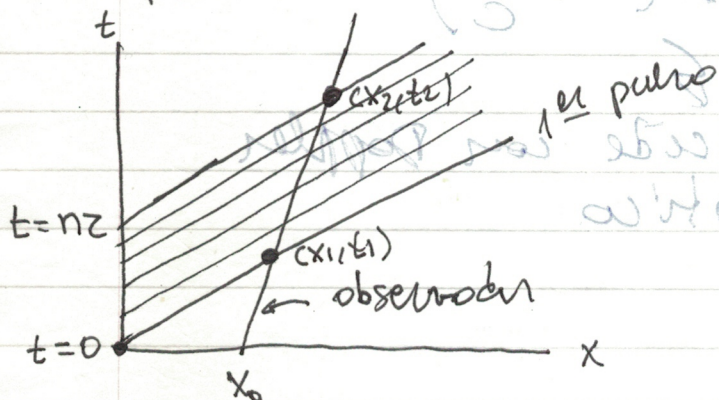


Efecto Doppler relativista



Fuente (S) emite pulsos de luz en $t = n\tau \Rightarrow$ en S, la frec. medida es $\nu = 1/\tau$

El 1º pulso es emitido en $t=0$, cuando el receptor (S') está en $x = x_0$



$$x_1 = ct_1 = x_0 + vt_1$$

$$x_2 = c(t_2 - n\tau) = x_0 + vt_2$$

$$t_2 - t_1 = \frac{cn\tau}{c-v}$$

$$x_2 - x_1 = \frac{vcn\tau}{c-v}$$

En S': $t_2' - t_1' = \gamma \left[(t_2 - t_1) - \frac{v}{c^2} (x_2 - x_1) \right]$

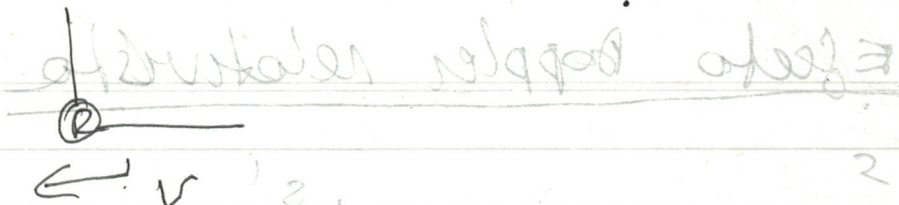
$$= \gamma \left(\frac{cn\tau}{c-v} - \frac{v}{c^2} \cdot \frac{vcn\tau}{c-v} \right) = \frac{\gamma cn\tau}{c-v} \left(1 - \frac{v^2}{c^2} \right)$$

$$\Rightarrow \tau' = \frac{\gamma cn\tau}{c-v} \left(1 - \frac{v^2}{c^2} \right) = \frac{\gamma \tau}{1 - \frac{v}{c}} \left(1 - \frac{v^2}{c^2} \right) = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \frac{1}{1 - \frac{v}{c}}$$

$$= \frac{\tau \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}} = \tau \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

$$\Rightarrow \nu' = \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2} \nu$$

Doppler longitudinal $\rightarrow$



$$v' = v \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2}$$

where $v/c \ll 1$

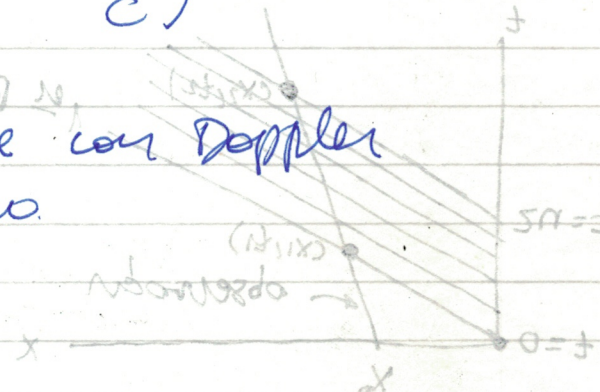
$$v' \approx v \left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} + \left(\frac{v}{c} \right)^2 \right)^{1/2}$$

$$\approx v \left[\left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} \right) \right]^{1/2} = v \left(1 - \frac{v}{c} \right)$$

coincide con Doppler
acustico

$$\frac{c-v}{c-v} = 1$$

$$\frac{c-v}{c-v} = 1$$



$$f' - f = \frac{v}{c} f \left[\left(1 - \frac{v}{c} \right) \left(1 - \frac{v}{c} \right) \right]^{1/2}$$

$$f' = f \left(\frac{c-v}{c-v} \cdot \frac{1}{1 - \frac{v}{c}} \right) = f \left(\frac{1}{1 - \frac{v}{c}} \right)$$

$$\Rightarrow f' = f \left(\frac{1}{1 - \frac{v}{c}} \right) = f \left(1 + \frac{v}{c} \right)$$

$$v' = v \left(\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}} \right)^{1/2}$$

Doppler relativistico

Momento Relativista

Idea: generalizar las leyes de Newton y las leyes de momento y energía.

Sup. que 2 cuerpos colisionen en el sist. S , en ausencia de $\vec{F}$ externos $\rightarrow$ se conserva el momento
 $\vec{p} = m_1 \vec{u}_1 + m_2 \vec{u}_2$.

* (con Lorentz)

sin embargo al transf. a otro sist. S' en movimiento S' , se encuentra que $\vec{p}$ no es conservado.
 $\vec{p}' = m_1 \vec{u}_1' + m_2 \vec{u}_2' \neq \vec{p}$

$\Rightarrow$ Re-definir el momento

(* T. de Galileo works fine)

(i) El momento relativista debe ser conservado en todas las colisiones.

(ii) El momento relativista debe reducirse a valor newtoniano cuando $v/c \ll 1$.

La definición correcta es (ejercicio)

$$\vec{p} = m \gamma \vec{u} = \frac{m}{\sqrt{1 - (v/c)^2}} \vec{u} \quad (1)$$

La fuerza relativista está definida por

$$\vec{F} = \frac{d\vec{p}}{dt} \quad (2)$$

Energía relativista

↓ trabajo

$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} \left(\frac{dp}{dt} \right) dx$$

$$\text{pero } \left(\frac{dp}{dt} \right) dx = \left(\frac{dp}{du} \frac{du}{dt} \right) dx = \left(\frac{dp}{du} \right) \frac{du}{dx} \frac{dx}{dt} dx$$

$$= \left(\frac{dp}{du} \right) u \frac{du}{dx} dx = \left(\frac{dp}{du} \right) u du$$

$$\text{como } \frac{dp}{du} = \frac{d}{du} \frac{mu}{\left(1 - \left(\frac{u}{c}\right)^2\right)^{1/2}} = \frac{m}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} + \frac{mu \cdot (-2) \frac{u}{c^2}}{2 \left(1 - \left(\frac{u}{c}\right)^2\right)^{3/2}}$$

$$= \frac{m}{\left(1 - \left(\frac{u}{c}\right)^2\right)^{3/2}}$$

$$\Rightarrow W = \int_0^u \frac{mu}{\left(1 - \left(\frac{u}{c}\right)^2\right)^{3/2}} du = mc^2 \left(1 - \left(\frac{u}{c}\right)^2\right)^{-1/2} \Big|_0^u$$

$$W = \frac{mc^2}{\sqrt{1 - \left(\frac{u}{c}\right)^2}} - mc^2 \quad \checkmark \quad (3)$$

= K (cambio en E. cinética de la partícula)

$$\frac{u}{c} \ll 1: W \approx mc^2 \left(1 + \frac{1}{2} \left(\frac{u}{c}\right)^2 + \dots\right) - mc^2 = \frac{m}{2} u^2 \quad \checkmark \quad (4)$$

se puede escribir $K = \gamma mc^2 - mc^2$ $\rightarrow$ energía en reposo

$\gamma mc^2 =$ energía en reposo + energía cinética

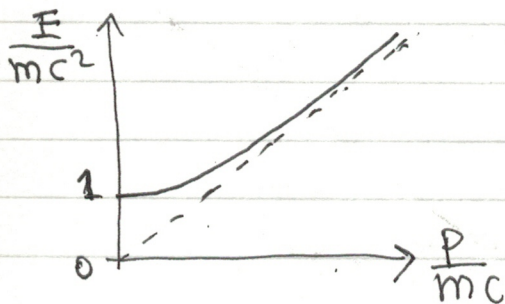
por lo tanto definimos $E = K + mc^2 = \gamma mc^2$

$$\therefore E = \frac{mc^2}{\sqrt{1 - (\frac{u}{c})^2}} \quad (5)$$

De $\vec{p} = m\gamma \vec{u}$ y $E = m\gamma c^2$

se obtiene $E = \sqrt{(pc)^2 + (mc^2)^2}$

relación de dispersión



comparar con

$$E = \frac{p^2}{2m}$$

Fotones: $m=0 \Rightarrow E = pc$ relativista

$E = h\nu$ (M.C.) $\rightarrow p = \frac{h}{\lambda}$

Ejercicio: Dem. que $E^2 - (pc)^2$ es invariante relativista.

$I = (E, c\vec{p})$ 4-momentos

$I \cdot I = m^2 c^4$ $\leftarrow$ evaluado en syst. en reposo.

Relación de Dispersión

$$E = \gamma mc^2 = mc^2 / (1 - (u/c)^2)^{1/2}$$

$$p = \gamma mu = mu / (1 - (u/c)^2)^{1/2}$$

$$p^2 = m^2 \gamma^2 u^2, \quad \text{pero } \gamma^2 u^2 = \frac{u^2}{1 - (u/v)^2} = \frac{c^2 (u/c)^2}{1 - (u/c)^2}$$

$$= c^2 \left[\frac{(u/c)^2 - 1 + 1}{1 - (u/v)^2} \right] = -c^2 \left[\frac{1 - (u/c)^2}{1 - (u/c)^2} \right] + \frac{c^2}{1 - (u/v)^2}$$

$$= -c^2 + c^2 \gamma^2$$

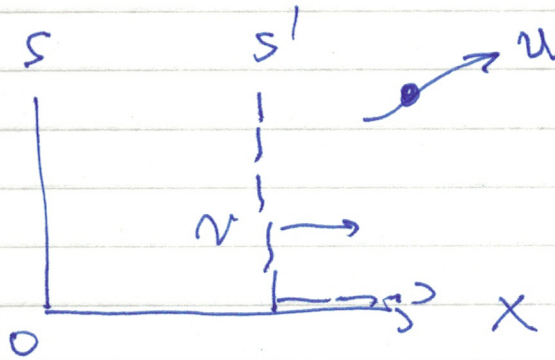
$$\Rightarrow p^2 = -m^2 c^2 + m^2 c^2 \gamma^2 \quad / c^2$$

$$c^2 p^2 = -m^2 c^4 + \underbrace{m^2 \gamma^2 c^4}_{E^2}$$

$$\therefore E^2 = m^2 c^4 + c^2 p^2$$

$$E = \sqrt{m^2 c^4 + c^2 p^2} //$$

T. de Lorentz para energía y momento



$$\gamma = (1 - (v/c)^2)^{-1/2}$$

$$u_x' = \frac{u_x - v}{1 - vu_x/c^2} \quad ; \quad u_y' = \frac{u_y}{1 - vu_x/c^2} \quad (1)$$

Sist. S

$$E = m_0 c^2 \gamma(u)$$

$$p_x = m_0 u_x \gamma(u)$$

$$p_y = m_0 u_y \gamma(u)$$

donde $\gamma(u) = (1 - (u/c)^2)^{-1/2}$

lo mas difícil es relacionar
gamma(u) con gamma(u')

Sist. S'

$$E' = m_0 c^2 \gamma(u')$$

$$p_x' = m_0 u_x' \gamma(u')$$

$$p_y' = m_0 u_y' \gamma(u')$$

con $\gamma(u') = (1 - (u'/c)^2)^{-1/2}$

Dem. que:

$$E' = \gamma(v)(E - vp_x)$$

$$p_x' = \gamma(v)(p_x - \frac{vE}{c^2})$$

$$p_y' = p_y$$

$$E' = mc^2 \gamma(u')$$

$$\gamma(u') = (1 - (u'/c)^2)^{-1/2}$$

$$\gamma(u') = [1 - (u_x'^2/c^2) - (u_y'^2/c^2)]^{-1/2}$$

$$(a) \quad 1 - (u_x'^2/c^2) = 1 - \frac{(u_x - v)^2}{c^2 (1 - vu_x/c^2)^2}$$

$$= \frac{c^2 (1 - \frac{vu_x}{c^2})^2 - (u_x - v)^2}{c^2 (1 - \frac{vu_x}{c^2})^2} =$$

$$= \frac{c^2 (1 + (\frac{vu_x}{c^2}) - \frac{2vu_x}{c^2}) - u_x^2 - v^2 + 2vu_x}{c^2 (1 - \frac{vu_x}{c^2})^2}$$

$$= \frac{c^2 + c^2 (\frac{vu_x}{c^2})^2 - u_x^2 - v^2}{c^2 (1 - \frac{vu_x}{c^2})^2}$$

$$= \frac{1 + (\frac{vu_x}{c^2})^2 - (u_x/c)^2 - (v/c)^2}{(1 - vu_x/c^2)^2}$$

$$= \frac{(1 - u_x^2/c^2)(1 - (v/c)^2)}{(1 - vu_x/c^2)^2}$$

(1)

$$\text{De } u_y' = \frac{u_y / \gamma(v)}{1 - v u_x / c^2} \quad ; \quad \gamma(v) = (1 - v^2/c^2)^{-1/2}$$

$$\Rightarrow \frac{u_y'^2}{c^2} = \frac{(u_y^2/c^2) \gamma^2(v)}{(1 - v u_x/c^2)^2} = \frac{(u_y^2/c^2)(1 - v^2/c^2)}{(1 - v u_x/c^2)^2} \quad (2)$$

$$(1) + (2) \Rightarrow$$

$$\begin{aligned} 1 - \frac{u'^2}{c^2} &= \frac{(1 - u_x^2/c^2)(1 - v^2/c^2) - (u_y/c^2)^2(1 - v^2/c^2)}{(1 - v u_x/c^2)^2} \\ &= \frac{(1 - v^2/c^2)(1 - u_x^2/c^2 - u_y^2/c^2)}{(1 - v u_x/c^2)^2} = \frac{(1 - v^2/c^2)(1 - u^2/c^2)}{(1 - v u_x/c^2)^2} \end{aligned}$$

$$\text{v sep, } \frac{1}{\gamma^2(u')} = \frac{1}{\gamma^2(v)} \cdot \frac{1}{\gamma^2(u)} \cdot \frac{1}{(1 - v u_x/c^2)^2}$$

$$\Rightarrow \boxed{\gamma(u') = \gamma(v) \gamma(u) (1 - v u_x/c^2)} \quad (3)$$

$$\text{we get } E' = \gamma(u') m c^2$$

$$= \gamma(v) \left[\gamma(u) m c^2 - \gamma(u) m c \frac{v u_x}{c^2} \right]$$

$$\therefore \boxed{E' = \gamma(v) [E - v p_x]} \quad (4)$$

$$P_x' = \gamma(u') m u_x' = \gamma(v) \gamma(u) \left(1 - \frac{v u_x}{c^2}\right) m \cdot \frac{(u_x - v)}{\left(1 - \frac{v u_x}{c^2}\right)}$$

$$P_x' = \gamma(v) \gamma(u) m (u_x - v)$$

$$\text{O See } P_x' = \gamma(v) \left(P_x - \frac{v E}{c^2} \right) \quad (5)$$

$$P_y' = \gamma(u') m u_y' = \gamma(v) \gamma(u) \left(1 - \frac{v u_x}{c^2}\right) m \cdot \frac{u_y / \gamma(v)}{\left(1 - \frac{v u_x}{c^2}\right)}$$

$$= m \gamma(u) u_y = P_y$$

$$\therefore \boxed{P_y' = P_y} \quad (6)$$



energia foton=?

$u = ?$

$$E_\gamma + mc^2 = 3m\gamma_u c^2 \quad (1)$$

$$\frac{E_\gamma}{c} = 3m\gamma_u u \quad (2)$$

$$\frac{(2)}{(1)} \Rightarrow \boxed{\frac{E_\gamma}{E_\gamma + mc^2} = \frac{u}{c}}$$

$$(2) \Rightarrow \left(\frac{E_\gamma}{c}\right)^2 = (3m)^2 (\gamma_u u)^2 = 9m^2 (\gamma_u^2 - 1)$$

$$\left(\frac{E_\gamma}{3mc^2}\right)^2 = \gamma_u^2 - 1 \Rightarrow \gamma_u^2 = 1 + \left(\frac{E_\gamma}{3mc^2}\right)^2$$

$$\Rightarrow (1) \quad (E_\gamma + mc^2)^2 = 9m^2 c^4 \left(1 + \left(\frac{E_\gamma}{3mc^2}\right)^2\right)$$

$$\cancel{E_\gamma^2} + (mc^2)^2 + 2mc^2 E_\gamma = (3mc^2)^2 + \cancel{E_\gamma^2}$$

$$2mc^2 E_\gamma = 8(mc^2)^2$$

$$\boxed{E_\gamma = 4(mc^2)}$$

$$\Rightarrow \frac{u}{c} = \frac{4(mc^2)}{4(mc^2) + mc^2} = \frac{4}{5} \therefore \boxed{\frac{u}{c} = \frac{4}{5}}$$