Quantum system
with finite potential

→ bound states

→ continuum modes with
positive energies if $\lim_{r\to\infty} V(r) \equiv 0$

Bound states: localized in space and square-integrable

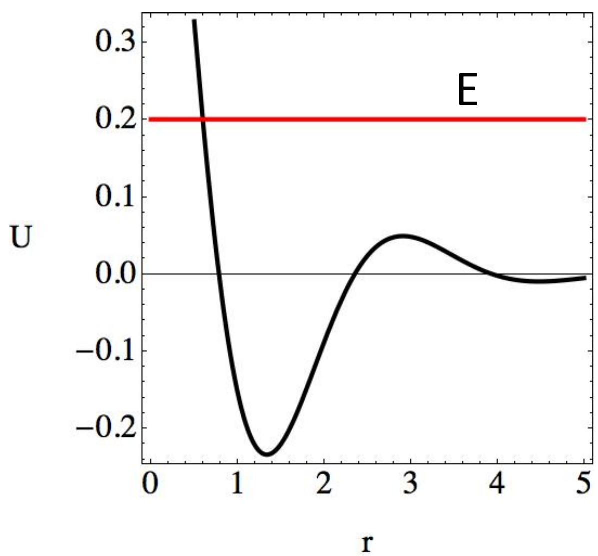
Continuum modes: extended in space and non-normalizable

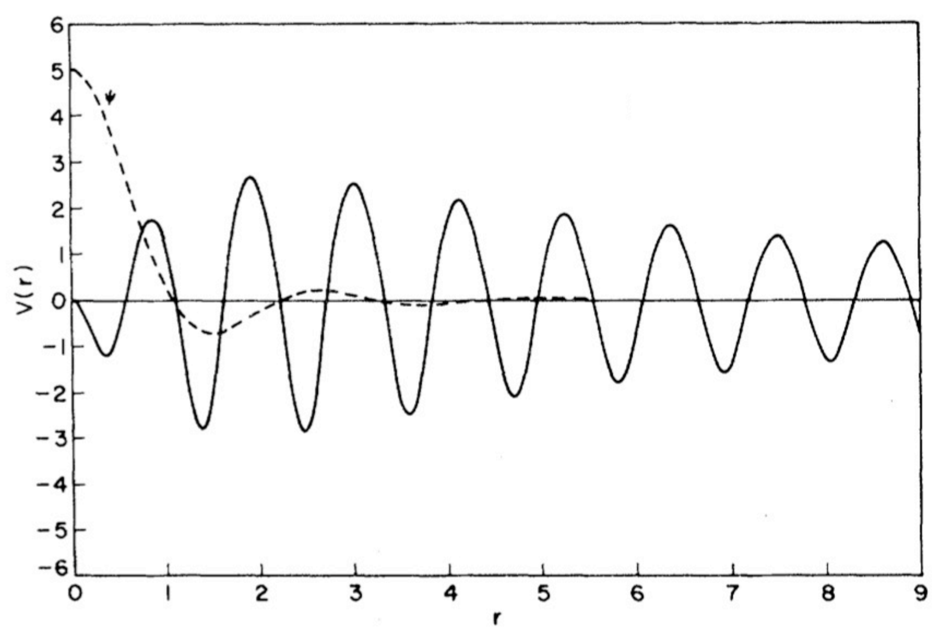
Bulk and surface bound states in the continuum

1929: Wigner and von Neumann:
Eq. Schrodinger with localized radial eigenstates embedded in continuous band of extended states (J. von Neumann and E. Wigner, Phys. Z. 30, 465 (1929))

Built potential supporting a bound state in the continuum with energy above the potential barrier

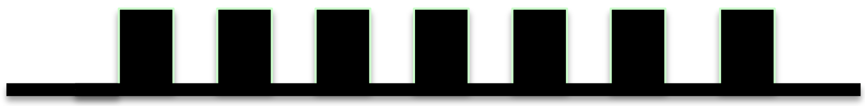
Stillinger and Herrick, PRA 11, 446 (1975)





Bulk and surface bound states in the continuum

Idea: To extend the concept to periodic optical waveguide arrays



Index of refraction

$$E(x,z) = \sum_n C_n(z)\psi(x - na)$$

$$C_n(z) = C_n \exp(i\lambda z)$$

$$(-\lambda + \epsilon_n)C_n + \sum_{m \neq n} V_{nm} C_m = 0,$$

$$\epsilon_n = \lambda - \sum_{m \neq n} V_{nm}(C_m / C_n).$$

Index of refraction

The procedure

$\epsilon_n = 0$ solve $\lambda C_n = \sum_{m \neq n} V_{nm} C_m$

select λ^o and $\{C_n\} \equiv \{\phi_n\}$

modulate: $C_n = \phi_n f_n$ with eigenvalue λ^o

need $\epsilon_n = \lambda^o - \sum_{m \neq n} V_{nm} \left(\frac{f_m}{f_n}\right)\left(\frac{\phi_m}{\phi_n}\right)$.

$$\lim_{n\to\infty} f_n = 0 \text{ and } \sum_n |f_n|^2 < \infty$$

$$\left(\frac{f_{n+1}}{f_n}\right) = 1 - \delta_n \implies f_n = \prod_m^{|n|-1} (1 - \delta_m)$$

choose $$\delta_n = \frac{a}{1 + |n|^b} N^2 \phi_n^2 \phi_{n+1}^2$$

$$\text{when } \phi_n \to 0 \implies \delta_n \to 0 \implies \epsilon_n \to 0$$

Asymptotics: $$f_n \sim \exp(-\alpha_n n^{1-b}), \qquad n \to \infty$$

$$\epsilon_n \sim \frac{A_n V a}{n^b}, \qquad n \to \infty$$

once in possesion of $\{\epsilon_n\}$ solve

$$(-\lambda + \epsilon_n)C_n + \sum_{m \neq n} V_{nm} C_m = 0,$$

examine structure of all $\{\lambda, \{C_n\}\}$

participation ratio $R = \dfrac{(\sum_n |C_n|^2)^2}{\sum_n |C_n|^4}$

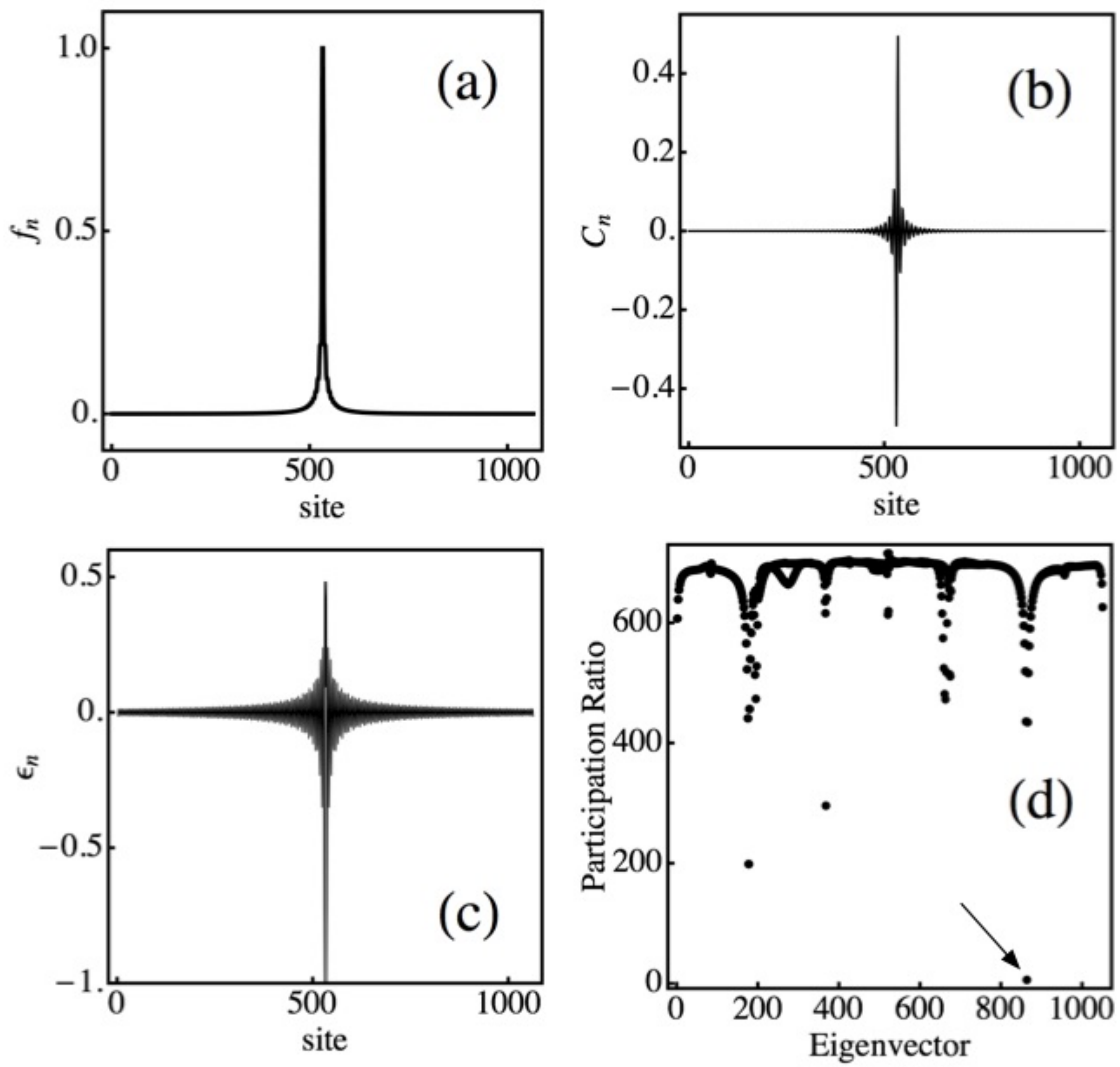
localized $R = O(1)$ extended $R = O(N)$

Results

Bulk mode; nearest-neighbor interactions

N=1065
λ=1.6946
a=1/2
b=3/4

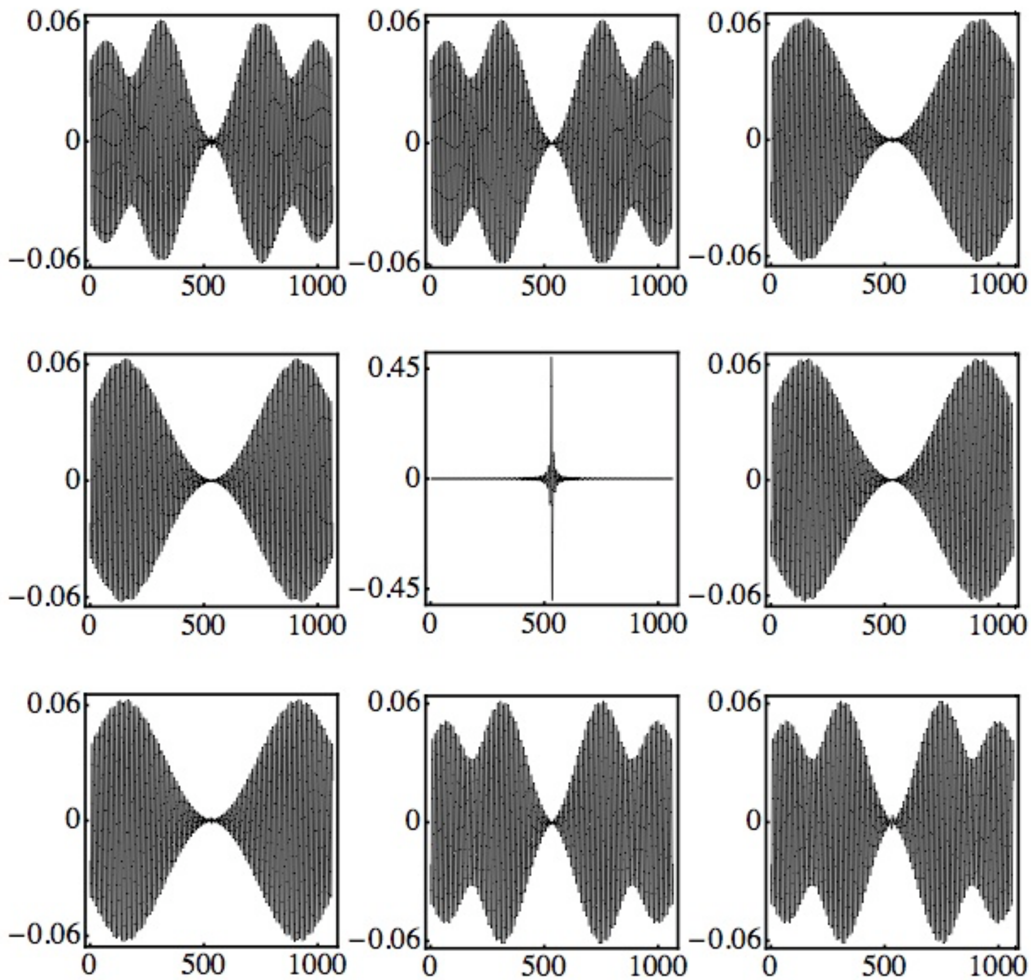
Some states pushed outside the band



Results

Bulk mode; nearest-neighbor interactions

N=1065
λ=1.6946
a=1/2
b=3/4

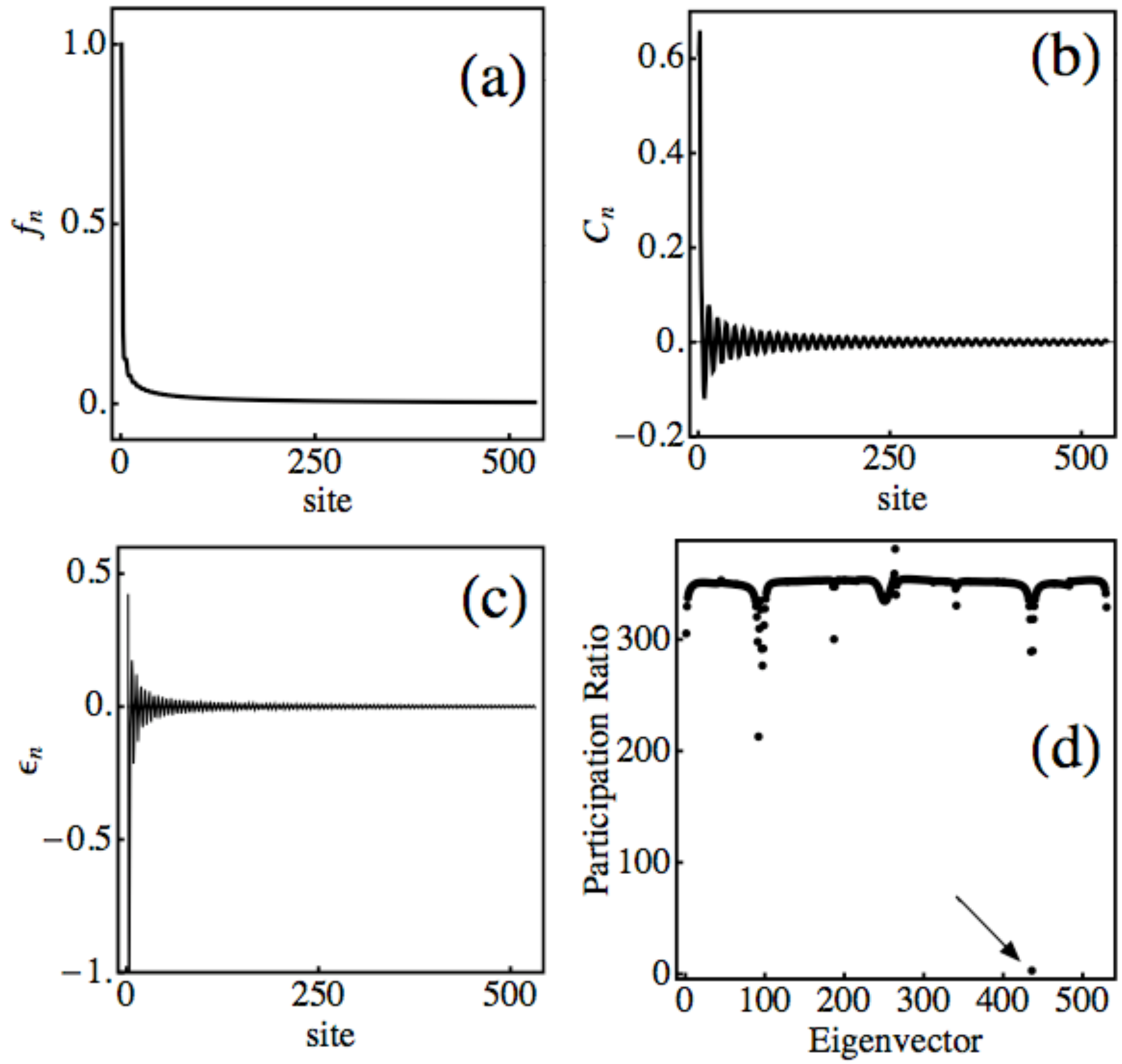


Results

Surface mode; nearest-neighbor interactions

N=533
λ=1.69568
A=2/5
B=9/10

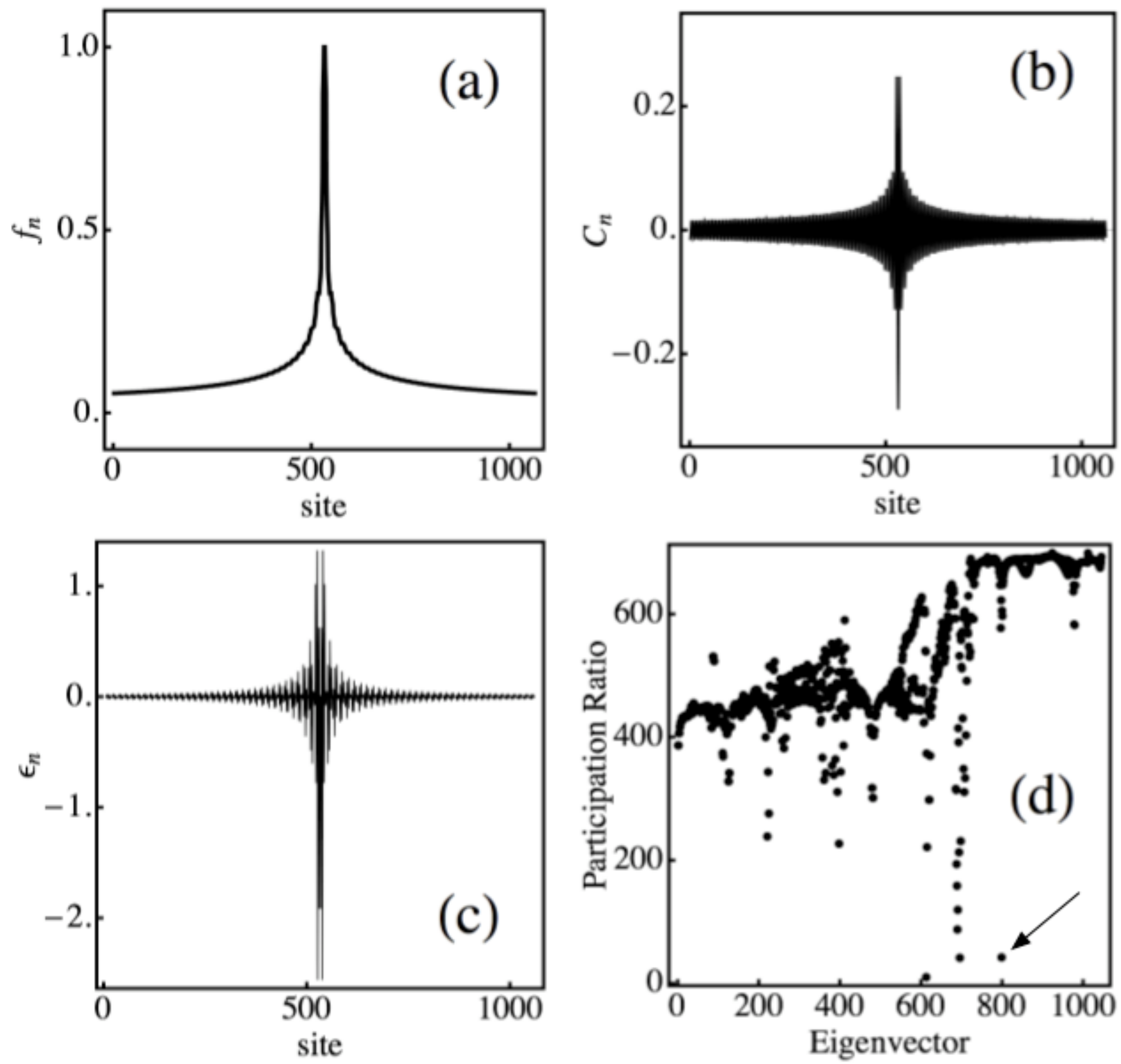
Some states pushed outside the band



Results

Bulk mode; first-and second nearest-neighbor interactions

N=1065
λ=1.69357
V1=1
V2=1
A=3.5
B=0.99



Some states pushed outside the band

Onset of resonance-like states

Results

Surface mode; first-and second nearest-neighbor interactions

N=533
λ=1.5655
A=2.5
B=0.75

Some states pushed outside the band

Onset of resonance-like states