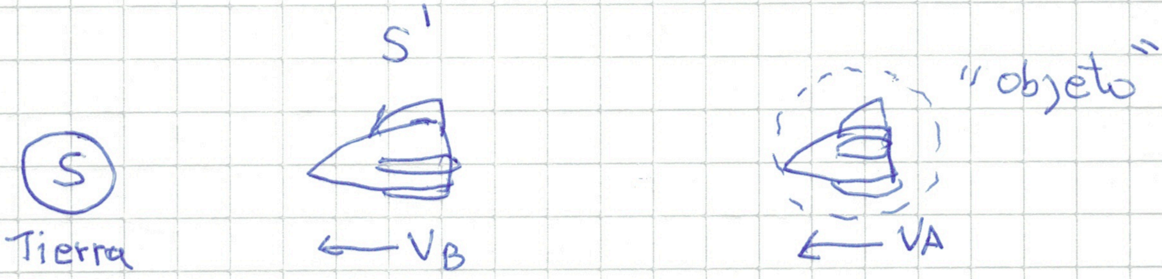


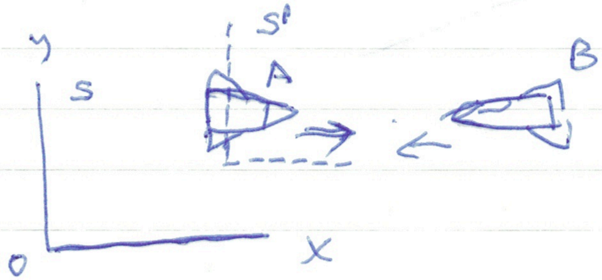
En gral, $\vec{v}_A = \dfrac{\vec{v}_B + \vec{v}_A'}{1 + \dfrac{\vec{v}_A' \cdot \vec{v}_B}{c^2}}$ $\Rightarrow$

$\vec{v}_A' = \dfrac{\vec{v}_A - \vec{v}_B}{1 - \dfrac{\vec{v}_A \cdot \vec{v}_B}{c^2}}$. In our case $\vec{v}_A = -0.5c\,\hat{\imath}$
$\vec{v}_B = -0.8c\,\hat{\imath}$

$$\Rightarrow v_A' = \frac{-0.5c - (-0.8c)}{1 - (-0.5)(-0.8)} = \boxed{0.5c}$$

EJ: 2 naves A y B se mueven en direcciones opuestas

S (tierra) $V_A = 0.750c$
$V_B = 0.850c$



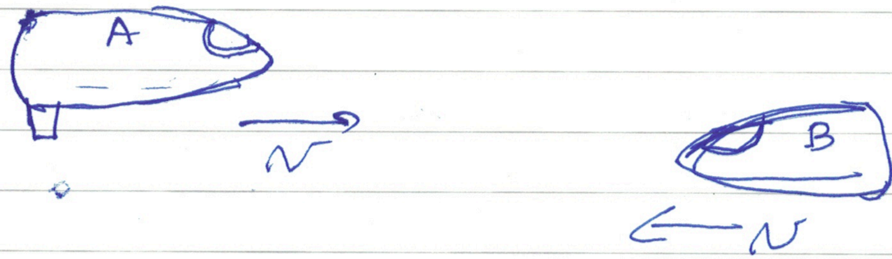
Hallar la veloc. de B c/r a A.

<u>solución</u> tomar S' en la nave A, con $v = 0.750c$ la veloc. entre S y S'. La nave B se toma como un objeto moviéndose hacia la izquierda con velocidad $u_x = -0.850c$ c/r a la tierra (S)

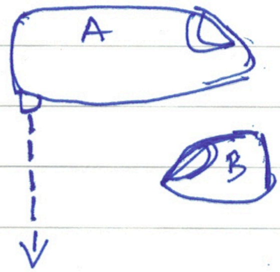
$\Rightarrow$ la veloc. de B c/r a A (S') será

$$u_x' = \frac{u_x - v}{1 - \frac{u_x v}{c^2}} = \frac{-0.850c - 0.750c}{1 - \frac{(-0.850c)(0.750c)}{c^2}} = -0.9771c$$

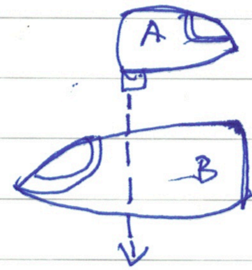
Típica paradoja



(i)

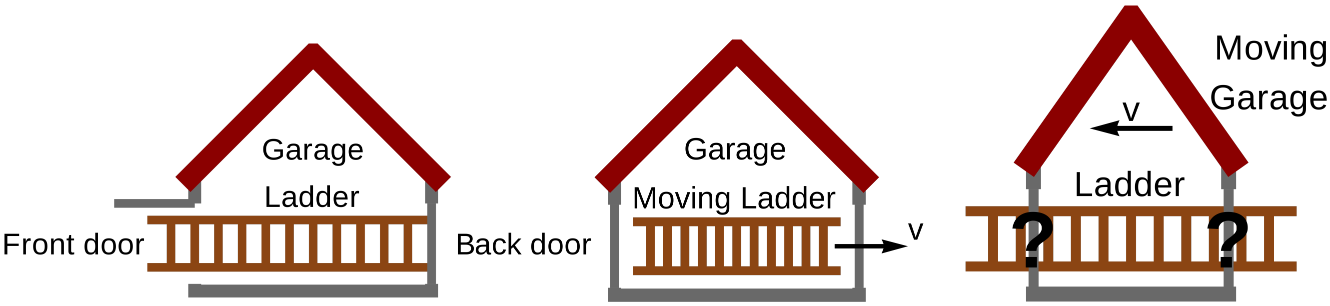


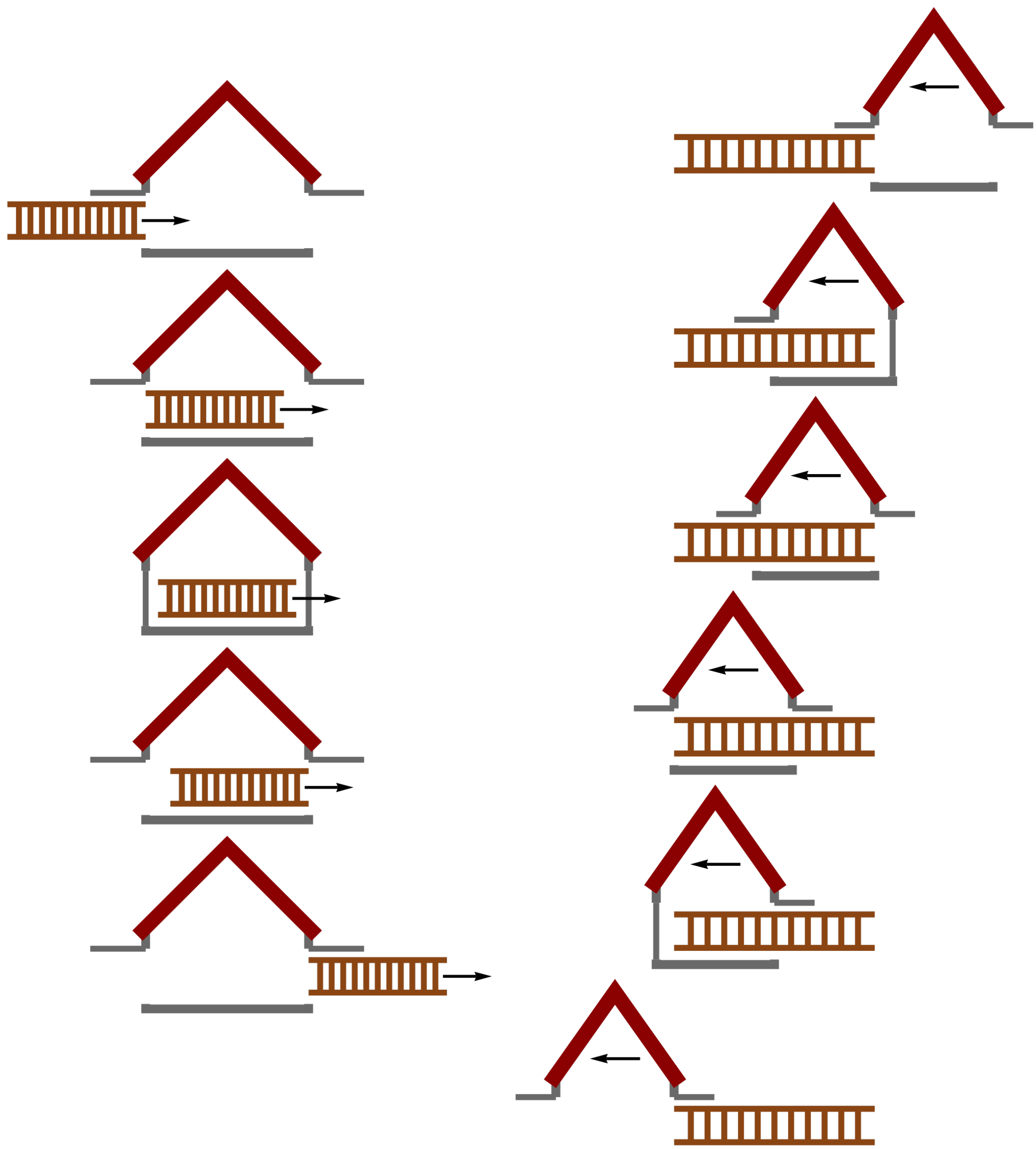
(ii)



ES DESTRUIDA LA NAVE B ?

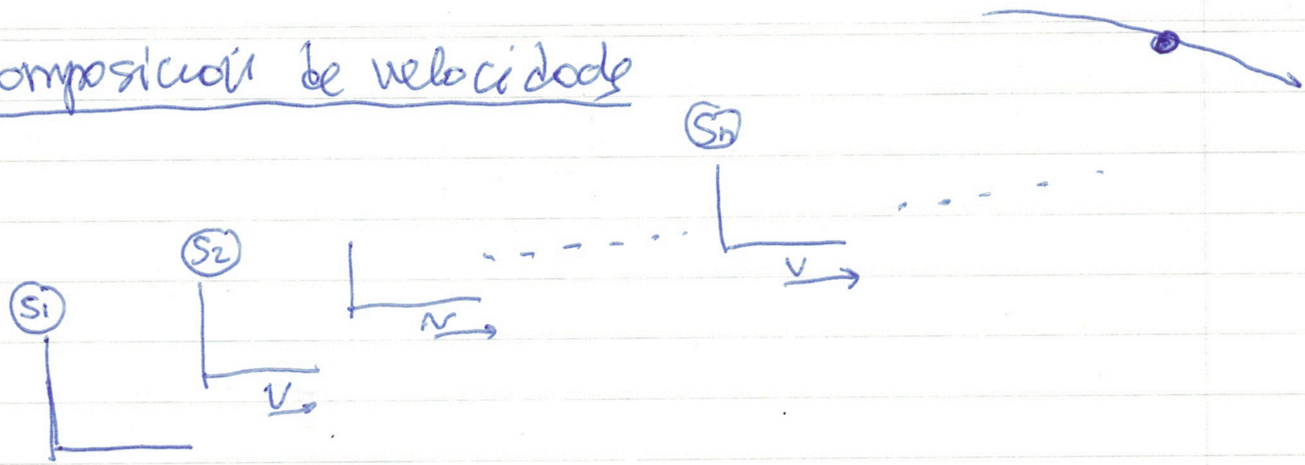
Paradoja de la escalera





Simultaneidad !

composición de velocidade



$$u_n = \frac{u_{n+1} + v}{1 + \frac{u_{n+1} v}{c^2}}$$

pero $n \to \infty$, $u_n \to u$, $u_{n+1} \to u$

$$u = \frac{u + v}{1 + \frac{uv}{c^2}} \Rightarrow u\left(1 + \frac{uv}{c^2}\right) = u + v$$

$$\frac{u^2 v}{c^2} = v \Rightarrow u^2 = c^2$$

$$\boxed{u \to c}$$

Muones

$$N(t) = N_0 e^{-t/\tau}$$

τ = vida media $\simeq 2\mu s$ (sist. reposo c/r al muón)
$v \simeq 0.998c$ y son creados en la alta atmosfera.

La distancia recorrida (desde S) debería ser $h \simeq v\tau = 0.998c \cdot 2\mu s = 600$ m.

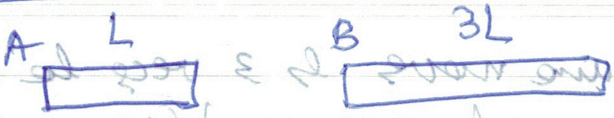
Desde la tierra, $\tau \to \gamma\tau \simeq 15 \times 2\mu s = 30\mu s$

$\Rightarrow h \simeq 9.000$ m

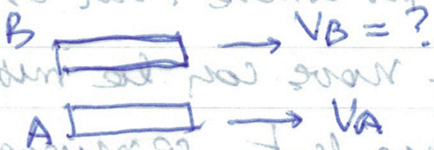
Desde el muón, τ no cambió, pero el suelo se aproxima a $0.998c$ $\Rightarrow$ se contrae la altura y $9000\,m \to \frac{9000}{\gamma}m = \frac{9000}{15} = 600$ m.

$\therefore$ El muón llega al suelo ya sea analizando desde S ó S'.

Prob. 26 (Serway) →



The proper length of one spaceship is three times that of another. The two spaceships are traveling in the same direction and, while both are passing overhead, an Earth observer measures the two spaceships to have the same length. If the slower spaceship is moving with a speed of 0.35c, determine the speed of the faster spaceship.



(S) ∴ $L_A = \dfrac{L}{\gamma_A}$

$$L_B = \frac{3L}{\gamma_B}$$

but $L_A = L_B \Rightarrow 1 = \dfrac{\frac{L}{\gamma_A}}{\frac{3L}{\gamma_B}} = \dfrac{\cancel{L}\gamma_B}{\gamma_A \cdot 3\cancel{L}} = \dfrac{\gamma_B}{3\gamma_A}$

$$\Rightarrow \gamma_B = 3\gamma_A \Rightarrow \frac{1}{\gamma_B} = \frac{1}{3\gamma_A} \Rightarrow \sqrt{1-\left(\frac{V_B}{c}\right)^2} = \frac{1}{3}\sqrt{1-\left(\frac{V_A}{c}\right)^2}$$

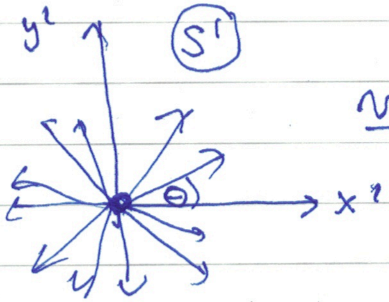
$$1 - \left(\frac{V_B}{c}\right)^2 = \frac{1}{9}\left(1 - \left(\frac{V_A}{c}\right)^2\right) = \frac{1}{9} - \frac{1}{9}\left(\frac{V_A}{c}\right)^2$$

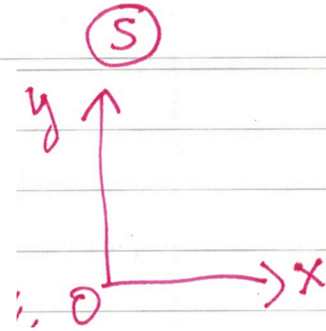
$$\frac{8}{9} - \left(\frac{V_B}{c}\right)^2 = -\frac{1}{9}\left(\frac{V_A}{c}\right)^2 \Rightarrow \left(\frac{V_B}{c}\right)^2 = \frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2$$

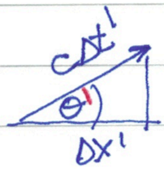
$$\frac{V_B}{c} = \sqrt{\frac{8}{9} + \frac{1}{9}\left(\frac{V_A}{c}\right)^2} < 1$$

$$= 0.95$$

Haces de luz







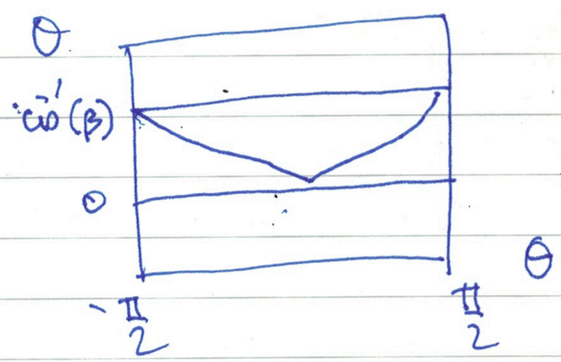
$$\cos\theta' = \frac{\Delta x'}{c\Delta t'}$$

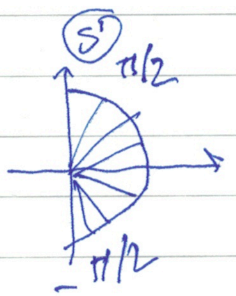
$$\cos\theta = \frac{\Delta x}{c\Delta t}$$

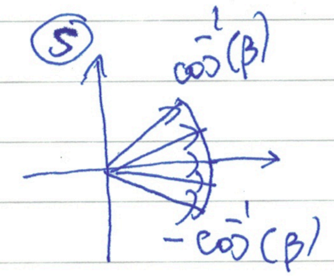
$$\cos\theta = \frac{\Delta x}{c\Delta t} = \frac{\cancel{\gamma}(\Delta x' + v\Delta t')}{c\cancel{\gamma}(\Delta t' + \frac{v}{c^2}\Delta x')} = \frac{\left(\frac{\Delta x'}{\Delta t'}\right) + v}{c\left(1 + \frac{v}{c^2}\frac{\Delta x'}{\Delta t'}\right)}$$

$$= \frac{\frac{\Delta x'}{c\Delta t'} + \frac{v}{c}}{1 + \frac{v}{c}\frac{\Delta x'}{c\Delta t'}} = \frac{\cos\theta' + v/c}{1 + \left(\frac{v}{c}\right)\cos\theta'}$$

$$\boxed{\cos\theta = \frac{\cos\theta' + \beta}{1 + \beta\cos\theta'}}$$



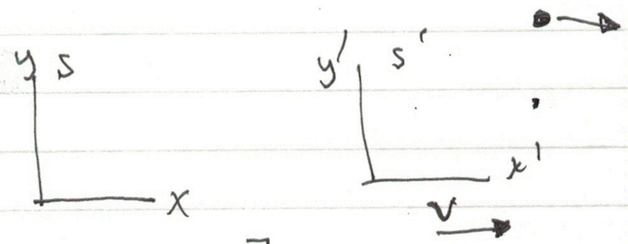




Movimientos Acelerados

$$u_x = \frac{u_x' + v}{1 + \frac{v u_x'}{c^2}} \quad ; \quad u_y = \frac{u_y'/\gamma}{1 + \frac{v u_x'}{c^2}}$$

$$t = \gamma\left(t' + \frac{v x'}{c^2}\right)$$



$$du_x = \frac{du_x'}{1 + \frac{v u_x'}{c^2}} - \left[\frac{(u_x' + v)}{\left(1 + \frac{v u_x'}{c^2}\right)^2} \cdot \frac{v}{c^2} du_x'\right]$$

$$= \frac{1 + \cancel{\frac{v u_x'}{c^2}} - \cancel{u_x' \frac{v}{c^2}} - \left(\frac{v}{c}\right)^2}{\left(1 + \frac{v u_x'}{c^2}\right)^2} du_x' = \frac{\left[1 - \left(\frac{v}{c}\right)^2\right] du_x'}{\left(1 + \frac{u_x' v}{c^2}\right)^2}$$

también $dt = \gamma\left(dt' + \frac{v}{c^2} dx'\right) = \gamma\, dt'\left(1 + \frac{v u_x'}{c^2}\right)$

$$\Rightarrow a_x = \frac{du_x}{dt} = \frac{(du_x'/dt')}{\gamma^3 \left(1 + \frac{v u_x'}{c^2}\right)^3}$$

$$\therefore \boxed{a_x = \frac{a_x'}{\gamma^3 \left[1 + \frac{v u_x'}{c^2}\right]^3}}$$

similarmente [Ejercicio]

$$a_y = \frac{a_y'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^2} - \frac{(v u_y'/c^2)\, a_x'}{\gamma^2 \left(1 + \frac{v u_x'}{c^2}\right)^3}.$$

Ejemplo cinemática relativista

$a_{x'} = g$

$u_{x'} = 0$

$$a_x = \frac{a_{x'}}{\gamma^3\left(1+\frac{v u_{x'}}{c^2}\right)^3} = \frac{g}{\gamma^3(u_x)}$$

$$\frac{du_x}{dt} = g\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}$$

$$\underbrace{\int_0^{u_x} \frac{du_x}{\left(1-\left(\frac{u_x}{c}\right)^2\right)^{3/2}}} = \int_0^t g\,dt$$

$$\frac{u_x}{\sqrt{1-\left(\frac{u_x}{c}\right)^2}} = gt$$

$$\Rightarrow \frac{u^2}{1-\frac{u^2}{c^2}} = (gt)^2 \Rightarrow u^2 = (gt)^2 - (gt)^2\frac{u^2}{c^2}$$

$$u^2\left(1+\frac{(gt)^2}{c^2}\right) = (gt)^2$$

$$u^2 = \frac{(gt)^2}{1+\left(\frac{gt}{c}\right)^2} \Rightarrow \boxed{u = \frac{gt}{\sqrt{1+(gt/c)^2}}} \quad (*)$$

si $u = c/2$

$$\Rightarrow \left(\frac{c}{2}\right)^2\left(1+\left(\frac{gt}{c}\right)^2\right) = (gt)^2 \Rightarrow (c/2)^2 = (gt)^2 - \left(\frac{gt}{2}\right)^2 = \frac{3}{4}(gt)^2$$

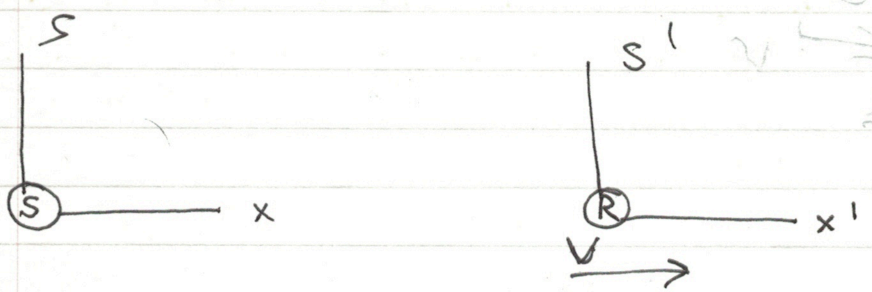
$$c^2 = 3(gt)^2 \Rightarrow c = \sqrt{3}\,gt$$

$$\Rightarrow \boxed{t = \frac{c}{\sqrt{3}g}} = 6.8 \text{ months}$$

$$(*) \Rightarrow x(t) = \frac{c^2}{g}\left[-1+\sqrt{1+(gt/c)^2}\right]$$

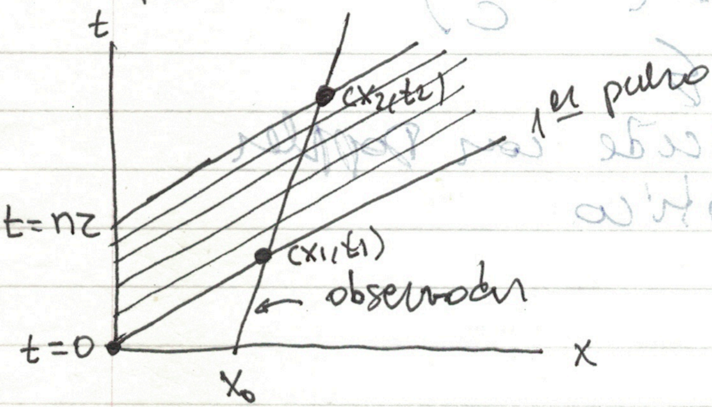
si $c \gg 1$, $x \to \frac{1}{2}gt^2$

Efecto Doppler relativista



Fuente (S) emite pulsos de luz en $t = n\tau$ $\Rightarrow$ en S, la frec. medida es $\nu = 1/\tau$

El 1er pulso es emitido en $t=0$, cuando el receptor (S') está en $x = x_0$



$$x_1 = ct_1 = x_0 + v t_1$$
$$x_2 = c(t_2 - n\tau) = x_0 + v t_2$$

$$\Rightarrow t_2 - t_1 = \frac{cn\tau}{c-v}$$

$$x_2 - x_1 = \frac{vcn\tau}{c-v}$$

<u>En S'</u>: $t_2' - t_1' = \gamma\left[(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)\right]$

$$= \gamma\left(\frac{cn\tau}{c-v} - \frac{v}{c^2}\cdot\frac{vcn\tau}{c-v}\right) = \frac{\gamma c\tau n}{c-v}\left(1 - \frac{v^2}{c^2}\right)$$

$$\Rightarrow \tau' = \frac{\gamma c\tau}{c-v}\left(1 - \frac{v^2}{c^2}\right) = \frac{\gamma\tau}{1-\left(\frac{v}{c}\right)}\left(1-\frac{v^2}{c^2}\right) = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)\frac{1}{1-\frac{v}{c}}$$

$$= \frac{\tau\sqrt{1-\frac{v^2}{c^2}}}{1-\frac{v}{c}} = \tau\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \Rightarrow \boxed{\nu' = \left(\frac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{1/2}\nu}$$

Doppler longitudinal $\longrightarrow$

$$\nu' = \nu \left(\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} \right)^{1/2}$$

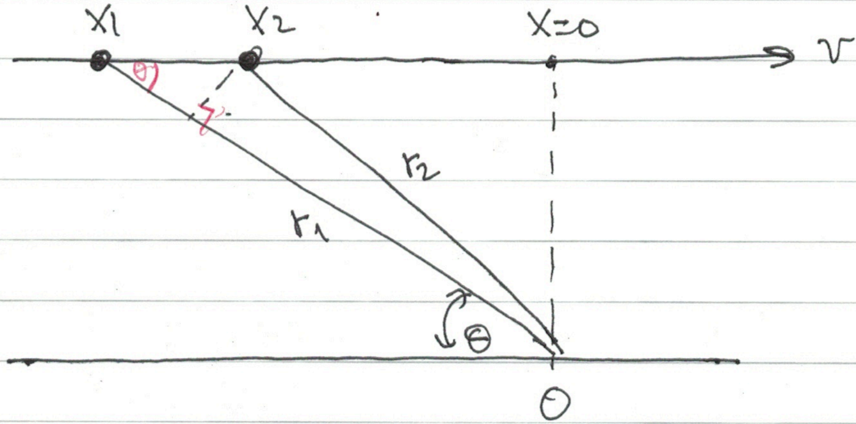
caso $v/c \ll 1$

$$\nu' \approx \nu \left[\left(1 - \frac{v}{c}\right) \left(1 - \frac{v}{c} + \left(\frac{v}{c}\right)^2 - \dots \right) \right]^{1/2}$$

$$\simeq \nu \left[\left(1 - \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) \right]^{1/2} = \nu \left(1 - \frac{v}{c}\right)$$

· coincide con Doppler acústico

Doppler Transversal



2 pulsos sucesivos son emitidos en $x = x_1$ y $x = x_2$ en los instantes $t = t_1$ y $t = t_2$.

En el sist. en reposo c/r al satélite, el intervalo entre pulsos es τ. $\Rightarrow t_2 - t_1 = \gamma \tau$ (por dilatación temporal)

El pulso #1 demora r_1/c en llegar a O
" " #2 " r_2/c " " " O

$\Rightarrow$ Intervalo entre pulsos: $\tau' = t_2 + \frac{r_2}{c} - (t_1 + r_1/c)$

si $|x_2 - x_1| \ll r_1 \Rightarrow r_1 - r_2 \simeq (x_2 - x_1)\cos\theta$

$= (v t_2 - v t_1)\cos\theta = v(t_2 - t_1)\cos\theta = v\gamma\tau\cos\theta$

$\therefore \tau' = (t_2 - t_1) + \frac{1}{c}(r_2 - r_1) = \gamma\tau - \gamma\frac{v}{c}\tau\cos\theta = \gamma\tau\left(1 - \frac{v}{c}\cos\theta\right)$

$$\Rightarrow \nu' = \frac{\nu}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)} = \boxed{\frac{\nu\left(1 - (v/c)^2\right)^{1/2}}{\left(1 - (v/c)\cos\theta\right)}}$$