpha \to \frac{2 m_y c^2 Q}{2\left(m_\alpha c^2 + m_y c^2\right)} = \left(\frac{m_y}{m_\alpha + m_y}\right) Q$$

2

Umbral para producción de partículas

$A \rightarrow B \qquad C \rightarrow D \rightarrow$

calc. energia cinetica incidente minima para la reaccion

$$P_i = P_f$$

$$\Rightarrow P_i^2 = P_f^2$$

$$P_i = P_A + P_B = (E_A, \vec{p}_A) + (m_B, \vec{0}) = (E_A + m_B, \vec{p}_A)$$

$$\Rightarrow P_i^2 = (E_A + m_B)^2 - p_A^2$$ INVARIANT (calc en LAB)

$$P_f = P_C + P_D = (E_C, \vec{p}_C) + (E_D, \vec{p}_D)$$

$$(E_C + E_D, \vec{p}_C + \vec{p}_D)$$

$$P_f^2 = (E_C + E_D)^2 - (p_C + p_D)^2 \overset{\text{C-M en umbral}}{=} (m_C + m_D)^2$$

$$(m_C + m_D)^2 = (E_A + m_B)^2 - p_A^2$$

$$= E_A^2 + m_B^2 + 2 m_B E_A - p_A^2$$

$$= m_A^2 + m_B^2 + 2 m_B E_A$$

$$\Rightarrow \frac{(m_C + m_D)^2 - m_A^2 - m_B^2}{2 m_B} = E_A = T + m_A$$

$$T = \frac{(m_C + m_D)^2 - m_A^2 - m_B^2}{2 m_B} - m_A$$

$$\therefore \boxed{T = \frac{(m_C + m_D)^2 - (m_A + m_B)^2}{2 m_B}}$$